GAP CLOSING

Relating Situations to Mathematical Operations

Junior / Intermediate Facilitator's Guide
Module 6
Relating Situations to Mathematical Operations

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**RELATING SITUATIONS TO MATHEMATICAL OPERATIONS**

**Relevant Expectation for Grade 6**

- solve problems that arise from real-life situations and that relate to the magnitude of whole numbers up to 1,000,000
- solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms);
- represent relationships using unit rates

**Possible reasons a student might struggle in relating situations to mathematical operations**

Students may not recognize that
- every subtraction problem can be solved by adding
- subtraction situations might involve take away, comparing or determining a missing addend
- multiplying and dividing only apply when there are equal groups
- every division problem can be solved by multiplying
- multiplication situations involving two numbers might involve equal groups, areas, combinations or rates
- division situations might involve sharing, determining the number of equal groups, or rates

**Additional considerations**

Any problem that can be solved by multiplication can also be solved by addition. Any problem that can be solved by division can also be solved by subtraction, or thinking in reverse, by multiplication or addition.

It may be valuable for students to whom you assign the Open Question to have a look at the meanings for each operation described in the Think Sheet. This may round out their own more limited understandings.
Administer the diagnostic

If students need help in understanding the directions of the diagnostic, you are free to clarify an item’s intent.

Using diagnostic results to personalize interventions

Intervention materials are included on each of these topics:

- Recognizing subtraction situations
- Recognizing multiplication situations
- Recognizing division situations

You may use all or only part of these sets of materials, based on student performance with the diagnostic.

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Solutions

1. 342 – 174 or 174 + 342
2. e.g., (342 + 174 + 200) – 400 OR (400 – 342) + 174 + 200. [If students omit the parentheses, clarify with them what is actually subtracted.]
3. 2 × 174 OR 174 + 174
4. 342 ÷ 9 OR 9 × 342 = 342 [Some students might use repeated subtraction or addition, but this is not expected.]
5. 400 – 342 or 342 + 400
6. 342 – 43 OR 43 + 342
7. 200 ÷ 5 (or 5 × 200)
8. 200 × 2.5 [Repeated addition could be used, but this is not expected.]
9. 2394 ÷ 342 OR 342 × 3 = 2394
10. 95 × 60 [Repeated addition could be used, but this is not expected.]
11. 7500 ÷ 100 or 100 × 7500 [Repeated addition or subtraction could be used, but this is not expected.]
12. 3 × 4 [Repeated addition could be used, but this is not expected.]
13. 8 × 9 [Repeated addition could be used, but this is not expected.]
14. 8 × 9 + 6 × 11 [Solely addition could be used, but this is not expected.]
15. 342 + 174 + 200 all multiplied by 2 and then the result divided by 3 [Repeated addition could be used, but this is not expected.]
The purpose of the suggested work is to help students build a foundation for work in proportional reasoning and integer and fraction computations.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:
• Questions to ask before using the approach
• Using the approach
• Consolidating and reflecting on the approach
Recognizing Subtraction Situations

Learning Goal

• connecting the various meanings of subtraction to real-life situations.

Open Question

Questions to Ask Before Using the Open Question

◊ Suppose Jeff’s mother is 35 years old and his grandmother is 62 years old. How do you find out how much older his grandmother is than his mother? (e.g., I would figure out how far it is from 35 to 62 on a number line.)
◊ How would you do that? (e.g., I would say that it is 5 to get to 40 and then 22 more to get to 62, so that is 27.)
◊ How did you get the 27? (I added 5 and 22.)
◊ Is there any other way to solve this? (e.g., You could subtract 35 from 62.)
◊ What makes this a subtraction problem? (You want to know how far apart 35 and 62 are.)
◊ When else might you subtract? (e.g., If you are taking something away.)

Using the Open Question

Provide base ten blocks or number lines for students to use.

Make sure they realize that the problems could be about the same thing (e.g., computers or TV sets or packs of paper) or different things. They do not have to solve the problems since the focus in this lesson is just on recognizing what kind of problem it is.

Assign the tasks.

By viewing or listening to student responses, note if they:
• recognize that subtraction applies to take away, but also applies to determining differences or determining how much more is needed
• think of addition as linked to subtraction

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ What made your problems similar? (They are all subtraction problems.)
◊ Could they have been solved using a different operation? (Yes, addition)
◊ Tell me how. (e.g., Instead of taking 389 from 1000, I could have added up from 389 until I got to 1000.)
◊ What made your problems different? (The stories were different.)
◊ Did subtraction mean something different? (e.g., I guess so–once it was take away, but the other times it wasn’t.)
◊ What kinds of problems would you call subtraction problems? (Problems where there is a whole and a part, and you want the other part. Sometimes the part was taken away, but not always. There are also subtraction problems where there are two separate things and you want to know how much more one is than the other.)

