Comparing Fractions

Junior / Intermediate Student Book
Module 2
Comparing Fractions

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1. Circle the greater shaded amount. If the fractions are equal, circle both.

   a) ![Fraction A](image1)

   b) ![Fraction B](image2)

   c) ![Fraction C](image3)

   d) ![Fraction D](image4)

2. Circle the greater fraction. Tell why it is greater.

   a) \(\frac{7}{12}\)  \(\frac{5}{12}\) because

   b) \(\frac{3}{4}\)  \(\frac{3}{2}\) because

   c) \(\frac{5}{6}\)  \(\frac{5}{8}\) because

   d) \(\frac{4}{6}\)  \(\frac{1}{3}\) because

3. Circle all of the fractions below that are greater than \(\frac{2}{5}\).

   a) \(\frac{4}{5}\)  b) \(\frac{1}{5}\)  c) \(\frac{5}{5}\)  d) \(\frac{2}{3}\)  e) \(\frac{2}{7}\)  f) \(\frac{8}{3}\)  g) \(\frac{20}{6}\)  h) \(\frac{7}{8}\)
4. Circle the fractions below that are equivalent (equal) to \( \frac{4}{6} \).
   
   a) \( \frac{2}{3} \)  
   b) \( \frac{3}{5} \)  
   c) \( \frac{5}{7} \)  
   d) \( \frac{20}{30} \)  
   e) \( \frac{32}{48} \)  

5. Complete the statement: The best way to create an equivalent fraction to \( \frac{3}{5} \) is to

6. Circle all of the fractions below that are between 0 and \( \frac{1}{2} \).
   
   a) \( \frac{2}{5} \)  
   b) \( \frac{5}{6} \)  
   c) \( \frac{4}{9} \)  
   d) \( \frac{2}{3} \)  
   e) \( \frac{1}{10} \)  
   f) \( \frac{4}{7} \)
Comparing Fractions Using Pictures

Learning Goal

• reasoning about how two fractions shown pictorially can be compared.

Open Question

1. Choose 4 of the 6 fraction circles to put in order from least to greatest depending on how much is shaded. Describe your strategies.

   A
   B
   C
   D
   E
   F

2. Choose 4 of the 6 fraction rectangles to put in order from least to greatest depending on how much is shaded. Describe your strategies.

   A
   B
   C
   D
   E
   F
Comparing Fractions Using Pictures (Continued)

Think Sheet

Think of comparing two different fractions.

Each is a part of the same whole.

\[
\frac{2}{3} \quad \frac{1}{4}
\]

Sometimes, it’s just easy to see which is greater.

\[\frac{2}{3} > \frac{1}{4}\] since \(\frac{2}{3}\) is more than half the circle and \(\frac{1}{4}\) is less and it’s easy to see.

Sometimes we have to work a little harder. One fraction is greater than another if its picture has “extra” when we overlap them as much as possible.

\[
\frac{3}{5} \quad \frac{2}{3}
\]

For example, \(\frac{3}{5} > \frac{2}{3}\) since we can move the shaded area from one fraction \(\left(\frac{3}{5}\right)\) into the unshaded area of the other fraction \(\left(\frac{2}{3}\right)\) and see that \(\frac{3}{5}\) takes up less area than \(\frac{2}{3}\).

We can only use pictures to compare fractions if the fractions are parts of the same whole.
Comparing Fractions Using Pictures (Continued)

Circle the fraction that is greater.

1. \(\frac{4}{5}\)  \(\frac{1}{3}\)

2. \(\frac{4}{5}\)  \(\frac{3}{4}\)

3. \(\frac{5}{6}\)  \(\frac{3}{5}\)

4. \(\frac{3}{8}\)  \(\frac{1}{3}\)

5. \(\frac{2}{5}\)  \(\frac{2}{8}\)

6. \(\frac{1}{5}\)  \(\frac{1}{3}\)

7. Shade in \(\frac{5}{8}\) of a rectangle. Use these pictures to show at least six other fractions greater than \(\frac{5}{8}\).
Comparing Fractions with the Same Denominator

Learning Goal

• reasoning about how two fractions with the same denominator can be compared.

