PATTERNING TO ALGEBRA
LESSONS
K – 3

Repeating Patterns
Patterns around us
Patterns can be recognized & named

Growing Patterns
Patterns around us
Patterns can be extended & generalized
Patterns can be created & named

Patterns around us
Patterns can be recognized & named
Patterns can be extended & generalized
Patterns can be created & named

K-1
Patterns can be created & named

2-3
Patterns can be created & named
Patterns can be extended & generalized

Research Informed
Patterning to Algebra K - 3

Background:

Three years ago a team of five primary grade teachers from the Kawartha Pine Ridge District School Board, with the support of their district mathematics coaches and a mathematics education research team from Trent University embarked in Collaborative Action Research, a job embedded professional learning model to improve student achievement in mathematics, organized by the Elementary Teachers’ Federation of Ontario. The lessons focused on developing students’ understanding of concepts in the Patterning to Algebra strand and the use of the mathematical processes. This strand warrants time and attention as it has rich connections to Number Sense, Geometry, Measurement and Data Management. The team’s research also examined the impact of intervention lessons on student learning when lessons focused on rich problems and teacher questioning.

They learned that developing a Math Talk Learning Community was key to moving the learning forward for the whole class. Establishing classroom norms and routines involved classroom management, respect of student answers and getting students to talk to one another promoted responsibility to all for the learning. Student ideas were allowed to stand longer, including incorrect or flawed ideas. It was important to explicitly teach students how to listen to one another and how to use strategies such as ‘turn and talk’. ‘Sense Making’ became a daily habit promoted through teacher ‘Questioning’ and examining other’s ideas. Teachers learned the importance of ‘Wait’ time before and after ideas were presented.

As a result of these practices the learning team observed a great deal of growth in the students’ learning and in their ability to justify their reasoning in patterning.

The lessons provided are samples of an instructional trajectory, designed to promote early stages of algebraic reasoning. They are the fundamental lessons which will allow students to analyze and create increasingly complex patterns. It is intended that they be started early in the year and revisited with opportunities in-between and beyond these lessons to practice and deepen understanding at centres and through games, to create, analyze and name a variety of patterns generated by students using a variety of materials (e.g. concrete manipulatives, interactive white board, virtual manipulatives).

For example time in-between lesson 1 and 2 needs to be spent developing an understanding of attributes which comes from the Data Management and Geometry strands.

Future Plans

It is intended that teachers and learning teams use these research-informed lessons, adapting them to their students’ needs. These lessons also provide a potential professional learning opportunity when teaching colleagues test these lessons in their classrooms, and meet to discuss the resulting student work, and future adjustments they would make to the lessons.

Currently there is a Curriculum & Connections document for Patterning to Algebraic Modelling for grades 7-10. A new Curriculum & Connections document is being written to include K-12 materials. It will be posted in the Learning Materials shelf in the Math GAINS library.

Research Informed Key Concepts

- There are repeating and growing patterns all around us- in music, nature, stories and numbers.
- Patterns can be recognized, named, extended and created.
- Repeating patterns have a core identified by the unit of repeat (e.g., AB, ABC, AAB).
- Repeating patterns can be generalized by recognizing pattern families that can look different but have the same core.
- Repeating patterns can be named in different ways depending on which attributes are considered. (e.g., a pattern can belong to 2 families- shape & colour; Data Management)- hyperlink to picture RBY, RBY,… and square, triangle, square, triangle, …)
- Patterns can be created and named using a variety of multi-attribute materials. (ex. shapes, animals, attribute blocks, … but not calendars due to the limitation of completing the core)
- Growing patterns can be represented with numbers or as spatial patterns. (See pg. 35)
- Linear growing patterns have a sequence of elements that increase or decrease systematically when viewed as a recursive pattern using additive reasoning (ex. 2, 4, 6, 8,…; 1, 3, 5, 7,…).
- Linear growing patterns can also be examined using multiplicative reasoning when analyzing the relationship between the position number and its value, which will assist students to generalize the pattern or position rule.

Research Informed Instructional Considerations to Develop Algebraic Thinking in Early Years

<table>
<thead>
<tr>
<th>The teacher should:</th>
<th>It allows students to:</th>
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<tbody>
<tr>
<td>label and sort repeating patterns with letters like AB, ABC, AABB, ABCD, ABBC, etc. into pattern families.</td>
<td>analyze and compare the underlying structure to examine similarities and differences.</td>
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<td>use language of growing pattern and position pattern.</td>
<td>translate from one form to another. e.g. Action pattern (\rightarrow) colour pattern.</td>
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<td>see difference between the two ways of considering growing patterns- additively and multiplicatively</td>
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<td>Use multiple approaches and strategies to teach patterning (e.g. show the core in different ways: hands, string)</td>
<td>Demonstrate their learning in a variety of ways</td>
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<td>Use ‘building mats’.</td>
<td>Develop concept of a term.</td>
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<td>Notice patterning opportunities in other strands</td>
<td>Analyze the relationship between the position number and the number associated with it.</td>
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<td>Make connections in Number Sense for near and far predictions; Geometry when examining attributes, Measurement and Data Management when graphing</td>
<td>To deepen their understanding (John Van de Walle)</td>
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<td>Post anchor charts in the room</td>
<td>Have repeated experiences in patterning</td>
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<td>Make connections to prior learning</td>
<td>Make their learning visible</td>
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<td>Make their learning visible</td>
<td>Build on others’ thinking</td>
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<td>Build on others’ thinking</td>
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**The teacher should**

