Grade 9 Applied
# Unit 1: Measurement: 2-D and 3-D

## Grade 9 Applied

### Lesson Outline

#### BIG PICTURE:

English language learners will:
- create their personal word study notebook;
- begin to work productively in flexible student groupings.

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson Title</th>
<th>Language Goals</th>
<th>Expectations</th>
</tr>
</thead>
</table>
| 1   | Fill It Up   | • Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)  
• Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO)  
• Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)  
• Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO) | MG2.04, NA2.01, NA2.03, NA2.04, NA2.08 | CGE 5a |
| 2   | A Sweet Problem | • Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO)  
• Write in a variety of forms with teacher guidance. (ESLBO)  
• Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)  
• Write in a variety of forms. (ESLBO) | MG2.04, MG2.05, NA2.01, NA2.02, NA2.03, NA2.04, NA2.08 | CGE 3c |
| 3   | Diagnostic to Activate Prior Knowledge of Composite Figures | • Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO)  
• Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO)  
• Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO)  
• Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO) | MG2.03 | CGE 2c, 5a, 5b |

**ESLAO** – Beginning communication in English builds on students’ previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment.

**ESLBO** – English in daily life expands students’ essential English communication skills and introduces the language of classroom studies.
Math Learning Goals
- Develop through investigation the formulas for the volume of a pyramid and cone, based on the volume of the corresponding prism or cylinder of the same radius and height.

Materials
- 3-D relational solids
- filler beads or sand
- baggies
- paper plates
- BLM 1.1.1, 1.1.2

Assessment Opportunities
If a sufficient number of sets is not available, consider sorting as a whole class and a carousel for Action.

Minds On ...

Small Groups → Exploration
Each group works with one set of solids. Orient students to the 3-D relational solids. Allow for exploration time.

Students establish their own criteria and rationale for sorting the various solids into groups.

Whole Class → Sharing
Each group shares its strategy.
Use this opportunity to review the terminology related to the geometric shapes.

Action!

Small Groups → Investigation
The groups compare the volumes of prisms (cylinder) and pyramids (cone) with congruent bases and equal heights (BLM 1.1.1).

Learning Skills (Teamwork)/Observation/Checklist: Observe how students interact as they investigate in their groups.

Consolidate Debrief

Whole Class → Journal: Representing
Give the class the following journal prompt: Using words, pictures, numbers, and symbols, describe the relationships you discovered today.

Pairs → Pair/Share
Students complete BLM 1.1.2, alternating as indicated between writing and coaching.

Differentiated Exploration Reflection

Home Activity or Further Classroom Consolidation
Use the 3-D relational sets and record as many paired relationships as you can.
For example, the small, triangular prism is half the volume of the small, square-based prism.

The relationship between the cone and the hemisphere is examined in Day 2.
Unit 1: Day 1: Fill It Up

Terminology
- volume
- area
- base
- height
- radius
- cone
- cylinder
- pyramid
- prism
- geosolids
- hypothesis

Language Goals
- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Demonstrate adaptations to school norms, key teacher expectations, and classroom routines.

Minds On…

Small Groups → Exploration
English-speaking students help English-language learners get involved in the sorting game by using key words and gestures to model the criteria for sorting the manipulatives.

Whole Class → Sharing
As each group shares their strategy, state the terms clearly as you write them on the board with diagrams. Have English language learners record the terms in their personal word study notebook accompanied by a picture of the geometric figures.

Action!

Small Groups → Investigation
Group English language learners with English-speaking peers so that they can observe the steps involved with investigating the first shape before they repeat the steps to complete the investigation. The English-speaking students talk about what they are doing and learning.

Learning Skills (Collaboration)/Observation Checklist: Observe how students interact as they investigate in their groups.

Consolidate

Whole Class → Journal: Representing
Write the journal prompt on the board connecting verbal, written, and pictorial representations of the same word.

Students put the new terms on the Word Wall.

Pairs → Pair/Share
Partner an English language learner with an English-speaking peer (and optimally, who also speaks their first language) so that they have opportunities to hear and see the terminology during oral and written aspects of the tasks.

Home Activity or Further Classroom Consolidation
English language learners share their work and new learning with parents/guardians in their first language.
Math Learning Goals
- Develop through investigation the formula for volume of a sphere based on the volume of a cylinder/cone.
- Consolidate volumes of prisms, pyramids, cylinders, cones, and spheres.
- Solve problems involving combinations of the figures using metric and imperial measure.

Materials
- 3-D relational solids
- filler beads
- BLM 1.2.1, 1.2.2
- computer/data projector

Assessment Opportunities

Minds On ... Pairs ➔ Think/Pair/Share/Discussion
Individually, students make hypotheses about the relative volumes of pairs of 3-D solids. In pairs, they compare and consolidate their lists.
Ask:
- Are there any other pairs where one is 3 times the other? 4 times? 2 times? [hemisphere \( \times 3 = \) cylinder; small, triangular-based prisms \( \times 4 = \) large, triangular prism; small cylinders \( \times 2 = \) large cylinder;]
- Is there a series of shapes which all compare? [square and rectangular-based prisms (all 4 doubling each time)]
- Are there any shapes with equal volume? [hemisphere = cone]
Demonstrate selected examples, emphasizing the connection between the hemisphere and cone.