Open Question

\[
\frac{2}{3} \quad \frac{1}{3} \quad \frac{3}{8} \quad \frac{5}{8} \quad \frac{9}{8} \quad \frac{4}{10} \quad \frac{11}{10} \quad \frac{9}{10}
\]

Choose two fractions with the same denominator.
How do you know that the two fractions are not the same size?
How could you decide which is greater without using a picture?
Show your work in the box.

Repeat with two other pairs of fractions.
Comparing Fractions with the Same Denominator (Continued)

Think Sheet

If two fractions have the same denominator, they are easy to compare.

For example, $\frac{3}{8} > \frac{2}{8}$

$\frac{3}{8}$ is 3 copies of $\frac{1}{8}$

$\frac{2}{8}$ is 2 copies of $\frac{1}{8}$

3 copies > 2 copies

$\frac{3}{8} > \frac{2}{8}$

The same is true even if the fractions are greater than 1.

$\frac{12}{5} > \frac{9}{5}$

$\frac{12}{5}$ is 12 copies of $\frac{1}{5}$

$\frac{9}{5}$ is 9 copies of $\frac{1}{5}$

12 copies > 9 copies

$\frac{12}{5} > \frac{9}{5}$

1. Circle the greater fraction in each pair.

a) $\frac{3}{5}$ $\frac{1}{5}$

b) $\frac{3}{5}$ $\frac{8}{5}$

c) $\frac{2}{9}$ $\frac{9}{9}$

d) $\frac{4}{3}$ $\frac{2}{3}$
2. How can you explain to your friend why $\frac{5}{12} < \frac{7}{12}$?

3. Choose different values for the two boxes to make each statement true.
   
   a) $\frac{2}{3} < \frac{\square}{3}$
   
   b) $\frac{5}{8} > \frac{\square}{8}$
   
   c) $\frac{\square}{12} < \frac{7}{12}$

4. Fill in the boxes. Use each of the numbers: 0, 1, 2, 3, 4, and 5.
   
   a) $\frac{\square}{5} < \frac{\square}{5}$
   
   b) $\frac{\square}{8} > \frac{2}{8}$
   
   c) $\frac{\square}{5} < \frac{\square}{\square}$
Comparing Fractions with the Same Numerator

Learning Goal
• reasoning about how two fractions with the same numerator can be compared.

Open Question

\[
\frac{4}{8} \quad \frac{4}{10} \quad \frac{4}{3} \quad \frac{5}{8} \quad \frac{5}{2} \quad \frac{5}{10} \quad \frac{6}{9} \quad \frac{6}{8} \quad \frac{6}{4}
\]

Choose two fractions from the list with the same numerator.

How do you know that the two fractions are not the same size?
How could you decide which is greater without using a picture?
Show your work in the boxes.

Repeat with two other pairs of fractions.
If two fractions have the same numerator, they are easy to compare.

For example, $\frac{3}{8} < \frac{3}{5}$

$\frac{3}{8}$ is 3 copies of $\frac{1}{8}$

$\frac{3}{5}$ is 3 copies of $\frac{1}{5}$

$\frac{1}{8} < \frac{1}{5}$ since the whole is shared into more pieces, so the pieces are smaller.

$\frac{3}{8} < \frac{3}{5}$

The same is true if the fractions are greater than 1.

$\frac{8}{5} < \frac{8}{3}$

$\frac{8}{3}$ is 8 copies of $\frac{1}{3}$

$\frac{8}{5}$ is 8 copies of $\frac{1}{5}$

$\frac{1}{5} < \frac{1}{3}$

$\frac{8}{5} < \frac{8}{3}$

1. Circle the greater fraction in each pair.

a) $\frac{3}{8}$ $\frac{3}{10}$

b) $\frac{4}{12}$ $\frac{4}{5}$

c) $\frac{2}{9}$ $\frac{2}{3}$

d) $\frac{5}{3}$ $\frac{5}{2}$
Comparing Fractions with the Same Numerator (Continued)

2. How can you explain to your friend why \( \frac{5}{12} < \frac{5}{8} \)? It doesn’t make sense to her since \( 12 > 8 \).

