Comparing Fractions

Relevant Learning Expectation for Grade 6

… compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire® rods, drawings, number lines, calculators) and using standard fractional notation

Possible reasons a student might struggle in comparing fractions or deciding if they are equal

Many students struggle with comparing fractions and/or determining their equivalence. Some of the problems include:

• not realizing that you may have to reorient one diagram to match another to determine which fraction is greater (e.g., thinking that $\frac{2}{3} < \frac{3}{5}$ since the blue is farther to the right with $\frac{3}{5}$)

• not realizing that you can only compare visually if the fractions refer to the same whole

• not realizing that if fractions are close in size, hand-drawn diagrams may not be a useful tool, e.g. to compare $\frac{3}{8}$ and $\frac{4}{10}$

• not recognizing that numerators are all that matter when comparing fractions with the same denominator

• assuming that if fractions have the same numerator, the greater one has the greater denominator

• showing more discomfort when comparing improper fractions than when comparing proper fractions

• thinking that if both numerator and denominator are larger, the fraction is automatically larger (e.g., thinking that $\frac{7}{10} > \frac{3}{5}$ since 7 and 10 are more than 4 and 5)

• not realizing that fractions are equal only if numerator and denominator are multiplied or divided by the same amount, not if the same amount is added or subtracted

• thinking that $\frac{5}{6} > \frac{2}{3}$ since a picture of 4 out of 6 items shows more items than a picture of 2 out of 3 items

• not realizing that comparisons to the benchmarks of $\frac{1}{2}$ and 1 are often an easy way to compare fractions and that this can be done simply by comparing the numerator to the denominator

Part-of-whole models are used to close the gap in student understanding about fractional comparisons and equivalence rather than part-of-set models because the latter requires more sophistication. Although students could figure out why, for example, $\frac{5}{6}$ of 12 is greater than $\frac{1}{2}$ of 12 by determining the count for each, it would be much more difficult to see why $\frac{5}{6}$ of 11 is greater than $\frac{2}{3}$ of 11. For this reason, parts of wholes are used; parts of measures could also be used.

Additional consideration

Before beginning the diagnostic or using the intervention materials, students need to know that > means “greater than” and < means “less than.” Post a reminder as reference.
Administer the diagnostic

If students need help in understanding the directions of the diagnostic, clarify an item’s intent.

If available, students can use concrete fraction pieces instead of the pictorial models provided in the templates. However, after Question 1, the purpose is to see how students work symbolically. If they are dependent on the materials for many questions, assume that they need the relevant intervention materials.

Using the diagnostic results to personalize interventions

Provide Template 1 for students to use if needed.

Intervention materials are included for each of these topics:
• comparing fractions using pictures
• comparing fractions with the same denominator
• comparing fractions with the same numerator
• equivalent fractions
• comparing fractions to \( \frac{1}{2} \) and 1

You can use all or only part of these sets of materials, based on student performance with the diagnostic.

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Solutions

1. a) - [diagram showing comparison of fractions]

b) - [diagram showing comparison of fractions]

c) - [diagram showing comparison of fractions]

d) - [diagram showing comparison of fractions]

2. a) \( \frac{7}{12} \) e.g., because the pieces are the same size but there are more of them; (students may draw pictures and show that \( \frac{7}{12} \) uses more space, but the wholes must be the same size)

b) \( \frac{3}{2} \) e.g., because \( \frac{3}{2} \) is more than 1 and \( \frac{3}{4} \) is less than 1

c) \( \frac{5}{6} \) e.g., because there are 5 pieces each time but sixths are bigger than eighths

d) \( \frac{4}{6} \) e.g., because \( \frac{4}{6} \) is more than \( \frac{1}{2} \) and \( \frac{3}{3} \) is less than \( \frac{1}{2} \).

3. a, c, d, f, g, h

4. a, d, e

5. e.g., draw a picture of 3 out of 5 and keep making more rows, etc.