Action! Pairs ➔ Investigation
Students complete BLM 1.2.1 using the 3-D relational solids. They read the problem and highlight important information needed to solve the problem.

Reasoning and Proving/Observation/Anecdotal: Listen to pairs discuss to determine their understanding of the relationships being investigated.

Whole Class ➔ Presentation
To develop further understanding of the formula of the volume of a sphere, show the electronic presentation, Developing the Formula for the Volume of a Sphere, which formalizes the relationship between the volume of a sphere and a cone.

Consolidate Debrief Pairs ➔ Practice
Students solve a variety of problems involving the volume of prisms, pyramids, and spheres (BLM 1.2.2).

Home Activity or Further Classroom Consolidation
Find pictures of buildings/structures in your community that are prisms or pyramids. Choose one of the pictures, estimate the dimensions, and present a complete solution for the volume of the structure.

Application
Differentiated Exploration

VolumeSphere.ppt.

Consider using the volume of a sphere as equivalent to two cones until students are ready for the traditional formula.
Unit 1: Day 2: A Sweet Problem

**Terminology**
- sphere
- hemisphere
- metric measure
- imperial measure

**Language Goals**
- Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO)
- Write in a variety of forms with teacher guidance. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Write in a variety of forms. (ESLBO)

**Materials**

**Assessment Opportunities**

**Minds On...**  
**Pairs ➔ Think/Pair/Share/Discussion**
Use contextual language and 3-D solids to assist English language learners to make a hypothesis about 3-D volumes.
Partner English language learners with peers who speak both English and their first language, if possible, or with a group of three so that they can listen to the other two.
English language learners record the new words in their personal word study notebook. Students put the new words on the Word Wall, including pictures and symbols.

**Action!**  
**Pairs ➔ Investigation**
Use the same pairs or groups of three as in Minds On.
Students highlight the important words and information in each problem to focus on essential mathematical terms needed to work through the investigation.
Encourage them to refer to the Word Wall.
Model or have students model the problem, using the 3-D solids.

**Reasoning and Proving/Observation/Anecdotal:** Listen to pairs discuss to determine their understanding of the relationships being investigated.

**Whole Class ➔ Presentation**
During the electronic presentation, point to key words and diagrams as you are speaking.

**Consolidate Debrief**  
**Pairs ➔ Practice**
Students work with first-language peers, if possible, so that they can discuss their understanding of the concepts as they work. The pair could also use the 3-D models and reference the Word Wall.

**Home Activity or Further Classroom Consolidation**
Model the Home Activity with exemplars of previous student work or teacher-prepared examples.

**Make It Language Rich**
Encourage English language learners to access bilingual dictionaries.

**Make Sure They Are Ready**
Observe student interaction and involvement and intervene, as necessary.

**Make It Explicit**
Post exemplars for students to use as models.
Math Learning Goals

- Activate students’ prior knowledge of terminology related to identifying geometry shapes.
- Determine students’ readiness to identify geometric figures in composition, and use appropriate calculations for perimeter and area.

Materials

- BLM 1.3.1, 1.3.2, 1.3.3
- placemat
- overhead projector

Assessment Opportunities

Minds On …

Whole Class ➔ Orientation
Outline the procedure for the day, including the purposes of each component of the lesson [Minds On – activate prior knowledge of shapes, Action – review their measurement skills, and Consolidate – demonstrate these skills in an activity]. Explain that assessment allows you to plan upcoming lessons according to their current levels of understanding and that the results do not influence their grade in the course.

Small Groups ➔ Pass the Paper
Each group lists the geometric figures in the diagram (BLM 1.3.1). Circulate and provide direction and guidance, as necessary.

Students pass one piece of paper around the group, writing a response each time they receive the paper. Responses should include rectangles, squares, triangles, trapezoids, circles (semi-circles) and may include adjectives, as in ‘equilateral triangle.’

Learning Skill (Teamwork)/Observation/Mental Note: Observe students as they work. Encourage effective communication by referring to groupwork skills.

Action!

Pairs ➔ Scale Drawings
Students answer the questions on BLM 1.3.2. Circulate to encourage discussion and to clarify information regarding the diagrams.

Whole Class ➔ Discussion
Using an overhead of BLM 1.3.2, lead a discussion in which students share their answers.

Consolidate Debrief

Individual ➔ Calculate Area
Students draw line segments on the composite figure on BLM 1.3.3. Point out that they must be able to calculate the area of each shape created.

Individual and Groups ➔ Placemat
Describe the procedure for completing a placemat activity. Students individually respond to the question on BLM 1.3.3 for 5 minutes. The members of each group share their procedures, then complete the centre portion of the placemat, providing a model solution to the question. Designate a scribe for each group. Collect the placemats for assessment.

Curriculum Expectation/Placemat/Checkbric: Circulate and observe students as they complete a solution, recording individual strengths and needs.

Whole Class ➔ Discussion
Facilitate group sharing.