<table>
<thead>
<tr>
<th>Know the Important Mathematics related to Patterning and Algebra</th>
<th>In order to</th>
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<tr>
<td>Ask probing questions like, “How do you know?” “What do you mean by that?”</td>
<td>Inform</td>
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<td>Use questions to develop a Math Talk Learning Community:</td>
<td>- Teacher questioning</td>
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<td>“Can you repeat or restate in your own words what ‘A’ said?”</td>
<td>- How to respond to student thinking</td>
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<tr>
<td>“Can anyone add to that idea?”</td>
<td>- Lesson planning</td>
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<tr>
<td>“Do you agree or disagree with ‘B’?”</td>
<td>- How to differentiate</td>
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<td>“Do you want to revise your thinking?”</td>
<td>Assess student understanding</td>
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<tr>
<td>Ask ‘Sense Making’ questions: “Does that make sense?” “Does anyone have a different idea?” “Can we take another look at this part?” “How do you know?” “Does everyone/agree?”</td>
<td>Bring clarity to student thinking</td>
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<td>Consider appropriate timing and sequence</td>
<td>Extend student thinking</td>
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<td>Be aware that if a student gives you AAAA… as a repeating pattern you can acknowledge that it does have some characteristics of a repeating pattern</td>
<td>Deepen student understanding</td>
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<tr>
<td>Be aware of the limitations of using the calendar in the early stages of patterning development.</td>
<td>Assess students’ thinking</td>
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**Calendar Math in Preschool and Primary Classrooms, Ethridge & King Vol 3 (5), Early Childhood Education Journal**

Repeat Patterns should be developed before Growing Patterns
- K/1 should spend more time Repeating Patterns
- K/1 needs time to explore using manipulatives for the first time
- Spend time in gr. 2/3 for developing Growing Patterns as relationships which will develop multiplicative thinking

A repeating pattern is a cyclical repetition of an identifiable core.
- AAAA has elements of a repeating pattern such as a core (A) and it has repetitions (more A’s).
- But it is impossible to identify the core compared to any other iteration because it could be an infinite set of A’s as the core rather than just one A. So in that way, it is not a repeating pattern.
- It is also not helpful for young children to think about AAAA as a repeating pattern because there are replication issues, pattern identification issues, and it doesn’t allow children to notice relationships.

Not form early misconceptions. When developed daily the core as a unit of repeat is incomplete.
- Allow for student-centered learning rather than teacher-directed
3. As with number, these concepts of algebra are linked to all areas of mathematics. Much of algebra builds on students’ extensive experiences with number. Algebra also is closely linked to geometry and to data analysis. The ideas of algebra are a major component of the school mathematics curriculum and help to unify it.” (p. 8); Principles and Standards for School Mathematics: An Overview (2000); National Council of Teachers of Mathematics.

5. From their research, the teacher team learned that for Repeating Patterns:
   - AB patterns are a starting point for students and the learning can build from there;
   - when labelling a pattern with letters, any letter is acceptable to use as long as it represents the pattern and is used consistently;
   - the concept of a core is important and can be explicitly taught in a variety of ways (with physical actions, grouping on the Smart Board, underlining, etc.);
   - we need to explicitly teach students how to name patterns in order to help them recognize that there are patterns that have the same structure or core (pattern families);
   - the term ‘pattern families’ is a way to make an abstract concept more accessible to young students and gives them language to use when identifying and describing patterns that are the same;
   - when modeling patterns for students it is important for the core to repeat at least 3 times;
   - the concept of attributes is important and needs to be taught to allow complex patterns to be created, read and named;
   - the sequence of copy, extend, create and translate is a useful scaffold for student learning;
   - using the calendar for teaching patterns can lead to misconceptions about the core (often the month ends before the core of the pattern is complete). Therefore, there are better ways to teach patterning than the calendar.