Solutions

e.g.,
You had $1000 and spent $389. How much money did you have left?
This is subtraction since it’s take away.

You need to raise $1000, but only have $389 so far. How much more money do you need?
This is subtraction since you are figuring out the missing part of the whole.

You have $1000, but your friend only has $389. How much more do you have?
I’m finding the difference between two numbers.
Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ Tell me a story problem where I might subtract 100 from a number. (e.g., There are 424 students in a school and 100 students went on a field trip and you want to know how many students are left in the school.)
◊ What made that a subtraction problem? (Some people went away.)
◊ Do you think all subtraction problems involve things or people going away? (No, e.g., If I ask how much the temperature went up from 2° to 10°, I could subtract but there is nothing going away.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand the various meanings of subtraction shown. Point out which number is written before and which number after the minus sign in various situations. [Many students do this incorrectly.]

Make sure students understand that:
• each meaning of subtraction is a bit different from the others but that they are essentially the same idea (that there was a whole and part of it is accounted for and you want to know the other part)
• that any subtraction problem could be considered a “missing addition” problem.

Assign the tasks.

By viewing or listening to student responses, note if they:
• distinguish situations involving division from those involving a single subtraction
• recognize the variation in the types of subtraction situations there are
• realize that any subtraction equation can be rewritten as an addition equation
• realize that some problems may require subtraction in part of the problem but other operations might also be useful
• can be efficient when several subtractions are involved in a problem

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What made situations d) and f) different from the others in Question 1? (They were asking about groups and not about the size of one part.)
◊ Could you have used subtraction to solve them? (Yes, but you are counting how many times you can subtract the group size from the whole and not anything about the group sizes.)
◊ Why can you solve every subtraction with an addition? (You just look for what you have to add to the part to make the whole.)
◊ Did you have to subtract 12 and 18 separately in Question 6? Explain. (No, you could have subtracted 30.)
◊ What do you look for to decide if a problem is a subtraction problem? (e.g., I look for situations where there is a whole and a part, and you want the other part. Sometimes the part was taken away, but not always. There are also subtraction problems where there are two separate things and you want to know how much more one is than the other.)
Solutions

1. a) $400 - 325$, b) $400 - 325$, c) $150 - 70$ or $150 - 70 = 70$, e) $2.5 - 1.2$.
   Some students may say that d) is about $400 - 325$ [although the answer is not 75, but 1–there was only 1 group of 325 you could remove]. [Some may say that f) is about how many 42s can be subtracted from 210 and this is correct.]

2. a) You are finding the missing part. 
   b) You are taking away $2.50$. 
   c) You are finding how much more one number is than another.

3. a) $1500 - 925 = \_ or 925 + \_ = 1500$ [The unknown could come first in the addition.] 
   b) $8.99 - 2.50 = \_ or 2.50 + \_ = 8.99$ 
   c) $350 - 200 = \_ or 200 + \_ = 350$

4. e.g., Both. You subtract to find out the size of the other package but you could add up from 35 to 320.

5. e.g., How many more cards did Jack have than Ian?

6. e.g., You have to subtract the 12 points she lost on one move and the 18 points she lost on the other move. You could subtract twice, once for the 12 and once for the 18, or you could realize that $12 + 18 = 30$ and just subtract once. You could also subtract $17 - 12$ to see how many points she had gained after the first move and then subtract 18 to see how many points she had left.

7. e.g., I look to see if there is something taken away or if I want to find out how far apart two numbers are or if I know one part and the whole and have to find the other part.
Recognizing Multiplication Situations

Learning Goal

• connecting the various meanings of multiplication to real-life situations.

Open Question

Questions to Ask Before Using the Open Question

◊ Suppose a game costs $17. How would you figure out how much 3 games cost? (e.g., I would multiply 3 × 17.)
◊ What makes this a multiplication problem? (You have 3 equal amounts to put together and you multiply when you are adding equal amounts.)
◊ When else might you multiply? (e.g., If there are 3 groups of people and you want to know how many there are in all.)

Using the Open Question

Provide base ten blocks or number lines for students to use, if they wish.

Make sure that they realize that the problems could be about the same thing (e.g., row of chairs or coloured tile arrangements) or about different things. They do not have to solve the problems since the focus in this lesson is just on recognizing what kind of problem it is.

Assign the tasks.

By viewing or listening to student responses, note if they:
• have a limited (or no) understanding of multiplication
• understand that multiplication applies to equal groups, rates, area, and combinations
• think of addition as linked to multiplication

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting on the Open Question

◊ What made your problems similar? (They are all multiplication problems.)
◊ Could they have been solved using a different operation? (Yes, addition.)
◊ Tell me how. (e.g., Instead of multiplying 3 × 40, I could have added 40 three times.)
◊ What made your problems different? (The stories were different.)
◊ Did multiplication mean something different? (e.g., I guess so—it was area one time, but not every time.)
◊ What kinds of problems would you call multiplication problems? (Problems where there are a bunch of equal groups and you want to know how many in all.)