Home Activity or Further Classroom Consolidation
Choose another example from daily life of a figure that is represented by more than one geometric shape. Include a sketch and estimate the dimensions for the figure used.
Unit 1: Day 3: Diagnostic to Activate Prior Knowledge of Composite Figures

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<td>• Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)</td>
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<tr>
<td>• Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO)</td>
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<td>• Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)</td>
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<tr>
<th>Assessment Opportunities</th>
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<tr>
<th>Minds On… Whole Class ➔ Orientation</th>
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<tbody>
<tr>
<td>Explain the purpose of diagnostic assessments and assure students that they will not be graded. Encourage English language learners to look up the term in their bilingual dictionaries (first language/English).</td>
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<tr>
<th>Small Groups ➔ Pass the Paper</th>
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<tbody>
<tr>
<td>Model the activity so that the steps are clear to the English language learners.</td>
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</table>

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<tr>
<th>Action! Pairs ➔ Scale Drawings</th>
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<tr>
<td>Students highlight the important words and information to help them focus on what the problem is.</td>
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<tr>
<td>Pair English language learners with a first-language or English-speaking peer so that they will have opportunities to discuss the problems and how to arrive at solutions.</td>
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<th>Whole Class ➔ Discussion</th>
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<td>To provide visual clues for the learners, record the students’ responses on the board as they are discussed, connecting the verbal, written, and pictorial representations.</td>
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<tr>
<th>Consolidate Debrief Individual ➔ Calculate Area</th>
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<tr>
<td>Clearly describe the procedure for Area Challenge (BLM 1.3.3). Circulate, asking questions to help learners sort out what they know and how they can use their knowledge to solve the problem.</td>
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<th>Individual and Groups ➔ Placemat</th>
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<tr>
<td>Model how students are to complete their portion of the placemat. Encourage the use of pictures and words for clarity.</td>
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| Curriculum Expectation/Placemat/Checkbrick: | Circulate and observe students as they complete a solution, recording individual strengths and needs. |

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<th>Application Differentiated Home Activity or Further Classroom Consolidation</th>
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<tr>
<td>Explain the Home Activity, using an example of a composite figure found in the classroom.</td>
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</table>
## BIG PICTURE:

English language learners will:
- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- begin to make short presentations.

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<th>Day</th>
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| 1   | Ratio Carousel     | • Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)  
• Write in a variety of forms, with teacher guidance. (ESLBO)  
• Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)  
• Write in a variety of forms. (ESLBO) | NA1.01, NA1.02, NA1.03, NA1.04, NA1.05  
CGE 5a, 5e |
| 2   | Growing Dilemma    | • Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)  
• Write in a variety of forms, with teacher guidance. (ESLBO)  
• Read texts with familiar content or vocabulary, using a variety of reading strategies. (ESLBO)  
• Write in a variety of forms. (ESLBO) | LR1.03, LR2.02, LR2.03, MG2.02, NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, NA2.02  
CGE 3c, 4b, 5a, 5b |
| 3   | Pondering Proportions | • Find specific information in straight forward reference materials, with teacher guidance. (ESLAO)  
• Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO)  
• Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO)  
• Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO) | NA1.01, NA1.02, NA1.03, NA1.04, NA1.05, MG2.02  
CGE 2c, 2d |
| 4   | I’d Rather Be Scaling | • Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)  
• Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO)  
• Communicate orally, using accepted word order, common tenses, and other features of English grammar with some accuracy and consistency. (ESLBO)  
• Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO) | NA1.01, NA1.02, NA1.03, NA1.04, NA1.05  
CGE 2a, 2c, 5b |

**ESLAO** – Beginning communication in English builds on students’ previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment.

**ESLBO** – English in daily life expands students’ essential English communication skills and introduces the language of classroom studies.
**Math Learning Goals**

- Investigate ratio as a tool for comparing quantities, both qualitative and quantitative.
- Estimate answers and devise and explain informal solutions (e.g., constant of proportionality, unit rate, equivalent ratios) in a variety of contexts (e.g., numerical, geometric, measurement, probability).

**Materials**

- 2 computers with GSP®4
- 60 colour tiles of 2 colours
- BLM 4.1.1, 4.1.2

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**Minds On ...**

**Groups of 3 ➔ Graffiti**

Use heterogeneous groupings. Prepare chart paper for each of the following terms: ratio, rate, unit rate, equivalent ratios. Each group uses a different-coloured marker, cycles through the chart paper stations (2 minutes per chart), and writes characteristics of the term.

**Whole Class ➔ Presentation**

As a class, summarize the important points for each term students need to know for the carousel activity.

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**Action!**

**Groups of 3 ➔ Carousel**

Prepare sufficient sets of each of the three stations. Students use a pencil and calculator and record their findings on BLM 4.1.1.

Using the same groups as in the Minds On section, students rotate through the three stations. Direct the groups to move to the next station after 15 minutes.

**Learning Skills (Teamwork)/Observation/Checklist:** Observe and record students’ collaboration skills.

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**Consolidate Debrief**

**Whole Class ➔ Summarizing**

Lead a class discussion using guiding questions (BLM 4.1.1). Using information from the discussion, define ratio, rate, and unit rate, using examples from the activity.

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**Application Concept Practice**

**Home Activity or Further Classroom Consolidation**

Find examples of ratio, rate, and unit rate in your environment to post on the bulletin board.
### Terminology
- ratio
- rate
- unit rate
- equivalent ratios
- midpoint

### Language Goals
- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Write in a variety of forms, with teacher guidance. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Write in a variety of forms. (ESLBO)

### Language Goals
- Write in a variety of forms. (ESLBO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)

### Assessment Opportunities
- Begin a Word Wall for this unit.
- Additional vocabulary may be necessary, e.g., carousel, graffiti.
- Make Sure They’re Ready: Circulate to make sure that English language learners understand the problem at each station.