The teachers learned that for Growing Patterns:
   - the copy, extend, create sequence is useful for growing patterns and is a necessary scaffold for some students;
   - using illustrated picture books and stories, ‘I Know an Old Lady Who Swallowed a Fly’ are a good way to introduce students to and explore growing patterns as well as to show that growing patterns are different from repeating patterns;
   - creating growing pattern cards with simple patterns on them (+1, +2) were a good way to introduce the copy, extend, create sequence. Students were able to lay tiles directly on top of the pattern cards in order to copy the pattern if they needed to;
   - using tag board cards or 8 ½ by 11 sheets of paper as ‘building mats’ was a very useful way to make the abstract concept of a ‘term’ concrete for young students (1 card for each term so the terms are separate and distinct);
   - teaching students to label growing patterns in two different ways; 1) recursive pattern=growing pattern and; 2) relationship between the term number (x-value) and the total number of items (y-value)=position pattern or term pattern, deepens student understanding of growing patterns. It allows students to be able to move beyond just understanding recursive patterns and to be able to predict the values for subsequent terms.
   - using a variety of materials to create growing patterns (commercial materials and ‘found materials such as popsicle sticks) stretches student thinking;
   - students as young as Kindergarten are able to understand complex growing patterns (patterns with a constant);
   - when representing patterns with a constant, it helps to use one colour (e.g., red tile) to show the constant and another colour (e.g., blue tiles) to show that part of the pattern that is changing.

6.
Overall Curriculum Expectations:
- explore, recognize, describe, and create patterns using a variety of materials in different contexts. (K)
- identify, describe, extend, and create repeating patterns. (gr.1)

Connections to Overall Curriculum Expectations in other strands:
Number Sense:
- demonstrate an understanding of number, using concrete materials to explore and investigate counting, quantity, and number relationships. (K)
- read, represent, compare, and order whole numbers to 50 (gr.1)
- demonstrate an understanding of magnitude by counting forward to 100 and backwards from 20. (gr.1) (esp. for near and far predictions)
Geometry:
- describe, sort, classify, and compare two-dimensional shapes and three-dimensional figures, and describe the location and movement of objects through investigation (K)
- identify common two-dimensional shapes and three-dimensional figures and sort and classify them by their attributes. (gr.1)
Data Management:
- sort, classify, and display a variety of concrete objects, collect data, begin to read and describe displays of data (K)
- collect and organize categorical primary data and display the data using concrete graphs and pictographs, without regard to the order of labels on the horizontal axis. (gr.1)

### Kindergarten – Grade 1 Lessons Curriculum Connections

### Kindergarten – Grade 1 Lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
<th>Math Learning Goals</th>
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</thead>
</table>
| 1      | Repeating Patterns Are All Around Us | Students are learning to:  
  - copy a repeating pattern.  
  - extend a given repeating pattern.  
  - read a given pattern.  
  - communicate their understanding that a repeating pattern repeats over and over. 

Students continue to work with repeating patterns.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
<th>Math Learning Goals</th>
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</table>
| 2      | Repeating Pattern Core | Students are learning to:  
  - identify the core in a repeating pattern.  
  - communicate their understanding of the core by highlighting the unit of repeat with string or their hands.  
  - develop reasoning skills when justifying and explaining their choices and thinking. 

Students continue to identify the core in repeating patterns.

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| 3      | Naming Patterns using Letters | Students are learning to:  
  - communicate their understanding of repeating patterns by naming them using letters.  
  - reason about similarities and differences between repeating patterns.  
  - recognize that patterns can look different but belong to the same “pattern family”.  
  - justify their algebraic thinking. 

Students continue to work with patterns that look different but belong to the same “pattern family”.

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<th>Math Learning Goals</th>
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</table>
| 4      | Translating or Re-Presenting a Pattern | Students are learning to:  
  - identify the core of a repeating pattern.  
  - copy and extend a given repeating pattern.  
  - translate or re-present a repeating pattern.  
  - communicate their algebraic thinking. 

Students continue to translate repeating patterns.

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<tr>
<th>Lesson</th>
<th>Title</th>
<th>Math Learning Goals</th>
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</table>
| 5      | Growing Patterns | Students are learning to:  
  - copy a growing pattern of ‘add one’.  
  - extend a given growing pattern of ‘add one’.  
  - communicate their understanding that a growing pattern gets bigger. 

Students continue to work with a variety of repeating and growing patterns.
Before starting Patterning Activities interview your students and note their current patterning abilities.

1. Ask: What is a pattern?


### Observation Checklist

<table>
<thead>
<tr>
<th>Name</th>
<th>1. Patterning Awareness</th>
<th>2. Repeating Pattern Recognition</th>
<th>3. Growing Pattern Recognition</th>
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Lesson 1: Repeating Patterns Are All Around Us

Math Learning Goals
Students are learning to:
- copy a repeating pattern.
- extend a given repeating pattern.
- read a given pattern.
- communicate their understanding that a repeating pattern repeats over and over.

Whole Class → Discussion
Gather students around the carpet or learning area of the room. Create an anchor chart with the students on chart paper entitled “Where Do We Find Patterns?”
Together do the Chicken Dance with the students. Discuss the repeating parts of the dance and the music.
Ask students to identify any new additions to the anchor chart. (e.g., songs, poems etc)
Show the class a simple AB repeating pattern on a pattern card. Pose the question: “How would I copy and then extend this pattern?”
Students discuss with an elbow partner.
They direct how to copy and extend the pattern on the pattern card.