6. a, c, e
The purpose of the suggested student work is to help them build a foundation with comparing fractions that they can use with ratios, percents, and decimals as they move into Grade 7.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:
• Questions to ask before using the approach
• Using the approach
• Consolidating and reflecting on the approach
Comparing Fractions Using Pictures

Learning Goal

- reasoning about how two fractions shown pictorially can be compared.

Open Question

Questions to Ask Before Using the Open Question

Place a blue pattern block rhombus on top of a hexagon and beside it a pattern block triangle on top of a hexagon. Ask:

◊ What fraction of the hexagon is each shape? (\(\frac{1}{3}\) and \(\frac{1}{6}\))
◊ Which fraction is greater? How can you be sure? (\(\frac{1}{3}\) since it takes up more space; I can be sure by putting the triangle right on top of it to show it’s smaller)

Place the rhombus in different positions on the hexagon.

◊ Does where I put the fraction change its size? (no)
◊ Suppose I had a picture of two fractions. How could I compare them to see which took up more space? (e.g., cut one out and put it on top of the other)
◊ Why do the wholes have to be the same size if you’re comparing the fractions? (e.g., even a little part of something really big is a lot more than a big part of something really little)

Using the Open Question

Provide the templates of circle and rectangle models for fractions.

Explain that they are to use only circles for Question 1 and only rectangles for Question 2.

By viewing or listening to student responses, note if they:

• use visual estimates when appropriate (when fractions are easily compared visually)
• appropriately match to compare when the comparison is not as obvious

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe the comparisons students made and the strategies they used. Ask:

◊ Why was it easy to see that \(\frac{1}{10}\) was the smallest? (e.g., It took up hardly any space)
◊ When did you turn the circles to line things up to compare the fractions? Why? (e.g., for \(\frac{3}{6}\) and \(\frac{4}{10}\) since they didn’t start at the same place)
◊ Which fractions were really close in size? Was it hard to compare them even when you moved the fraction? (e.g., \(\frac{2}{3}\) and \(\frac{7}{10}\) - I think \(\frac{7}{10}\) was more but it was so close, I’m not sure)
◊ When you are ordering a whole set of fractions, what’s a good strategy for starting? (e.g., start with really tiny ones at one end and really big ones at the other end and then compare two at a time in the middle)
**Solutions**

**Circles:** Students show only 4 of these circle fractions:

\[
\begin{align*}
\frac{1}{10} & \quad \frac{1}{4} & \quad \frac{2}{5} & \quad \frac{3}{4} & \quad \frac{5}{6} & \quad \frac{9}{10}
\end{align*}
\]

Sample strategies:

\(\frac{1}{10}\) is very tiny so I knew it was least.

I overlapped \(\frac{5}{6}\) and \(\frac{2}{5}\) and \(\frac{2}{5}\) was less.

I could see that \(\frac{9}{10}\) was the most since it was almost the whole thing.

**Rectangles:** Students show only 4 of these rectangle fractions:

\[
\begin{align*}
\frac{1}{10} & \quad \frac{1}{3} & \quad \frac{3}{5} & \quad \frac{5}{8} & \quad \frac{2}{3} & \quad \frac{7}{10}
\end{align*}
\]

Sample strategies:

\(\frac{1}{10}\) is very tiny so I knew it was least.

I overlapped \(\frac{2}{3}\) and \(\frac{7}{10}\) and \(\frac{2}{3}\) was less.

I could just see that \(\frac{1}{3} < \frac{2}{3}\).
**Think Sheet**

### Questions to Ask Before Assigning the Think Sheet

Place a blue pattern block rhombus on top of a hexagon and beside it a pattern block triangle on top of a hexagon. Ask: *What fraction of the hexagon is each shape?* (\(\frac{1}{2}\) and \(\frac{1}{3}\))

- Which fraction is greater? How can you be sure? (e.g., \(\frac{1}{2}\) since it takes up more space; I can be sure by putting the triangle right on top of it to show it’s smaller)

Place the rhombus in different positions on the hexagon. Ask:

- Does where I put the fraction change its size? (no)
- Suppose I had a picture of two fractions. How could I compare them to see which took up more space? (e.g., cut one out and put it on top of the other)
- Why do the wholes have to be the same size if you’re comparing the fractions? (e.g., even a little part of something really big is a lot more than a big part of something really little)

### Using the Think Sheet

Read the shaded introductory box with the students.