### Materials
- Make It Language Rich: Leave the graffiti charts posted as a reference. Provide an opportunity for English language learners to access their bilingual dictionaries.
- Make It Explicit: Ensure that the purpose of each station is clear to the learner.

### Minds On… Groups of 3 ➔ Graffiti
Include English language learners in groups with English-speaking or first-language students so that they will have opportunities to listen to the discussion and write on the chart paper. Review the terms on the charts explicitly so English language learners are comfortable with participating in the activity.

### Whole Class ➔ Presentation
In summarizing, point to and highlight the appropriate graffiti chart and the key ideas listed to focus learners on what is important about each term.

### Action! Groups of 3 ➔ Carousel
Describe how a “carousel” works so that all students are clear on how the class will cycle through the stations.

Adjust the groups, if necessary, so that the English language learners are included and can participate in the carousel. Group members should use the manipulatives and talk as they complete the stations to help their peers understand the concepts.

**Learning Skills (Teamwork)/Observation/Checklist:** When assessing the English language learners’ collaboration skills, look for growth.

### Consolidate Debrief
**Whole Class ➔ Summarizing**
Have English language learners record the terms in their personal word study notebook.

### Home Activity or Further Classroom Consolidation
Share some prepared examples to help English language learners understand their assignment.
**Math Learning Goals**
- Investigate and determine what a ratio is using examples and non-examples of proportional and non-proportional situations (e.g., two ordered quantities that share a multiplicative relationship).
- Determine the characteristics of the graph of a proportional relationship.

**Materials**
- colour tiles (250)
- linking cubes (480)
- BLM 4.2.1, 4.2.2

**Assessment Opportunities**
- Discussion around the length of the diagonal must include mention of the Pythagorean theorem.
- Provide limited resources so that students will infer results for larger models.

**Minds On ... Whole Class Discussion**
Lead a review of basic concepts needed for the investigation.
Students need to be familiar with the concepts of a square, perimeter, area, length of a diagonal, cube, volume, ratio, and lowest terms.

**Pairs Anticipation Guide**
Students, individually, complete the Before column on the anticipation guide (BLM 4.2.1) and discuss their choices with their partner.

**Action! Pairs Investigation**
Distribute 16 colour tiles and 27 linking cubes to each pair.
Students work in pairs on the four investigations (BLM 4.2.2).
Circulate to prompt, clarify, and focus the students on the task.

**Learning Skills (Collaboration)/Observation/Anecdotal:** Observe the students’ contributions to completing the task.

**Consolidate Debrief**
**Pairs Think/Pair/Share/Discussion**
Students complete the After column of the anticipation guide and share their choices with their partner, providing reasons for their choices.
Adjacent pairs of students compare and discuss the results.

**Whole Class Note Making**
Lead a discussion to bring out that the perimeter and diagonal investigation show proportional reasoning and the others do not. Students should be able to explain how first differences, the ratios, and the graphs can all show proportionality.

**Concept Practice Reflection**
**Home Activity or Further Classroom Consolidation**
- In your journal, write a personal example of proportional reasoning.
- Using the scenarios below, check for proportionality and justify your response.
  a) You are paid an hourly wage. If you work 3 times the number of hours, does your pay triple?
  b) Student council raffle tickets cost $0.50/each or 3 for $1. If you buy twice as many tickets, does your cost double?
TIPS for English Language Learners in Mathematics – Grade 9 Applied: Unit 4, 2005

Unit 4: Day 2: Growing Dilemma

Terminology
- lowest term ratio
- proportional
- reasoning
- non-proportion

Language Goals
- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Write in a variety of forms, with teacher guidance. (ESLAO)
- Read texts with familiar content or vocabulary, using a variety of reading strategies. (ESLBO)
- Write in a variety of forms. (ESLBO)

Materials

Assessment Opportunities

Minds On...

Whole Class → Discussion
Refer English language learners to their personal word study notebook so they can review terms from previous units: square, perimeter, area, length of a diagonal, cube, volume, Pythagorean theorem, first differences.
Define lowest term ratio, writing some numerical examples on the board.

Pairs → Anticipation Guide
English language learners can verbalize each statement in their own words before they respond on the worksheet.

Action!

Pairs → Investigation
Ask students to highlight words that are giving them difficulty. Support them by using questions to help them sort out what they understand and what they need to do next.

Learning Skills (Collaboration)/Observation/Anecdotal: Observe the students’ contributions to completing the task.

Consolidate Debrief

Pairs → Think/Pair/Share/Discussion
As they complete the anticipation guide, provide prompts to help them connect back to the investigation.

Whole Class → Note Making
Use a graphic organizer to help learners make the connections between first differences, ratios, graphs, and proportions.

Concept Practice Reflection

Home Activity or Further Classroom Consolidation
Clarify any questions students have about the assignment.
Students provide a concrete or pictorial example to demonstrate their understanding of proportion.
**Math Learning Goals**
- Explore and develop an understanding of proportions, estimate answers, and devise and explain informal solutions (e.g., constant of proportionality, unit rate) in a variety of contexts (e.g., numerical, geometric, measurement, probability, algebraic).
- Solve problems using the Pythagorean relationship to connect proportional reasoning to contexts.