3. Choose different values for the two boxes to make each statement true.
   a) \( \frac{2}{3} > \frac{2}{\phantom{3}} \)
   b) \( \frac{5}{8} > \frac{5}{\phantom{8}} \)
   c) \( \frac{3}{\phantom{5}} > \frac{3}{5} \)

4. Fill in the boxes. Use each of the numbers 1, 2, 3, 4, and 5.
   \( \frac{4}{10} > \frac{\phantom{4}}{12} \)
   \( \frac{3}{5} > \frac{\phantom{3}}{8} \)
   \( \frac{1}{2} > \frac{1}{\phantom{2}} \)
   \( \frac{\phantom{3}}{\phantom{3}} > \frac{3}{\phantom{3}} \)
Equivalent Fractions

Learning Goal
• representing the same fraction in different ways.

Open Question

Look at the Fraction Tower.

Two fractions are equivalent, or equal, if they take up the same amount of area. For example, use a ruler at the end of the \( \frac{1}{3} \) and \( \frac{2}{6} \) sections to see why \( \frac{1}{3} = \frac{2}{6} \).

Find as many sets of equivalent fractions on the Fraction Tower as you can.

What do you notice about the numerators and denominators of equivalent fractions?
Two fractions are equivalent if they describe exactly the same amount.

For example, \( \frac{2}{3} = \frac{4}{6} \)  The fractions are equivalent since:

- We can split the sections of \( \frac{2}{3} \) equally and it is the same part of the whole.

\[
\begin{array}{ccc}
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& & \\
\hline
& & \\
\hline
\end{array}
\]

2 out of every 3 is the same as 4 out of every 6.

- Notice that if we multiply both the numerator and denominator by the same amount (but not 0), we get an equivalent fraction. If we have 2 times or 3 times or 4 times as many pieces, we will have 2 times or 3 times or 4 times as many pieces of the type we want.

For example, \( \frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15} \)

We can either split all the fifths into 3 equal pieces or we can make 3 rows of 3 out of 5.

\[
\begin{array}{ccc}
\hline
& & \\
\hline
& & \\
\hline
\end{array}
\]

1. Which of these pairs of fractions are equivalent?

a) \( \frac{2}{3} \) \( \frac{12}{18} \)  

b) \( \frac{3}{5} \) \( \frac{8}{10} \)  

c) \( \frac{5}{8} \) \( \frac{10}{16} \)  

d) \( \frac{3}{10} \) \( \frac{12}{40} \)
Equivalent Fractions

2. a) Name three fractions equivalent to $\frac{5}{12}$.

b) Choose one of your fractions from part a). How would you convince someone who wasn’t sure why it is equivalent to $\frac{5}{12}$?

3. Jane listed equivalent fractions for $\frac{2}{3}$. She noticed that the denominators were always at least 3 apart. Do you agree? Explain.

4. How do you know that you cannot add 2 to the numerator and denominator of $\frac{3}{4}$ and end up with an equivalent fraction?

5. $\frac{6}{\square} = \frac{\triangle}{3}$ What can the values of $\square$ and $\triangle$ be?

$\square =$

$\triangle =$
Comparing Fractions to $\frac{1}{2}$ and to 1

Learning Goal

• reasoning about how two fractions can be compared using benchmarks of $\frac{1}{2}$ and 1.

Open Question

Choose values for the missing parts of the fractions so that they are less than $\frac{1}{2}$. Do not use any number twice. Tell how you chose the values.

$$\frac{\square}{8} \quad \frac{\square}{5} \quad \frac{\square}{10} \quad \frac{3}{\square} \quad \frac{5}{\square} \quad \frac{8}{\square}$$

Now choose values so that the fractions are greater than 1. Tell how you chose the values.

$$\frac{\square}{8} \quad \frac{\square}{5} \quad \frac{\square}{10} \quad \frac{3}{\square} \quad \frac{5}{\square} \quad \frac{8}{\square}$$
Comparing Fractions to $\frac{1}{2}$ and to 1 (Continued)

Think Sheet

Comparing to $\frac{1}{2}$

A fraction equals $\frac{1}{2}$ if the denominator is 2 times the numerator.