Make sure they understand that sometimes you can just “see” which fraction is more and at other times you have to cut out and rearrange.

Assign the tasks.

By viewing or listening to student responses, note if they:

- can visually compare quickly, when appropriate
- know how to rearrange visuals to compare, when that is needed
- recognize that when only part of a whole is coloured, then it is easy to get greater fractions by colouring in more parts

Depending on student responses, use your professional judgement to guide further follow-up.

### Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

- Why was it easy to answer Question 1? (e.g., You can just look to see \(\frac{1}{2}\) is not even \(\frac{1}{2}\) and \(\frac{1}{3}\) is almost the whole thing.)
- Why might someone think that Question 2 easier than Question 3? (e.g., The fractions were shaded starting at the same place at the top so it was easy to overlap them.)
- Does your strategy change if the shape was a rectangle and not a circle? (e.g., no; I still had to turn things sometimes or fold them to compare them.)
- Why is it easy to see that \(\frac{3}{4}\) is greater than \(\frac{5}{6}\) if you colour in \(\frac{3}{4}\) of a shape? (just colour in two more sections - if you colour more, the fraction is greater)
**Solutions**

1. \( \frac{4}{5} \)
2. \( \frac{4}{5} \)
3. \( \frac{5}{8} \)
4. \( \frac{3}{8} \)
5. \( \frac{2}{5} \)
6. \( \frac{2}{8} \)

7. Sample responses:

   I could see that \( \frac{6}{8}, \frac{7}{8} \) and \( \frac{8}{8} \) were bigger by using the same rectangle since more of it would be used.

   I could use the rectangle with 4 sections and colour 3 or 4 of them and line them up to see that \( \frac{3}{4} \) and \( \frac{4}{4} \) are also bigger.

   I lined up the rectangle in 6 sections underneath \( \frac{5}{8} \) and saw that I would have to go to \( \frac{4}{6} \) to get another bigger fraction.
Comparing Fractions with the Same Denominator

Learning Goal

• reasoning about how two fractions with the same denominator can be compared.

Open Question

Questions to Ask Before Using the Open Question

Ask students to use pattern blocks to show $\frac{1}{3}$ and then $\frac{2}{3}$. Ask:

◊ How do your models show why $\frac{2}{3} > \frac{1}{3}$? (e.g., they cover more of the hexagon)
◊ Did you find that surprising? (no, 2 of something is more than 1 of it)
◊ How could you use the fraction tower to show the same thing? (e.g., I go to the $\frac{1}{3}$ row to show both fractions. $\frac{2}{3}$ is two $\frac{1}{3}$s and that goes farther to the right than $\frac{1}{3}$ does)

Using the Open Question

Provide the Fraction Tower (1) template for students to use, if needed.

By viewing or listening to student responses, note if they require the support of the visuals or recognize that if fractions have the same denominator, the fraction with the greater numerator is the greater fraction.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ What two fractions did you compare first? (e.g., $\frac{3}{8}$ and $\frac{5}{8}$)
◊ How did you decide which was greater? (e.g., I knew $\frac{5}{8}$ was greater since it was more eighths.)
◊ Would it be easy to compare $\frac{3}{20}$ to $\frac{4}{20}$ without a picture? How? (e.g., yes, $\frac{4}{20}$ is more since there are more twentieths)

Solutions

Sample responses

$\frac{2}{3}$ and $\frac{1}{3}$ are not the same size since $\frac{2}{3}$ is two sets of $\frac{1}{3}$, so that’s more. It makes sense since two of anything is more than one of that thing.

$\frac{3}{8}$ and $\frac{5}{8}$ are not the same size since $\frac{3}{8}$ is 3 sections that are each $\frac{1}{8}$ long, but $\frac{5}{8}$ is 5 of those sections and 5 is more than 3. I don’t need a picture to see that.