**Materials**
- BLM 4.3.1, 4.3.2
- linking cubes
- colour tiles
- grid paper
- relational rods

**Assessment Opportunities**
An alternate context may be more appropriate depending on the classroom environment.

**Minds On ...**

**Whole Class → Discussion**
Lead a discussion in which students share the informal methods of solving proportions from the Home Activity. Students may need to be reminded about conversions between feet and inches. (Do this in the context of ratios.)

**Action!**

**Groups of 4 → Exploration**
Form heterogeneous groups based on students’ preferred learning style, using observations of their previous two days’ work.

Students use concrete materials and at least two different informal methods for their exploration (BLM 4.3.1).

**Tools and Strategies/Observation/Mental Note:** Assess the selection of tools and computational strategies.

**Consolidate Debrief**

**Whole Class → Discussion/Note Making**
Members from different groups share their solutions. Ensure that a variety of solution strategies are shared.

Discuss which strategies were effective for the various types of problems. Summarize the various methods including equivalent ratios, the constant of proportionality, and algebraic reasoning. The algebraic reasoning may have to be formally taught, using the questions on BLM 4.3.2.

**Application Concept Practice**

**Home Activity or Further Classroom Consolidation**
- Complete the questions on worksheet Television Dimensions.
- Complete Pythagorean theorem questions.
**Unit 4: Day 3: Pondering Proportions**

**Terminology**
- feet
- inches

**Language Goals**
- Find specific information in straightforward reference materials, with teacher guidance. (ESLAO)
- Use some simple sentence patterns and key conventions of standard Canadian English to write about classroom topics and activities. (ESLAO)
- Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO)
- Use a variety of simple sentence patterns and basic conventions of standard Canadian English with some accuracy in written work. (ESLBO)

**Minds On…**

**Whole Class ➔ Discussion**
Demonstrate equivalent measures using a yardstick and a 12-inch ruler. Write the corresponding equivalent ratios on the board for visual reference.

**Action!**

**Groups of 4 ➔ Exploration**
Model ways of using manipulatives for solving problems. English language learners restate other group members’ comments, asking questions or adding ideas of their own.

**Tools and Strategies/Observation/Mental Note:**
Assess the selection of tools and computational strategies focusing on growth in their choices.

**Consolidate Debrief**

**Whole Class ➔ Discussion/Note Making**
Encourage English language learners to participate in the sharing of their group’s solutions by modelling part of the solution or by presenting the statement in a visual format.

**Home Activity or Further Classroom Consolidation**
English language learners start the worksheet 4.3.2 with a partner to ensure they understand the contexts.
Unit 4: Day 4: I’d Rather Be Scaling (TIPS4RM)

Math Learning Goals
• Investigate a variety of methods for solving problems using proportions (e.g.,
scaling/tables, drawings, constant of proportionality, unit rate, cross products).
• Solve problems involving ratios, rates, and directly proportional relationships in
a variety of contexts.
• Use estimation and proportional reasoning to determine the population size
based on a random sample.

Materials
• photos of crowds
• grid paper
• 2 colours of
number cubes
• overhead
transparencies
• BLM 4.4.1, 4.4.2

Assessment
Opportunities

Minds On ...

Pairs → Peer Coaching
Students compare solutions and help each other with practice questions from
the home activity.

Whole Class → Discussion
Show an aerial photograph of a large crowd. Students suggest ways the number
of people could be counted.

Action!

Pairs → Estimating
Differentiate this activity by providing different photographs with varying
density and distribution of people and pairing students who are at similar
mathematical development levels. Students draw a grid on the photograph or
use an overhead overlay. The grid should have six rows and six columns. Use
one coloured number cube to randomly choose a row and the other number
cube for the column.

Pairs of students choose three grid squares by rolling the number cubes. They
count the number of people in one of the grid squares and estimate the total
number of people using a proportion. Repeat the process using the total for all
three grid squares. Pairs compare their results with other pairs who used the
same photograph.

Connecting/Oral Questions/Anecdotal: Observe students as they discuss and
compare their results with other groups.

Pairs → Scaling
Distribute BLM 4.4.1. Assign each pair of students one of the following
enlargements (12 × 10 double both vertically and horizontally, 6 × 10 double
only vertically, 12 × 5 double only horizontally, 12 × 15 double horizontally
and triple vertically, 18 × 15 triple horizontally and vertically). Pairs of students
make an enlarged figure on grid paper and write their scaling instructions
below it.

Whole Class → Discussion
Discuss the impact of sample size on the accuracy of the estimate.
• What limitations does this method have?
• Which of your enlarged figures are distorted? Why?

Discuss what constitutes a scale diagram, the constant of proportionality, and
work through examples on how to use them.

Students record definitions and examples in their notes.

Home Activity or Further Classroom Consolidation
• Complete the worksheet, More Scaling Problems.
• Complete the practice problems.
• Bring in examples of scale diagrams and aerial photos for the bulletin board.
• Bring in grocery flyers with prices of snacks and drinks for tomorrow’s
activity.

Application
Concept Practice
TIPS for English Language Learners in Mathematics – Grade 9 Applied: Unit 4, 2005

**Language Goals**

- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Communicate orally, using accepted word order, common tenses, and other features of English grammar with some accuracy and consistency. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

**Terminology**

- proportions
- scale
- population size
- random sample

**Minds On...**

**Pairs → Peer Coaching**

As pairs share their responses, give positive feedback, using appropriate prompts and praise.

