Exploring Volume – Surface Area Relationships

**Math Learning Goals**
- Review volume and surface area concepts then apply their understanding to investigating relationships between volume and surface area of rectangular prisms.
- Investigate the concept of optimization of measurements using the mathematical processes representing and connecting.

**Materials**
- connecting cubes
- chart paper/markers

**75 min**

### Whole Group → Student Investigation/Discussion
Pre-assemble three rectangular prisms containing 64 connecting cubes (one should be a large $4 \times 4 \times 4$ cube). Provide groups with connecting cubes (set of at least 64 per group). As students make descriptive statements about the prisms, record their comments on the board.

**Curriculum Expectations/Observation/Mental Note:** Identify who is struggling to describe the surface area and volume of the prisms, and be prepared to discuss strategies for finding volume and area (e.g., count the faces/cubes).

Ask probing and converging questions to build on students’ statements so students recognize that all of the prisms have equal volume, but different surface areas.

### Whole Group → Problem Set Up
Define optimization in ‘student-friendly’ language (e.g., Sometimes it is useful to know when we get the smallest or largest value of a measurement, when all other factors are kept the same.).

Pose a ‘big idea’ question: When does the surface area of a rectangular prism reach its smallest value? Is it always the same shape?

### Individual → Personal Solving Time
Pose the problem: Is it always true that for a rectangular prism with a fixed volume that the cube shape will have the smallest surface area? Experiment with different fixed volumes to prove your argument. As students solve the problem independently, they list strategies and/or representations they use.

Differentiate responses based on readiness: Visit students as they solve the problem to check for confusion and to reinforce good attempts to list strategies for solving. Coach struggling students with possible strategies and representations.

### Pairs → Poster Preparation for Congress
Students work with a partner to solve the problem, answer the questions, and provide proof of their argument. They create a poster of their findings and prepare to share with the class.

During this time, identify two or three solutions or posters that will be used to begin the congress.

### Whole Group → Student Presentations
Invite pairs to present their work to the class. The following important concepts need to be developed, discussed, and consolidated by the group:
- The cube is the rectangular prism that has the smallest surface area for a given volume. This is true no matter how many cubes are used.
- Charts or tables can be used to show the relationship between the volume and surface area.

Lead a discussion about optimization and its applications once a relationship is established.

### Home Activity or Further Classroom Consolidation
Despite the knowledge that the cube always results in the smallest surface area when the volume remains constant, many packages are not in the shape of a cube. Why is that? What other considerations need to be taken in deciding the best shape to use?