Pairs → Exploration
Pairs of students are given a pattern card based on student readiness. They work co-operatively to copy and extend their pattern.
As they finish they trade in for a new pattern card.
Circulate throughout the groups of pairs, stopping to listen and question the students’ thinking and learning. Encourage the children to read the pattern (oral labelling using the attributes that students are able to use)
Scaffolding/Probing Questions:
Tell me about your repeating pattern.
Along with the student, have student point to the manipulative one at a time and verbalize the attribute (red, blue, Up, down)
What comes next? How did you know?
How do you know it is a repeating pattern?
Can you read it another way?
Would it help you to place objects directly on top of the pattern card?
Curriculum Expectations/Observation/Checklist: Assess and note on BLM 1.1 students’ ability to copy, extend, and read repeating patterns.

Whole Class → Discussion
Select a pair of students’ work involving a simple pattern to have a guided discussion around.
Gather the students around the selected students’ pattern card.
Ask the owners of the work: How did you know what came next? Are the different ways you could read your pattern? What is a repeating pattern?”
Create an anchor chart with the students on chart paper entitled “Where are Repeating Patterns?”
Ask the whole class:
Do you agree with that? Why?
Would you like to add to their thinking?
Can you say what ‘___’ said in your own words?

Home Activity or Further Classroom Consolidation
Teacher places the pattern cards in the classroom math centre for further practice and consolidation.
Students look around their environment at school and at home to find repeating patterns to share with their class the following day. Post new ideas on the anchor chart.
**Samples: Patterns Cards**

AB Repeating Pattern

![AB Repeating Pattern](image1)

ABB Repeating Pattern

![ABB Repeating Pattern](image2)
### BLM 1.1: Observation Checklist

<table>
<thead>
<tr>
<th>Name</th>
<th>Copies</th>
<th>Extends</th>
<th>Simple Pattern Card</th>
<th>Complex Pattern Card</th>
<th>Language used to Read Pattern</th>
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**Lesson 2: Repeating Pattern Core**

### Math Learning Goals

Students are learning to:

- identify the core in a repeating pattern.
- communicate their understanding of the core by highlighting the unit of repeat with string or their hands.
- develop reasoning skills when justifying and explaining their choices and thinking.

### Materials

- pattern cards from Lesson 1
- manipulatives from Lesson 1
- yarn or string
- BLM 2.1

### Minds On…

10 min.

**Think-Pair-Share → Reflection**

Gather students around the anchor charts ‘Where Do We Find Patterns?’ and ‘What are repeating patterns?’.

Pairs discuss new learning from their homework.

Ask if there are new ideas/examples for the anchor charts with a particular focus on repeating patterns. Add new ideas to the anchor charts using a different colour marker.

Point to one of the new repeating patterns on the anchor chart.

Pose the following questions and introduce new vocabulary:

- “Which part of this repeating pattern do you see again and again? Which part repeats? Let’s call this part **the core**. “ Show the core by framing with your hands. (See sample.)
- Display a simple pattern card on the floor, overhead or Interactive White Board.
- Ask for a student volunteer to identify the core by framing it with their hands.
- Ask all the students:
  - “Do you agree? Does that make sense? Is that the core? How do you know? Does anyone else have a different idea?”

Demonstrate the core can also be identified by circling it with string. (See sample.)

### Action!

10 min.

**Pairs → Exploration**

Pairs of students are given a pattern card based on student readiness. They work cooperatively to copy and extend their pattern and circle the core with the string.

As students finish, they trade in for a new pattern card. This can lead to self-differentiation through the choice of new pattern card.

Circulate around the pairs, stopping to listen and question the students’ thinking and learning. Encourage the children to read the pattern (oral labelling using the known attributes). Ask each student to frame the core with their hands.

Scaffolding/Probing Questions:

- Where is the core of your repeating pattern? How do you know it is the core?
- Can you frame the core with your hands?
- How many times does the pattern repeat? Does that core repeat again? Show me.

Curriculum Expectations/Observation/Checklist: Assess students’ understanding of the concept of core in a repeating pattern. Note on BLM 2.1

### Consolidate Debrief

10 min.

- Ensure that students are reading the pattern in a left to right direction.
- Explicitly use the term “core” when talking with students.
- If scaffolding needed, use string to highlight the core with the student.
- Use data to inform which students need further guided practice at the math centre.
- Look for the responses: I used my hands. I used the string. I noticed the part that repeats.
- Concept Attainment: examples & non-examples
- Allow time for students to revise their thinking.

**Whole Class → Discussion**

Gather the students around one of the simple pattern cards investigated by a pair.

Ask them: “How did you find the pattern core?” “What strategy did you use?”