**Action!**

**Pairs → Estimating**

English language learners should be paired with peers who are at similar mathematical development levels.

**Connecting/Oral Questions/Anecdotal:** Observe students as they discuss and compare their results with other groups.

**Pairs → Scaling**

Use prompts to help learners understand their specific instructions for scaling.

**Assessment Opportunities**

- Additional vocabulary may be necessary, e.g., *aerial photograph, density, distribution, limitations, distortions.*

**Make It Language Rich**

- Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

**Make It Explicit**

- Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.

**Consolidate Debrief**

**Whole Class → Discussion**

Post all the scale diagrams and sort to provide a visual summary of the key concepts, allowing students to see what scaling instructions lead to a proportional relationship.

**Home Activity or Further Classroom Consolidation**

Show examples of scale diagrams, aerial photographs, and appropriate grocery flyers so that learners will understand what they are to find.
### Unit 7: Algebraic Models

#### Lesson Outline

**BIG PICTURE:**

English language learners will:
- continue to build their own personal word study notebook;
- continue to work productively in flexible student groupings;
- continue to make short presentations;
- begin to participate in full class discussions.

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson Title</th>
<th>Language Goals</th>
<th>Expectations</th>
</tr>
</thead>
</table>
| 1   | Linear and Non-linear         | • Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)  
    | Investigations (Part 1)       | • Write in a variety of forms, with teacher guidance. (ESLBO)                  | NA2.07, LR3.02,   |
|     |                               | • Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO)   | LR3.04, LR4.03, LR4.05 |
|     |                               | • Write in a variety of forms. (ESLBO)                                             | CGE 5a, 7i         |
| 2   | Linear and Non-linear         | • Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO) | NA2.07, LR3.02,   |
|     | Investigations (Part 2)       | • Demonstrate adaptation to some key teacher expectations and school routines. (ESLBO)  | LR3.04, LR4.03, lr4.05 |
|     |                               | • Understand and use some subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO) | CGE 5a, 7i         |
|     |                               | • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO) |                    |
| 3   | Building Models               | • Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)  
    |                               | • Participate in conversations on familiar topics in some social situations. (ESLAO)   | NA2.01, NA2.05,   |
|     |                               | • Read texts with familiar content or vocabulary related to classroom studies. (ESLBO)               | LR4.03, LR4.04, LR4.05 |
|     |                               | • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO) | CGE 5a, 7i         |
| 4   | Simplifying Algebraic         | • Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)  
    | Models                        | • Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO) | NA2.01, NA2.05,   |
|     |                               | • Participate in conversations on familiar topics in some social situations. (ESLAO)               | NA2.06, LR4.03, LR4.04, LR4.05 |
|     |                               | • Demonstrate adaptation to school norms, key teacher expectations and classroom routines. (ESLBO) | CGE 2a, 2b         |

**ESLAO** – Beginning communication in English builds on students’ previous education and language knowledge to introduce the English language and help students adjust to their new cultural environment.

**ESLBO** – English in daily life expands students’ essential English communication skills and introduces the language of classroom studies.
Unit 7: Day 1: Linear and Non-Linear Investigations (Part 1) (TIPS4RM)

Math Learning Goals
- Investigate linear and non-linear relationships.
- Examine first differences and the shape of the graph.
- Explore the effects of changing the conditions.
- Write equations for linear relationships and describe non-linear relationships.

Materials
- BLM 7.1.1, 7.1.2
- BLM 7.1.3 (Teacher)
- see BLM 7.1.3 for additional materials

Assessment Opportunities

Minds On ...

Whole Class → Discussion
Explain what the students will be doing at each station.
Review terminology: linear and non-linear; rate of change and initial value (refer to Word Wall).

Action!

Small Groups → Carousel of Activities

Learning Skill (Teamwork)/Observation/Checkbric and Curriculum Expectations/Investigation/Rubric: Observe and record students’ contributions to the group as they complete the activities.

Arrange the four stations by placing the appropriate materials and one colour-coded copy of the experiment (BLM 7.1.2) at each station.
Students complete each of the four experiments and record their answers on BLM 7.1.1 (You will need four copies per group).

Consolidate Debrief

Whole Class → Connecting
After students have completed all four of the experiments, help them make the connection between the first differences and the type of relationship (linear and non-linear). If students have not finished all four of the experiments, allocate more time the next day and make connections then. (See Day 2 for guiding questions.)

See Answers to Experiments (BLM 7.1.3).

Home Activity or Further Classroom Consolidation

Complete the following journal entry:
Sally was not in class today. She doesn’t know how to use differences to determine if a relationship is linear or non-linear. Use words, pictures, and symbols to explain it to her.
Unit 7: Day 1: Linear and Non-Linear Investigations (Part 1)

Terminology
linear, non-linear rate of change initial value mathematical models: numerical, graphical, algebraic

Language Goals
- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Write in a variety of forms, with teacher guidance. (ESLAO)
- Demonstrate knowledge of English vocabulary related to classroom studies. (ESLBO)
- Write in a variety of forms. (ESLBO)

Minds On… Whole Class → Discussion
Model how to complete the record sheet and demonstrate how to use the manipulative for each investigation.
English language learners can refer to their personal word study notebooks and the Word Wall for terminology.

Action! Small Groups → Carousel of Activities
Arrange groups so that the English language learners work with English-speaking students who can model using the manipulatives and assist with the vocabulary for performing the investigation and recording the results.