$\frac{4}{10}$ and $\frac{11}{10}$ are not the same size since $\frac{4}{10}$ is less than a whole and $\frac{11}{10}$ is more than a whole and $\frac{9}{10}$ is a whole. Something more than a whole is greater than something less than a whole.
Think Sheet

Questions to Ask Before Assigning the Think Sheet

Students to use pattern blocks to show \( \frac{1}{3} \) and then \( \frac{2}{3} \). Ask:

◊ How do your models show why \( \frac{2}{3} > \frac{1}{3} \)? (e.g., They cover more of the hexagon.)
◊ Did you find that surprising? (no, 2 of something is more than 1 of it)
◊ How could you use the fraction tower to show the same thing? (I go to the \( \frac{1}{3} \) row to show both fractions. \( \frac{2}{3} \) is two \( \frac{1}{3} \)s and that goes farther to the right than \( \frac{1}{3} \) does)

Using the Think Sheet

Read the shaded introductory box with the students.
Make sure they understand that the numerator tells how many sections and the denominator indirectly indicates the size of the section.
Assign the tasks.
Indicate that students are free to use the Fraction Tower (2) template, if needed.
By viewing or listening to student responses, note if they observe that when fractions have the same denominator, the fraction with the greater numerator is the greater fraction and why.
Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Is there more than one way to figure out, for Question 1b, which fraction is greater? (e.g., Yes, since \( \frac{2}{5} \) is more fifths so that makes it greater, but it’s also more than 1 and \( \frac{2}{5} \) is less than 1, so that’s another reason it’s greater.)
◊ For Question 3, how did you know that 1 could not be an answer for 3a? (e.g., I know that the number has to be more than 2.)
◊ How did you start Question 4? Why did you start that way? (e.g., I knew that the missing number for the second one couldn’t be 0, 1, or 2. I know 0 of anything is small, so it had to be the last numerator or the first numerator.)
◊ What rule could you give for comparing fractions with the same denominator and why does it work? (e.g., The fraction with the greater numerator is greater since there are more pieces and all the pieces are the same for both fractions.)

Solutions

1. a) \( \frac{3}{5} \)    b) \( \frac{2}{5} \)    c) \( \frac{9}{9} \)    d) \( \frac{4}{3} \)
2. e.g., \( \frac{5}{12} \) is 5 sections of \( \frac{1}{12} \) and \( \frac{7}{12} \) is 7 sections of \( \frac{1}{12} \) so there are two extra sections; that makes it more.
3. a) e.g., 3 and 4    b) e.g., 1 and 2    c) e.g., 3 and 5
4. e.g., \( \frac{1}{5} < \frac{2}{5} \)    \( \frac{3}{8} > \frac{2}{8} \)    \( \frac{6}{5} < \frac{4}{5} \)
Comparing Fractions with the Same Numerator

Learning Goal

• reasoning about how two fractions with the same numerator can be compared.

Open Question

Questions to Ask Before Using the Open Question

◊ Which do you think is more, \(\frac{3}{3}\) or \(\frac{3}{4}\)? (e.g., \(\frac{3}{3}\) since \(\frac{3}{3}\) is a whole and \(\frac{3}{4}\) is less)

Have the student look at the \(\frac{1}{3}\) and the \(\frac{1}{8}\) on the tower. Ask:

◊ Which fraction is greater? Why? (\(\frac{1}{3}\) since it is a bigger piece)

◊ Why is it bigger? (since the whole is only 3 sections instead of 8 so there is more in each section)

◊ What about \(\frac{2}{3}\) and \(\frac{2}{8}\) which is more? Why? (\(\frac{2}{3}\), if there are two bigger sections and two littler sections, the two bigger ones are worth more)

Using the Open Question

Students can use the Fraction Tower (3) template, if needed.