Teacher says: “Now I’m going to show you some other examples. Can you help me make sure I’ve found the core in each of these patterns?”

Use a simple pattern card to show a variety of correct and incorrect cores using the string.

Ask the students: “Do you agree with my math thinking? Is this the core of the pattern? Does it make sense? If you agree, give a thumbs up. If you disagree, give a thumbs down.”

Lead a class discussion around thinking to justify their response.

Ask the whole class: “What can we say about repeating patterns?”

Listen for student language that suggests- “All repeating patterns have a core that repeats. ” Write this statement on the anchor chart.

### Further Classroom Consolidation

Place the pattern cards, manipulatives and string in the math centre for further practice and consolidation.
Samples: Framing the Core using hands or string
# BLM 2.1: Repeating Pattern Core Assessment Notes

<table>
<thead>
<tr>
<th>Name</th>
<th>Frames the Core</th>
<th>Articulates Strategy</th>
<th>Observations</th>
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Lesson 3: Naming Patterns using Letters

### Math Learning Goals

Students are learning to:
- communicate their understanding of repeating patterns by naming them using letters.
- reason about patterns looking different but belonging to the same “pattern family”.
- justify their algebraic thinking.

### Whole Class → Discussion

#### Minds On…
10 min.

Display an AB pattern card and a photocopy of it. Ask a pair of students to identify the core; one is framing on the concrete card and one is circling on the photocopy card. Whole class votes with ‘agree’ or ‘disagree’ with their thumbs.

Ask the class how this pattern can be read. Expect students will name using attributes (e.g., up, down; black, white)

Ask how this pattern could be shown using actions. For a 'black, white' pattern students might 'jump, clap'.

Repeat using a pattern with different attributes but with the same AB structure.

Ask students what they notice.

Introduce using letters as a means to name patterns which is helpful when comparing.

Place a sticky note labelled with A’s under the ‘A elements' on the pattern card. Ask what they notice about all of the A’s.

Place a ‘B’ under the first 'B element' on the pattern card. Ask why not an 'A'? Remove all the sticky notes except the first A and B. Ask for a volunteer to come and label the rest of the pattern with the sticky notes. Together read aloud the pattern two ways- using the attribute and then the letters.

Stresses that the A is always represented by the same element (e.g., the black button in the AB pattern black, white, black, white)

Look for: “We always did the same actions.”; “All the patterns look different but the actions were the same.”

Using letters to represent an element of a repeating pattern allows you to examine the underlying structure of repeating patterns and to compare them.

### Pairs → Exploration

#### Action!
10 min.

Hold up an ABC pattern card and ask: “Would the AB letters work for this pattern?” Pairs brainstorm solutions. Whole class share.

Each student is given a photocopy of a pattern card based on their readiness factor.

Pairs of students with the same pattern card work cooperatively to decide on the pattern core and repetitions. On their own photocopy students circle the core and underline each repetition. They label the core and the repetition using letters. (See sample.) They compare their work. Pairs meet in a group of four to share their patterns’ core and label with each other.

**Curriculum Expectations/Observation/Note:** Listen and question the students’ thinking about the circled core and the naming of the pattern. Place observations on photocopy pattern card. Determine were scaffolding is needed.

Scaffolding/Probing Questions:
- Is the core always repeated?
- How did you label your pattern?
- How can you check that you’ve labelled it accurately.
- Show me your As, are they all the same? What about the Bs, are they all the same?

### Groups of Four → Whole Class → Group Sort

Call on a student with an AB pattern to read their pattern label core aloud. Place their model on the floor or on the blackboard with magnets. Invite other students with this pattern to add to the first model.

Repeat two more times with different pattern families increasing in complexity.

Ask the students: “What do you notice about our sorts? What kind of patterns did we make today? Do all repeating patterns look the same? How do you know?”

Tell the students these are called pattern families because the core is the same in each repeating pattern. The first family is called the AB family (show AB student samples). Repeat with the other two pattern families

Using algebraic symbols and reasoning to compare and contrast repeating patterns.

### Further Classroom Exploration

Place pattern cards, their photocopies, string and sticky notes in the math centre for further exploration over a minimum of two weeks.

Provide blank index cards for creating family names.

S: Some students are given simple AB pattern cards while others are given more complex repeating patterns based on math centre observations.
Sample: Labelling Patterns
### Lesson 4: Translating or Re-Presenting a Pattern

**Math Learning Goals**
Students are learning to
- identify the core of a repeating pattern.
- copy and extend a given repeating pattern.
- translate or re-present a repeating pattern.
- communicate their algebraic thinking.