Learning Skill (Teamwork)/Observation/Checkbric and Curriculum Expectations/Investigation/Rubric: Observe and record students’ collaboration skills focusing on the English language learners’ progress and accountability to the group.

Consolidate Debrief Whole Class → Connecting
Use an overhead of a completed record sheet to highlight the connections among the first differences, the graph, and the algebraic model. Summarize the connections using a graphic organizer or a diagram.

Home Activity or Further Classroom Consolidation
Clarify any questions students have about the assignment.

Materials

Assessment Opportunities
Begin a Word Wall for this unit.
Additional vocabulary may be necessary, e.g., figure, “burning the candle at both ends.”

Make Sure They’re Ready
Circulate to make sure that English language learners understand each investigation.

Make It Language Rich
Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

Make It Explicit
Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.
Unit 7: Day 2: Linear and Non-Linear Investigations (Part 2) (TIPS4RM)

Math Learning Goals
- Investigate linear and non-linear relationships through investigation.
- Examine first differences and the shape of the graph.
- Explore the effects of changing the conditions.
- Write equations for linear relationships and describe non-linear relationships.

Materials
- graph paper
- BLM 7.2.1
- BLM 7.2.2 (Teacher)

Assessment Opportunities

Minds On ...
Whole Class ➔ Discussion
Summarize how to identify whether a relationship is linear or non-linear using first differences. (BLM 7.2.1)

Action!
Small Groups ➔ Carousel of Activities
Students continue to complete the experiments if not completed from Day 1.

Learning Skill (Initiative)/Observation/Checkbric and Curriculum Expectations/Investigation/Rubric: Observe and record students’ initiative as they work in their groups.

Consolidate Debrief
Whole Class ➔ Connecting
Use the following guiding questions:
- Which experiments had a linear relationship? (Take up equations using the graph, and identify the rate of change and the initial value.)
- Identify the rate of change and initial value for each linear relation. Write the equation for each relation.
- How can you use the table of values to predict if a relationship will be linear or non-linear? (Emphasize that the x values are increasing by 1, and that the differences are all the same.)

Discuss how changing the conditions of the experiments affects the graph (linear only).
Discuss with the students whether or not it makes sense to join the points on the graph based on whether the relationship is discrete or continuous.

Home Activity or Further Classroom Consolidation
Graph the relationships from worksheet 7.2.1 and identify the rate of change and the initial value for the linear relationships. Write the equation for each relation.

Solutions to BLM 7.2.1 are provided on BLM 7.2.2.
Unit 7: Day 2: Linear and Non-linear Investigations (Part 2)

Terminology

Language Goals
- Use high-frequency words and simple sentence patterns to communicate meaning. (ESLAO)
- Demonstrate adaptation to some key teacher expectations and school routines. (ESLAO)
- Understand and use some key subject-specific vocabulary in classroom discussions when visual aids are used. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials

Assessment Opportunities

Minds On… Whole Class Discussion
Model the process on the board, connecting verbal, written and pictorial representations of the words.

Action! Small Groups Carousel of Activities
Circulate to make sure that learners are recording all the representations for the investigations.

Learning Skill (Initiative)/Observation/Checkbrick and Curriculum Expectations/Investigation/Rubric: Observe and record students’ initiative as they work in their groups. Observe how they seek clarification from their group.

Consolidate Debrief Whole Class Connecting
English language learners can participate in the discussion by restating other students’ comments, asking questions, or adding their own ideas.

English language learners may wish to share explanations using an overhead or by drawing/writing on the board.

Home Activity or Further Classroom Consolidation
Have learners check their solutions to the first questions on BLM 7.2.1 to verify that they understand the Home Activity.

Additional vocabulary may be necessary, e.g., changing conditions, multiple representations.

Make it Language Rich
Encourage English language learners to access their bilingual dictionaries and their personal word study notebook.

Make it Explicit
Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.
TIPS for English Language Learners in Mathematics – Grade 9 Applied: Unit 7, 2005

Unit 7: Day 3: Building Models (TIPS4RM)

Math Learning Goals
- Use multiple representations (physical, numerical, algebraic).
- Develop an understanding that simplification is necessary to determine if two algebraic expressions are equivalent.

Materials
- computer/data projector
- BLM 7.3.1, 7.3.2
- algebra tiles
- pattern blocks

Assessment Opportunities

Minds On ...
Whole Class → Presentation
Use the electronic presentation Patterns to introduce number patterns and terminology.

Pairs → Discussion
Students complete BLM 7.3.1 in pairs, comparing and refining responses. Students work with another pair to compare/refine their responses to the problems.

Curriculum Expectations/Observation/Mental Note: Circulate while students are working to assess prior knowledge.

Action!
Whole Class → Setting Context
Introduce the task (BLM 7.3.2) and establish a purpose for finding a pattern (e.g., Frieda may want to know how many chairs she needs for 32 tables or how many tables she needs for 108 people).

Small Groups → Guided Exploration
Students complete BLM 7.3.2.

Consolidate Debrief
Whole Class → Discussion
Debrief the Feeding Frenzy activity to determine that students can build an algebraic model from a number pattern and that students recognize that there may be more than one correct algebraic model. Compare the equivalent models and simplify them to demonstrate that they are the same. Discuss and compare the patterns in both parts.