By viewing or listening to student responses, note if they:

• require the support of the visuals

• recognize that if fractions have the same numerator, the fraction with the greater denominator is the lesser fraction

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ What two fractions did you compare first? (e.g., \(\frac{4}{8}\) and \(\frac{4}{10}\))

◊ How did you decide which was greater? (e.g., I knew \(\frac{4}{8}\) is \(\frac{1}{2}\) and \(\frac{4}{10}\) is not even \(\frac{1}{2}\) since \(\frac{2}{5}\) is \(\frac{2}{10}\))

◊ How could you use the section sizes to help figure out that \(\frac{4}{8}\) is more than \(\frac{4}{10}\)? (e.g., I know that eighths are bigger than tenths and so 4 eighths is more than 4 tenths.)

◊ Why is it difficult to tell, without a picture, whether \(\frac{3}{2}\) or \(\frac{4}{2}\) is greater, but easier to tell if \(\frac{3}{2}\) or \(\frac{3}{8}\) is greater? (e.g., I know fifths are bigger than sixths so if I have the same number of each, I know \(\frac{3}{8}\) is more. But even though sixths are smaller, if I have more of them (like with 4 sixths compared to 3 fifths), I don’t know which is more in total.)

◊ How could you advise someone to compare two fractions like \(\frac{4}{10}\) and \(\frac{4}{10}\) if you don’t know exactly what the denominators are? (e.g., I would tell them that whichever is the bigger denominator is the smaller fraction since the pieces are bigger and 4 big pieces is always more than 4 little ones.)

Solutions

Sample responses:

\(\frac{3}{5}\) and \(\frac{4}{10}\) are not the same size since if the whole is divided into 10, the pieces are smaller than when it is divided into 8. So I am comparing 4 smaller pieces to 4 bigger ones. That means that \(\frac{3}{5}\) > \(\frac{4}{10}\). I don’t need a picture to tell.

\(\frac{3}{5}\) > \(\frac{2}{3}\) since \(\frac{2}{3}\) is more than one whole (which is only \(\frac{2}{2}\)) and \(\frac{3}{5}\) is less than a whole, since it’s not \(\frac{8}{8}\)

\(\frac{5}{6}\) < \(\frac{5}{8}\) since ninths are smaller than eighths so six small pieces are less than six bigger ones
Questions to Ask Before Assigning the Think Sheet

◊ Which do you think is more \( \frac{3}{4} \) or \( \frac{2}{3} \)? (e.g., \( \frac{2}{3} \) is a whole and \( \frac{3}{4} \) is less)

Have the student look at the \( \frac{1}{3} \) and the \( \frac{1}{8} \) on the tower. Ask:

◊ Which fraction is greater? Why? (e.g., \( \frac{1}{3} \) since it is a bigger piece)

◊ Why is it bigger? (e.g., since the whole is only 3 sections instead of 8 so there is more in each section)

◊ Which is more \( \frac{2}{3} \) or \( \frac{3}{2} \)? Why? (\( \frac{3}{2} \), e.g., if there are two bigger sections and two littler sections, the two bigger ones are worth more)

Using the Think Sheet

Read the shaded introductory box with the students.

Make sure they understand that the numerator tells how many sections and the denominator indirectly indicates the size of the section.

Assign the tasks.

Indicate that students can use the Fraction Tower (4) template.

By viewing or listening to student responses, note if they observe that when fractions have the same numerator, the fraction with the greater denominator is the lesser fraction and why.

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Is there more than one way to figure out, for Question 1c, which fraction is greater? (e.g., yes, since \( \frac{2}{3} \) is way less than half and \( \frac{3}{5} \) is more, I know \( \frac{3}{5} \) is bigger but I also know that 2 thirds is more since thirds are bigger than ninths, so \( \frac{2}{3} > \frac{3}{5} \))

◊ For Question 3, how did you know that 1 could not be an answer for 3a? (e.g., I know that the number has to be more than 3.)

◊ How did you start Question 4? (e.g., I know it’s easiest to compare if the denominators or numerators are the same, so I used 4 for the first fraction and 3 for the second one.)

Solutions

1. a) \( \frac{3}{8} \)  b) \( \frac{3}{5} \)  c) \( \frac{2}{3} \)  d) \( \frac{5}{2} \)

2. e.g., If something is divided into 12 pieces, each piece is smaller since there are more pieces. That means twelfths are smaller than eighths, so \( \frac{5}{12} \) is less than \( \frac{5}{8} \).