<table>
<thead>
<tr>
<th><strong>Pair/Share → Activation of Prior Knowledge</strong></th>
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<tbody>
<tr>
<td>Select three AB pattern cards from Lesson 3 sorting activities.</td>
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<tr>
<td>Students turn to a partner and go “Eye to eye, knee to knee” and discuss with their partner what they notice about the As and the Bs in the repeating pattern.</td>
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<tr>
<td>Select one of the pattern cards for the Focus Question: “How many different ways can you show this pattern using the manipulatives at the centre?” Share 2-3 responses.</td>
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</table>

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<thead>
<tr>
<th><strong>Small Group in Centre → Guided Exploration</strong></th>
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</thead>
<tbody>
<tr>
<td>Over several days student groups rotate through the math centre with the teacher until all students have worked in the guided group.</td>
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<tr>
<td>Assign or allow students to select a pattern card at their readiness level. Students copy and extend the pattern using manipulatives.</td>
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<td>Repose the focus question: How else can you show this pattern using the manipulatives at the centre? Students use a different manipulative than given on the pattern card. (See sample.)</td>
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<tr>
<td>Probing and Scaffolding Questions</td>
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<tr>
<td>Can you frame the core with your hands; with string? Where is the repeat?</td>
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<td>How many times does the pattern repeat?</td>
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<td>How can you name your pattern?</td>
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<td>What family does your pattern belong to? How do you know?</td>
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<td>Can you read your pattern another way? (e.g., using the attributes)</td>
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<tr>
<td>Curriculum Expectations/Observation/Checklist: Assess students' understanding of the concept of core in a repeating pattern and ability to translate or re-present. Note data on BLM 4.1</td>
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</tbody>
</table>

| **Mini Consolidation with Small Group** |
| How did you name your patterns? |
| Were the other patterns from the same pattern family or different ones? How do you know? |
| What did you find out about repeating patterns? |

| **Small Group Sharing with Whole Class** |
| At the end of centre time gather the whole class around the students with their patterns and translations. Students share their pattern cards, identify the pattern family and their re-presentations with the whole group. |
| Post their work to develop a Pattern Families anchor chart. |

| **Whole Group → Discussion** |
| Once all students have completed the math centre activity gather them around the Pattern Families anchor chart and pose the questions: |
| What did you find out about repeating patterns? |
| How did you know when they belong to the same pattern family? Does it make sense? |
| What are your pattern names? |

| **Further Classroom Development** |
| Place the pattern cards, string, and manipulatives in the math centre for student to practice and consolidate. Encourage students to explore different and creative translations. Provide opportunities for students to sort new patterns into pattern families and add to the Pattern Families anchor chart. |

**Materials**
- manipulatives (new attributes, eg. keys, big/small paper clips)
- pattern cards
- string
- BLM 4.1

*Students build on a partner’s ideas. Students are translating or re-presenting their pattern into another form. For example a colour pattern: red, blue, red, blue may be translated into circle, square, circle, square, or clap, snap, clap, snap. It is important that the core remains the same.*

> **Alternate representations promote algebraic thinking.**

This is a quick sharing to expose students to different ways to translate patterns.
Sample: Translating or Re-Presenting Patterns
BLM 4.1: Repeating Patterns Assessment Data

<table>
<thead>
<tr>
<th>Name</th>
<th>Frame the core with hands or string</th>
<th>Name the core using attributes</th>
<th>Name the core using letters</th>
<th>Pattern Family Comfort (AB, ABC, AAB)</th>
<th>Translate the core using new manipulatives</th>
<th>Level of Support needed to be successful</th>
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Lesson 5: Growing Patterns

Math Learning Goals

Students are learning to:
- copy a growing pattern of ‘add one’.
- extend a given growing pattern of ‘add one’.
- communicate their understanding that a growing pattern gets bigger.

Whole Class → Read Aloud/Think Aloud/ Discussion

At the learning area or carpet read a poem like “over in the Meadow” or a story that has a growing pattern of ‘one more’. It should be familiar to the students and have been read prior to the lesson. Stop after you read the first part in the poem or story and do a ‘think aloud’: “I think there is a pattern here. I wonder if there’s a way we can record it? I’m going to use this green sticker to show the baby toadie.” Place a green sticker on the chart paper. (See sample.) As you continue to read the poem or book stop at each part and build a growing pattern using the stickers to record each animal; i.e. use 2 yellow stickers for the fishes, 3 blue stickers for the birdies, 4 red stickers for the ratties etc. Start a new vertical column of stickers for each new animal.

Stop reading the poem after the ratties (or 4th term) and ask: “How many stickers do you think I’ll need for the next animal?” Do a ‘thumbs up - agree’, ‘thumbs down - disagree’ survey. Ask: “Does this make sense? How can we check? Have a student volunteer place the predicted number of stickers on the chart. Ask: “What is the pattern?” Students may say it goes up by one, or getting bigger by one. Rephrase this as the pattern is growing by one more or add one.