Home Activity or Further Classroom Consolidation
- Use algebra tiles to show that the given three expressions are equivalent:
  (i) $2 + 4n$  
  (ii) $1 + 2n + 2n + 1$  
  (iii) $6 + (n – 1)(4)$
- Journal entry: Jason thinks that both of the Feeding Frenzy examples show a linear relationship between the number of tables and number of chairs. What evidence can you offer to support his claim?
- Practise your skills with algebraic expressions.

Provide appropriate practice questions.
Unit 7: Day 3: Building Models

Terminology
- patterns
- algebraic
- expression
- models:
  - numerical
  - algebraic
  - graphical

Language Goals
- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Participate in conversations on familiar topics in some social situations. (ESLAO)
- Read texts with familiar content or vocabulary related to classroom studies. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Minds On…

Whole Class → Presentation
Reinforce the information during the electronic presentation using words and pointing to or circling key ideas on the screen.

Pairs → Discussion
Use questions to help English language learners sort out what they understand.

Curriculum Expectations/Observation/Mental Note: Circulate while students are working to assess prior knowledge.

Action!

Whole Class → Setting Context
Model how to complete the first portion of the worksheet using visual aids.

Small Groups → Guided Exploration
Arrange groups so English language learners work with English-speaking peers to help them structure their explanations.

Consolidate Debrief

Whole Class → Discussion
During the discussion have learners explain their thinking using the manipulatives as part of their explanation.
To increase English language learners’ comfort in participating, have them present the group’s statement visually, e.g., with diagrams or manipulatives.

Home Activity or Further Classroom Consolidation
Clarify any questions students have about the assignment.

Concept Practice

Materials

Assessment Opportunities

Additional vocabulary may be necessary, e.g., frenzy, catering.

Make It Explicit
Ask students to highlight words that are giving them difficulty. Support their understanding, as needed.
Unit 7: Day 4: Simplifying Algebraic Models (TIPS4RM)

Math Learning Goals
- Use multiple representations (physical, numerical, algebraic).
- Simplify algebraic expressions.

Math Learning Goals

Materials
- overhead projector
- algebra tiles
- computer/data projector
- BLM 7.4.1, 7.4.3 (Teacher)
- BLM 7.4.2, 7.4.4

Assessment Opportunities

Minds On ...
Small Groups → Practice
Students found a relationship between the term number and the expression for the term (Day 3). Provide each group with a set of answer cards. Hold up a cue card with an expression and students match it with one of their set. Students explain their choices. Clear up any misunderstandings.

Whole Class → Practice
Do several examples like the following:
- What is an algebraic expression for “3 more than double a number”? (Answer: 3 + 2x)
- Use algebra tiles to create a model for the expression. (Answer: Use the overhead to show 2 x-tiles and 3 one-tiles.)

Action!
Whole Class → Presentation
Use electronic presentations Collecting Terms Using Algebra Tiles, and Expanding and Simplifying Algebraic Expressions.

Pairs → Practice
Students complete BLM 7.4.2.

Learning Skill (Work Habits)/Observation/Rating Scale: Observe and record how students work to complete the assignment.

Consolidate Debrief
Whole Class → Connecting
Use Debrief Notes to consolidate learning and make connections between numerical, algebraic, and graphical models (BLM 7.4.3).

Home Activity or Further Classroom Consolidation
Complete worksheet 7.4.4 We’re All Correct!

How can you determine if two expressions are equivalent? Students may need additional practice questions.
Unit 7: Day 4: Simplifying Algebraic Models

Terminology
- sum
- product
- quotient
- increased
- decreased
- difference
- times

Language Goals
- Participate in oral learning tasks and engage in social interaction in the classroom. (ESLAO)
- Use some key reading strategies for decoding and comprehension, with teacher guidance. (ESLAO)
- Participate in conversations on familiar topics in some social situations. (ESLBO)
- Demonstrate adaptation to school norms, key teacher expectations, and classroom routines. (ESLBO)

Materials
- toothpicks

Assessment Opportunities
- Additional vocabulary may be necessary, e.g., cue cards, double.

Minds On… Small Groups → Practice
Explain the meanings of homophones using a math context. English language learners record the terms in their personal word study notebook.
Show the cue card and read it slowly, more than once, orally emphasizing and pointing to the key words. When an incorrect response is given, read the cue card again, pointing to the math symbols on the correct answer card. Post the cue cards with the answer for reference.

Whole Class → Practice
Model a few questions using visual aids and oral clues.

Action!

Whole Class → Presentation

Pairs → Practice
There are two parts to this practice: simplifying the algebraic expression and translating the algebraic expression into a word statement. Circulate to determine the portion of the practice with which students may be having difficulty, and use appropriate prompts to guide their learning.
To determine that students understand there are multiple ways to state an algebraic expression, have them share different word statements, e.g., \( x + 26 \) is twenty-six more than a number, the sum of a number and twenty-six, a number increased by twenty-six,…

Learning Skill (Work Habits)/Observation/Rating Scale: Observe and record how students work to complete the assignment, noting their growth.

Consolidate Debrief

Whole Class → Connecting
To describe the process that transforms \( 3(x - 1) \) into \( 3x - 3 \), learners use arrows or write the steps in numerical form on the board.

Home Activity or Further Classroom Consolidation
Students can model their thinking of the solution with the toothpicks.