3. a) e.g., 4 and 5  b) e.g., 9 and 10  c) e.g., 3 and 4

4. \( \frac{4}{10} > \frac{4}{12} \)  \( \frac{3}{8} > \frac{3}{8} \)  \( \frac{1}{2} > \frac{1}{5} \)  \( \frac{3}{1} > \frac{3}{2} \)
Equivalent Fractions

Learning Goal

• representing the same fraction in different ways.

Open Question

Questions to Ask Before Using the Open Question

◊ Suppose a pie was cut into 8 equal pieces. How many pieces would be half of it? Why? (4 pieces since there would be two equal shares of 4)
◊ How does that tell you that $\frac{1}{2} = \frac{4}{8}$? ($\frac{4}{8}$ is 4 pieces if there are 8 pieces in the pie and it’s half the pie)
◊ We call fractions that are equal equivalent fractions. Can you think of another fraction equivalent to $\frac{1}{2}$? (e.g., $\frac{5}{10}$)

Help the student see why $\frac{1}{2} = \frac{2}{4}$ on the tower by showing how a vertical line down the right edge of $\frac{1}{2}$ passes through the right edge of $\frac{2}{4}$.

Using the open Question

Encourage students to list as many possible equivalent fractions as they can, using the Fraction Tower (4) template.

Encourage them to look at the values of the equivalent fractions to see what they notice.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ Which fraction did you find the most equivalent fractions for? Why do you think that happened? (e.g., 1 had the most equivalent fractions since each line was another name for 1)
◊ Once you knew that $\frac{1}{2} = \frac{2}{4}$, how did that help you predict other equivalent fractions? (e.g., I just doubled the numerators to get $\frac{2}{3} = \frac{4}{6}$ and then I also doubled the numerator and denominator of $\frac{2}{3}$ to get $\frac{4}{6}$.)
◊ Look at all the equivalent fractions for $\frac{1}{3}$. What do you notice about the relationship between the numerator and denominator? (The denominator is always 3 times the numerator.)
◊ How could that help you get more equivalent fractions? (e.g., I would just write any numerator I wanted and multiply by 3 to get the denominator.)
◊ What rule could you give someone for getting equivalent fractions? (e.g., multiply the numerator and denominator by the same thing.)
Solutions

Any or all of:

\[
\begin{align*}
1 &= \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{8}{8} = \frac{9}{9} = \frac{10}{10} = \frac{12}{12} = \frac{15}{15} = \frac{18}{18} = \frac{20}{20} \\
\frac{1}{2} &= \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{9}{18} = \frac{10}{20} \\
\frac{1}{3} &= \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} \\
\frac{2}{3} &= \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} \\
\frac{1}{4} &= \frac{2}{8} = \frac{3}{12} = \frac{5}{20} \\
\frac{3}{4} &= \frac{6}{8} = \frac{9}{12} = \frac{15}{20} \\
\frac{1}{5} &= \frac{2}{10} = \frac{3}{15} = \frac{4}{20} \\
\frac{2}{5} &= \frac{4}{10} = \frac{6}{15} = \frac{8}{20} \\
\frac{3}{5} &= \frac{6}{10} = \frac{9}{15} = \frac{12}{20} \\
\frac{4}{5} &= \frac{8}{10} = \frac{12}{15} = \frac{16}{20} \\
\frac{1}{6} &= \frac{2}{12} = \frac{3}{18} \\
\frac{5}{6} &= \frac{10}{12} = \frac{15}{18} \\
\frac{1}{9} &= \frac{2}{18} \\
\frac{4}{9} &= \frac{8}{18} \\
\frac{5}{9} &= \frac{10}{18} \\
\frac{7}{9} &= \frac{14}{18} \\
\frac{8}{9} &= \frac{16}{18} \\
\frac{1}{10} &= \frac{2}{20} \\
\frac{3}{10} &= \frac{6}{20} \\
\frac{7}{10} &= \frac{14}{20} \\
\frac{9}{10} &= \frac{18}{20}
\end{align*}
\]