Solutions

e.g., Jane had 4 dimes. Elena had 3 times as many dimes. How many did she have?
If you have 3 times as many, it is like having 3 groups of 40 cents.

Three baskets each held 40 sugar packets. How many packets were there altogether?
There are equal groups and you want a total.

A rectangular shape was made with 3 rows of 40 tiles. How many tiles were used?
This shows area. There are actually 3 equal groups of 40 tiles.
Think Sheet

Questions to Ask Before Assigning the Think Sheet

Put out 3 groups of 4 red counters and 6 groups of 5 blue counters.
◊ How would you find out how many red counters? (e.g., I would multiply \(3 \times 4\).)
◊ How would you find out how many blue counters? (I would multiply \(6 \times 5\).)
◊ How would you find out how many counters in all? (I would add the two answers.)
◊ Why wouldn’t you multiply? (The groups aren’t all equal.)
◊ Here are some square tiles. What might you do with them that would lead you to multiply? (e.g., I might make a rectangle and figure out its area by multiplying.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand:
• the various meanings of multiplication shown
• that each meaning of multiplication is a bit different from the others but that they are essentially the same idea (that there are equal groups and you want a total)
• that a multiplication problem could be considered a repeated addition problem

Assign the tasks.

By viewing or listening to student responses, note if they:
• distinguish situations involving multiplication from those involving only addition
• realize that words like “twice” connote multiplication
• realize that multiplication describes a variety of different but related types of situations
• recognize that some problems might include multiplication as well as other operations

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What made situation a) different from situation b) in Question 1? (One involved equal groups and the other didn’t.)
◊ Why is 2b) a multiplication situation? Where are the equal groups? (e.g., There are 3 groups of 2 since for each shirt, there are 2 outfits (one with each pair of pants.).)
◊ Which operations would you use to solve Question 5? (multiplication and addition)
◊ Why do you need both operations? (The bread and tuna cost different amounts)
◊ What kinds of problems would you call multiplication problems? (Ones where there are a bunch of equal groups and you want to know how many in all.)

Materials
• red and blue counters
• Square tiles

Recognizing Multiplication Situations (Continued)

By viewing or listening to student responses, note if they:
• realize that words like “twice” connote multiplication
• distinguish situations involving multiplication from those involving only addition
• recognize that some problems might include multiplication as well as other operations

Rates: If we know how much one item costs or the measurement of several of those items.

For example:
• 145 mL of milk cost 25¢
• 200 mL of juice cost 57¢
• 300 mL of juice cost $1.71
• 2 litres of juice cost $3.42

For example, if there are 3 kinds of ice cream cones and 6 flavors of ice cream, 3 \times 6 different treats can be created. That’s because there are 6 groups of 8 squares

For example, if there are 3 kinds of ice cream cones and 6 flavors of ice cream, 3 \times 6 different treats can be created. That’s because there are 6 groups of 8 squares

The groups could be arranged in many different ways.

For example:
The groups are a bunch of equal groups and you want to know how many in all.)

Remember to check to make sure you still have both operations.

Remember to check to make sure you still have both operations.

Remember to check to make sure you still have both operations.
**Solutions**

1. b), c) (optional), d) (optional), e), f)

2. a) It is like having 31 groups of 35 (for most months).
   b) It is a combination problem.
   c) If you have twice as much, it is like having two groups. Then you have 52 groups of that amount since there are 52 weeks in a year.

3. a) $31 \times 35$ for some months or $30 \times 35$ or $29 \times 35$ or $28 \times 35$.
   b) $3 \times 2$
   c) $2 \times 52 \times 25$

4. e.g., She could knit a different amount of minutes each day.

5. e.g., It is both. You multiply $5 \times 1.17$ to get the cost of the tuna and you multiply $2 \times 2.12$ to get the price of the bread. But you add to get the total bill.

6. e.g., She wants to make three shirts. How much fabric does she need?

7. e.g., I look to see if there are equal groups somehow and I want a total. If that is true, then it is a multiplication situation.
Recognizing Division Situations

Learning Goal

• connecting the various meanings of division to real-life situations.

Open Question

Questions to Ask Before Using the Open Question

◊ There are 36 students in the art club. Each table only holds 4 students. How would you figure out how many tables are needed? (e.g., I would divide 36 by 4.)
◊ What makes this a division problem? (You have a whole amount that you want to separate out into equal groups.)
◊ When else might you divide? (e.g., If you are sharing.)
◊ Why would you divide then? (e.g., You are still forming equal groups.)
◊ How related do you think multiplication and division are? (e.g., Very related. When you divide, you can multiply the smaller number by the answer, and you should get the bigger number.)