Continue to build the growing pattern on the chart using the stickers with the class. Stop at the 7th animal and ask “What would happen if there were 10 babies? Assuming the students predict 10, place 10 stickers where the tenth term would go leaving space for the 8th and 9th terms. Ask how to check and once again what is the pattern. Summarize by identifying this as a growing pattern that grew by one more each time.

Small Group in Centre → Guided Exploration

Over several days student groups rotate through the math centre with the teacher until all students have worked in the guided group.

Revisit the growing pattern chart and ask students to recall the pattern type. One at a time, for the first three terms, re-present the sticker pattern using tiles of one colour on ‘building mats’, blank sheets of paper for each term. Each time ask if it makes sense; shows the same number of animals. ?” Repeat for the first three terms. (See sample.) Students use tiles to copy the pattern on their own ‘building mats’. Then they extend the pattern on a blank building mat placed at the end of their copied model. Ask them to predict what would come next and to build it. Repeat two more times.

Curriculum Expectations/Observation/BLM 5.1: Record students’ ability to copy and extend a growing pattern and how they communicate their understanding about growing patterns.

Small Group Mini Consolidation → Discussion

How did you know how many tiles you needed each time? Does that make sense? How do you know? How did you figure it out? Is there a pattern? How is the pattern changing? By how many tiles? How do you know you’ve followed the pattern? How can you show me that you have continued the pattern the right way?

Whole Class → Discussion

Gather the students on the carpet. Display the growing chart with stickers and a student created growing pattern card with tiles.

Ask the students: “What kind of pattern is this? How do you know? What did we learn about growing patterns?”

Home Activity and Further Classroom Explorations

Students copy and extend other ‘add one’ growing patterns using different contexts and manipulatives. Introduce horizontal arrangements “like a snake”.

Students look around their environment at school and at home to find growing patterns to share with their class. Send home a picture of the first three terms of the pattern with a note to parents to “Ask me” what I’ve learned about growing patterns. Ask me what would come next in this pattern. Ask me how I know. Ask me what would come next if we repeated this 5 times?
Samples: ‘Growing by One’

Teacher builds growing pattern on chart paper with students as the story is read.

Teacher creates ‘growing pattern cards’. Elements of the pattern are represented by tiles on ‘building mats’ (blank index cards or sheets of paper).

Student copies the growing pattern.

Teacher adds a ‘building mat’ to prepare for extension of growing pattern.

Student extends the growing pattern by one.
### BLM 5.1: Growing Patterns Observation Checklist