For example, $\frac{3}{6} = \frac{1}{2}$ since $6 = 2 \times 3$.

A fraction is more than $\frac{1}{2}$ if the numerator is more than half the denominator.

For example, $\frac{5}{6} > \frac{1}{2}$ since half of 6 is 3. So $\frac{3}{6} = \frac{1}{2}$ and $\frac{5}{6} > \frac{3}{6}$.

A fraction is less than $\frac{1}{2}$ if the numerator is less than half of the denominator.

$\frac{4}{12} < \frac{1}{2}$ since half of 12 is 6. So $\frac{1}{2} = \frac{6}{12}$ and $\frac{4}{12} < \frac{6}{12}$.

Comparing to 1

A fraction equals 1 if the numerator and denominator are equal.

For example, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$.

A fraction is more than 1 if the numerator is greater than the denominator.

For example, $\frac{6}{5} > 1$ since $1 = \frac{5}{5}$ and $\frac{6}{5} > \frac{5}{5}$.

A fraction is less than 1 if the numerator is less than the denominator.

For example, $\frac{4}{5} < 1$ since it is only 4 out of 5 parts, not the whole $\frac{5}{5}$.

1. Circle the fractions that are less than $\frac{1}{2}$.

   $\frac{3}{8}$, $\frac{4}{10}$, $\frac{2}{5}$, $\frac{2}{9}$, $\frac{2}{3}$

2. Circle the fractions that are between $\frac{1}{2}$ and 1.

   $\frac{7}{8}$, $\frac{6}{8}$, $\frac{2}{10}$, $\frac{3}{5}$, $\frac{5}{3}$
3. What possible values could the numerator have if \( \frac{\square}{9} < \frac{1}{2} \)? Why would the numerator have just these values?

4. Replace the missing values with counting numbers (1, 2, 3,…).
   Why are there more solutions to \( \frac{\square}{6} > 1 \) than to \( \frac{\square}{6} < 1 \)?

5. Fill in the boxes with 2, 4, 6, 8, 10. Use each number once to make these statements true.
   \[
   \frac{1}{2} < \frac{\square}{4} \quad \frac{8}{\square} > \frac{1}{\square} \quad \frac{\square}{2} > 1 \quad \frac{6}{\square} > \frac{1}{2}
   \]
Fraction Circles and Rectangles (1)
Fraction Circles and Rectangles (2)

1.

2.
Pairs of Fractions

1.

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\begin{array}{c}
\frac{1}{5} & \frac{1}{5} \\
\frac{1}{5} & \frac{1}{5} \\
\frac{1}{5} & \frac{1}{5}
\end{array}
\]

\[
\begin{array}{c}
\frac{1}{3} & \frac{1}{3}
\end{array}
\]

2.

\[
\begin{array}{c}
\frac{1}{5} & \frac{1}{5} \\
\frac{1}{5} & \frac{1}{5} \\
\frac{1}{5} & \frac{1}{5}
\end{array}
\]

\[
\begin{array}{c}
\frac{1}{4} & \frac{1}{4}
\end{array}
\]

3.

\[
\begin{array}{c}
\frac{1}{6} & \frac{1}{6} \\
\frac{1}{6} & \frac{1}{6} \\
\frac{1}{6} & \frac{1}{6}
\end{array}
\]

\[
\begin{array}{c}
\frac{1}{5} & \frac{1}{5} \\
\frac{1}{5} & \frac{1}{5}
\end{array}
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4.

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\begin{array}{cccc}
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8}
\end{array}
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\begin{array}{ccc}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
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5.

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\begin{array}{c}
\frac{1}{5} \\
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6.

\[
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7.

\[
\begin{array}{cccc}
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
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Comparing Fractions
### Fraction Tower (1)

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