I notice that every time you need to multiply the numerator and denominator of one fraction by the same amount to get the other fraction.
Questions to Ask Before Assigning the Think Sheet

◊ Suppose a pie was cut into 8 equal pieces. How many pieces would be half of it? Why? (4 pieces since there would be two equal shares of 4)
◊ How does that tell you that $\frac{1}{2} = \frac{4}{8}$? (e.g., 4 pieces if there are 8 pieces in the pie and it’s half the pie)
◊ We call fractions that are equal “equivalent fractions”. Can you think of another fraction equivalent to $\frac{1}{2}$? (e.g., 5 out of 10)

Using the Think Sheet

Read the shaded introductory box with the students.

Make sure they understand that an equivalent fraction either uses the same space or represents the same ratio, i.e., _____ out of every _____.

Assign the tasks.

By viewing or listening to student responses, note if students:
• can identify equivalent fractions
• can create equivalent fractions either by multiplying both terms or dividing both terms by the same amount
• recognize patterns in equivalent fractions
• recognize the difference in the effect of adding the same amount to both terms of a fraction as compared to multiplying both terms by the same amount

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why are $\frac{5}{6}$ and $\frac{15}{18}$ equal? (e.g., divide each third into 6 equal sections and you have $\frac{15}{18}$)
◊ Why are $\frac{3}{4}$ and $\frac{8}{10}$ not equal even though 8 is 5 more than 3 and 10 is 5 more than 5? (e.g., $\frac{3}{4}$ is just a little bit over half but $\frac{8}{10}$ is almost the whole thing)
◊ List some equivalent fractions for $\frac{2}{3}$. How far apart are the denominators? Why? (e.g., $\frac{1}{3}$, $\frac{2}{6}$ - the denominators are 4 apart - if you split the fourths into smaller pieces, you get 4 extra pieces each time)
◊ For the last question, how did you know that if □ is big, then ▲ is little? (e.g., if □ is big, then the fraction $\frac{6}{□}$ is little so you need a little numerator for 3.)

Solutions

1. a, c, d

2. a) e.g., $\frac{10}{24}, \frac{15}{36}, \frac{20}{48}$
   b) e.g., I would draw $\frac{5}{12}$ and break up each twelfth into 2 equal parts. Then there would be 24 equal parts and 10 parts would be shaded, so that’s $\frac{10}{24}$.

3. yes e.g., The equivalent fractions come from multiplying 2 and 3 by the same amount. When you multiply by 3, there is an extra group of 3 for each extra amount in the number, so there is always 3 or 6 or 9, …

4. e.g., I know that $\frac{5}{6}$ is $\frac{1}{3}$ away from 1, but $\frac{2}{3}$ is $\frac{1}{3}$ away and since $\frac{1}{6}$ is less, it has to be closer to 1.

5. e.g., $\frac{6}{3} = \frac{18}{9}$ or $\frac{6}{2} = \frac{9}{3}$ or $\frac{6}{3} = \frac{6}{6}$ or $\frac{6}{6} = \frac{3}{3}$
Comparing Fractions to \(\frac{1}{2}\) and 1

Learning Goal

- reasoning about how two fractions can be compared using benchmarks of \(\frac{1}{2}\) and 1.

Open Question

Questions to Ask Before Using the Open Question

Examine the fraction tower with the students. Ask:

- How many thirds do you need to be past \(\frac{1}{2}\)? (3)
- How many fourths? (3)
- How many fifths? (3)
- What do you notice? (e.g., There were two lines in a row that needed the same number of sections to get past \(\frac{1}{2}\) and then it changed.)
- What if you used sixths and sevenths? (It would be 4 both times.)
- How could you predict that \(\frac{7}{10}\) is more than \(\frac{1}{2}\) but \(\frac{1}{2}\) is less? (e.g., I know that \(\frac{1}{2} = \frac{5}{10}\) so I have to go higher than 5 in the numerator to be more than \(\frac{1}{2}\).)