[This language will be problematic when fractional values are involved but is reasonable for students at this point in their development. You may want to rephrase, e.g., Yes. When you multiply the divisor by the quotient, you do get the whole amount, the dividend.]

Using the Open Question

Provide base ten blocks or number lines for students to use, if they wish.
Make sure that they realize that the problems could be about the same thing (e.g., muffins, books, or people) or about different things. Students do not have to solve the problems since the focus in this lesson is just on recognizing what kind of problem it is.

Assign the tasks.

By viewing or listening to student responses, note if they:
• have a limited (or no) view of division
• recognize that division applies to creating equal groups, sharing, area and rates
• think of multiplication and/or subtraction as linked to division

Depending on student responses, use your professional judgment to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ What made your problems similar? (They are all division problems.)
◊ Could they have been solved using a different operation? (Yes, multiplication.)
◊ Tell me how. (e.g., When I divide 120 by 6, I am really figuring out how many 6s to multiply to get to 120.)
◊ What made your problems different? (The stories were different.)
◊ Did division mean something different? (e.g., Yes, since sometimes I was sharing and sometimes I was counting how many groups.)
◊ What kinds of problems would you call division problems? (Problems where there is a whole and you are trying to create equal groups somehow.)

Solutions

e.g., 120 students are put on 6 teams. The teams are the same size. How many students were on each team? This is a “sharing” problem.

120 muffins are baked in tins that hold 6 muffins. How many tins are needed? This is a “creating equal groups” problem.
If 6 books cost $120, how much does 1 book cost? This is a rate problem.
Questions to Ask Before Assigning the Think Sheet

• Tell students you have 24 counters in your hand. Put 8 of them down.
  ◊ How many are still in my hand? (16)
  ◊ How did you figure that out? (I subtracted.)
  ◊ How could you have used your multiplication facts to figure that out? (e.g., There are 3 eights in 24 and if I used 1 eight, there are 2 eights left.)
  ◊ Why is finding out how many eights are in 24 actually a division problem? (e.g., Division means you want to make equal groups and count how many and that’s what you’re doing.)
  ◊ Suppose I asked you to share the 24 counters among 8 people. Would I still be making equal groups? (Yes, since sharing means everyone should get the same.)
  ◊ Is it still division? (Yes, because there are equal groups. This time there are 8 instead of 3.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand:
• the various meanings of division shown
• that each meaning of division is a bit different from the others but that they are essentially the same idea (that there are equal groups and you know the total)
• that any division can be thought of as finding the missing multiplication factor

Assign the tasks.

By viewing or listening to student responses, note if they:
• understand that division might involve sharing fairly or counting the number of groups of a given size, as long as the groups are equal
• understand that any division problem could be considered a missing number multiplication problem or a repeated subtraction problem or even a repeated addition problem (where you have to figure out how many times you can repeatedly add)
• realize that division describes a variety of different, but related types of situations
• recognize that some problems might include division but other operations as well

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What made situation a) different from situation b) in Question 1? (One involved equal groups and the other didn’t.)
◊ Why is 2c a division situation? What do the equal groups represent? (e.g., The number of grams in each hamburger is what has to be equal.))
◊ Why were you able to write a multiplication or a division sentence for any of the problems in Question 2? (You could write a division sentence since, each time, a whole is divided into equal groups. You can write a multiplication sentence because, whenever you can write a division sentence, you can automatically write a multiplication sentence.)
◊ Which operations would you use to solve Question 5? (division and multiplication)
◊ Why do you need both operations? (I divided to find out how many beats in one minute, but multiplied to find out how many beats in 2 minutes.)
◊ What kinds of problems would you call division problems? (Problems where there is a whole and you are trying to create equal groups somehow.)
Solutions

1. b), c), d), f)  [e) could be thought of as division if you think of dividing into 3 equal groups where Jake’s share is 2 of those groups.]

2. a) This is a sharing problem. You are sharing the 153 people into 17 buses.

   b) This is a “creating equal groups” problem. The groups are all of size 20 and you are counting how many.

   c) This is a sharing problem.

3. a)  \(153 \div 17 = \underline{11} \text{ or } 17 \times \underline{11} = 153\)

   b)  \(220 \div 20 = \underline{11} \text{ or } 20 \times \underline{11} = 220\)

   c)  \(444 \div 6 = \underline{74} \text{ or } 6 \times \underline{74} = 444\)

4. e.g., There are small boxes and big ones. The small ones hold 10 books, but the big ones hold 20.

5. e.g., Yes and no. I would divide to find out how many beats in 1 minute, but then I would multiply to find out how many beats in 2 min.

6. e.g., The pencils were put in packages that each held 10 pencils. How many packages were needed?

7. e.g., I look for a situation where a total is being divided up into equal groups.