Using the Open Question

Students can use the Fraction Tower (4) template.

Encourage students to come up with several values for each numerator or denominator.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

- Was it easier to create fractions greater than 1 or less than \(\frac{1}{2}\) to make up the numerator? (e.g., greater than 1 since I just could add 1 to the denominator)
- How could you be sure your fraction was less than \(\frac{1}{2}\)? (If I took half of the denominator and the numerator was lower, I would be right.)
- Is it easier to create fractions greater than 1 if you are given the denominator or the numerator? (e.g., denominator, since I just keep adding 1s to get more and more numberators, but if I get the numerator, I can subtract, but I don't have as many possibilities)
- Is \(\frac{14}{20}\) more or less than \(\frac{1}{2}\)? How do you know? (e.g., more since \(\frac{10}{20}\) is \(\frac{1}{2}\) and \(\frac{14}{20}\) is more than \(\frac{10}{20}\))

Solutions

e.g., \(\frac{1}{8}\)  \(\frac{2}{5}\)  \(\frac{4}{10}\)  \(\frac{3}{10}\)  \(\frac{5}{30}\)  \(\frac{8}{20}\)

I had to make sure that the denominator was more than double the numerator.

e.g., \(\frac{9}{8}\)  \(\frac{6}{5}\)  \(\frac{11}{10}\)  \(\frac{3}{2}\)  \(\frac{5}{4}\)  \(\frac{8}{3}\)

I had to make sure that the numerator was more than the denominator.
**Questions to Ask Before Assigning the Think Sheet**

Examine the fraction tower with the students. Ask:

- How many thirds do you need to be past 1/2? (3)
- How many fourths? (2)
- How many fifths? (2)
- What do you notice? (e.g., There were two lines in a row that needed the same number of sections to get past 1/2 and then it changed.)
- What if you used sixths and sevenths? (It would be 4 both times.)
- How could you predict that 7/10 is more than 1/2 but 4/10 is less? (e.g., I know that 4/10 = 2/5 so I have to go higher than 5 in the numerator to be more than 1.)

**Using the Think Sheet**

Read the shaded introductory box with the students.

Indicate that the students can use the Fraction Tower (4) template.

Assign the tasks.

By viewing or listening to student responses, note if they realize that:

- to compare to 1, they simply compare numerator and denominator and a greater numerator means greater than 1
- to compare to 1/2, they can either take half the denominator and compare to the numerator or double the numerator and compare to the denominator

Depending on student responses, use your professional judgement to guide further follow-up.

**Consolidating and Reflecting: Questions to Ask After Using the Think Sheet**

- How did you know that 4/10 < 1/2? (I know that 1/2 = 5/10 is less.)
- How did you know that 5/8 was less than 1/2? (e.g., I know that half of 9 is about 4, so a little more than 4/8 = 1/2 is less.)
- Is it easy to decide if a fraction is greater than 1? (Yes, the numerator is more than the denominator.)
- What rule could you give for deciding if a fraction is between 1/2 and 1? (e.g., The numerator has to be less than the denominator or it would not be less than 1. But if you take half of the denominator, the numerator must be more than that or it wouldn’t be more than 1/2.)

**Materials**

- Fraction Tower (4) template

**Solutions**

1. \(\frac{3}{8}, \frac{4}{10}, \frac{2}{5}, \frac{2}{9}\)

2. \(\frac{7}{8}\) and \(\frac{3}{5}\)

3. 0, 1, 2, 3, or 4 since half of 9 is \(4\frac{5}{9}\), so 5 would be too much

4. e.g., Any number greater than 6 makes \(\frac{6}{10}\) = \(\frac{3}{5}\) >1 so you could go to the millions or higher but only 0, 1, 2, 3, 4, and 5 work if the fraction is less than 1.

5. Sample responses:
   
   e.g., \(\frac{1}{2} < \frac{4}{6} < \frac{8}{10} < \frac{1}{2} < \frac{4}{2} > 1 > \frac{8}{9} > \frac{1}{2}\)