<table>
<thead>
<tr>
<th>Name</th>
<th>Copies</th>
<th>Extends</th>
<th>Language used to Describe Growing Pattern</th>
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<tr>
<td>Grade 2/3 Lessons Curriculum Connections</td>
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<tr>
<td><strong>Overall Curriculum Expectations:</strong></td>
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<tr>
<td>- identify, describe, extend, and create repeating patterns, growing patterns, and shrinking patterns. (gr.2)</td>
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<tr>
<td>- describe, extend, and create a variety of numeric patterns and geometric patterns. (gr.3)</td>
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<td><strong>Connections to Overall Curriculum Expectations in other strands:</strong></td>
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<td><strong>Number Sense:</strong></td>
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<td>- read, represent, compare, and order whole numbers to 100. (gr.2)</td>
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<td>- demonstrate an understanding of magnitude by counting forward to 200 and backwards from 50, using multiples of various numbers as starting points. (gr.2) (esp. for near and far predictions)</td>
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<tr>
<td>- read, represent, compare, and order whole numbers to 1000. (gr.3)</td>
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<tr>
<td>- demonstrate an understanding of magnitude by counting forward and backwards by various numbers and from various starting points. (gr.3) (esp. for near and far predictions)</td>
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<td><strong>Geometry:</strong></td>
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<td>- identify two-dimensional shapes and three-dimensional figures and sort and classify them by their geometric properties. (gr.2)</td>
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<tr>
<td>- compose and decompose two-dimensional shapes and three-dimensional figures. (gr.2)</td>
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<td>- compare two-dimensional shapes and three-dimensional figures and sort them by their geometric properties. (gr.3)</td>
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<tr>
<td>- describe relationships between two-dimensional shapes, and between two-dimensional shapes and three-dimensional figures. (gr.3)</td>
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<td><strong>Data Management:</strong></td>
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<td>- when graphing collect and organize categorical or discrete primary data and display the data, using tally charts, concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers, with labels ordered appropriately along horizontal axes, as needed. (gr.2)</td>
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<tr>
<td>- read and describe primary data presented in tally charts, concrete graphs, pictographs, line plots, simple bar graphs, and other graphic organizers. (gr.2)</td>
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<tr>
<td>- collect and organize categorical or discrete primary data and display the data using charts and graphs, including vertical and horizontal bar graphs, with labels ordered appropriately along horizontal axes, as needed. (gr.3)</td>
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<tr>
<td>- read, describe, and interpret primary data presented in charts and graphs, including vertical and horizontal bar graphs. (gr.3)</td>
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</tbody>
</table>
## Grade 2/3 Lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
<th>Math Learning Goals</th>
</tr>
</thead>
</table>
| 1      | Repeating Patterns Around Us   | - recognize a repeating pattern.  
- identify the core in a repeating pattern by highlighting the unit of repeat.  
- extend a given repeating pattern.  
- communicate their understanding of the core in a repeating pattern.  
- represent their thinking by extending a given pattern using manipulatives.  
Students have additional opportunities to explore, extend and create repeating patterns |
| 2      | Labelling Pattern Families     | - represent patterns by labelling using symbols (e.g., letters).  
- recognize the underlying structure in a repeating pattern, i.e. there is repetition as identified by the core (unit of repeat).  
- sort patterns with the same core into pattern families.  
- make sense of different patterns that have the same core.  
Students continue to explore, create and solve problems involving repeating patterns.  
e.g. The core in a repeating pattern is ○ ○ □. If there are 10 □’s how many ○’s will there be? |
| 3      | Patterns Can Belong to More Than One Family | - understand that patterns can be created using 2 or more attributes/properties.  
- understand that patterns can belong to more than one ‘family’.  
- represent patterns in different ways (with concrete materials, with letter symbols).  
- compare similarities and differences in a variety of patterns.  
- justify their reasoning about repeating patterns.  
Students continue work with attributes by playing “Changing Attributes Game”, BLM 3.2, and playing with Attribute Trains from the National Library of Virtual Manipulatives.  
They also continue to explore, create and solve problems involving more complex repeating patterns.  
e.g. What is the 21st element in this repeating pattern? ■ □ □ ○ ■ □ ○ □ ●  
Work with skip counting and multiplication needs to proceed lesson 4. |
| 4      | Growing Patterns              | - articulate the similarities and differences between repeating and growing patterns.  
- analyze and name the pattern rule in growing patterns.  
- use position numbers in growing patterns to indicate each term.  
- communicate their understanding of a growing pattern.  
Students have many opportunities to build and expand growing patterns at centres. Include patterns that involve spatial representations. |
| 5      | Two Ways to Analyze and Name Growing Patterns | - represent patterns using a ratio table.  
- examine growing patterns in more than one way- additively and multiplicatively.  
- communicate about pattern rules.  
- justify the pattern rule.  
Students have many opportunities to explore, create and solve context based problems involving growing patterns, including growing patterns with a constant, at their readiness level (DI).  
e.g. Solve the one of the ‘Chairs Around the Table’ problems.  
When using activities that have a constant part, show the constant part in one colour and the growing part in another colour. |
| 6      | Creating and Representing Growing Patterns | - extend a growing pattern.  
- identify growing pattern rules.  
- represent growing patterns using ratio tables.  
- create a growing pattern.  
- justify their reasoning for near and far predictions.  
Students continue to work with a variety of repeating and growing patterns. They are learning to represent their patterns using T-charts, ratio tables, graphs, pictures, 100’s chart, number line, input/output machines. |
Before starting Patterning Activities interview your students and note their current patterning abilities.

1. Ask: “What is a pattern?”

2.-4. For each pattern lay the tiles out. Ask: “What comes next? How do you know?”

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Observation Checklist

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<tr>
<th>Name</th>
<th>1. Patterning Awareness</th>
<th>2. Repeating Pattern Recognition</th>
<th>3.&amp;4 Growing Pattern Recognition</th>
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Math GAINS  Collaborative Action Research: Patterning to Algebra K - 3  July 2011  24/41
Lesson 1: There are Repeating Patterns All Around Us

Math Learning Goals
Students are learning to:
• recognize a repeating pattern.
• identify the core in a repeating pattern by highlighting the unit of repeat.
• extend a given repeating pattern.
• communicate their understanding of the core in a repeating pattern.
• represent their thinking by extending a given pattern using manipulatives.

Materials
• construction paper
• variety of stickers
• variety of manipulatives commonly found at home
• straws
• chart paper
• sticky notes
• tape

Pairs → Whole Group → Discussion
Pose the question: “Where can patterns be seen?” Elbow partners discuss. Ask for and record student responses on a board or chart paper.

Pose the question: “What is a repeating pattern?” Elbow partners record their ideas on sticky notes and post on a large teacher-created Frayer model in the facts/characteristics section. (Sample 1.1-1.2)

Pairs → Analyze
Elbow partners are given a ‘Pattern Card’ – a card containing an example or non-example of a repeating pattern. Elbow partners work to decide if their card is a repeating pattern or not. Students tape their card on Frayer model chart paper under Example or Non-Example. (Sample 1.3)

Pairs → Investigation
With a partner, students select from a variety of manipulatives to create 3 repetitions of a repeating pattern. Students show they are finished by placing a straw at the end of the 3rd repetition. (Sample 1.5)

Probing Questions:
Tell me about your pattern. How do you know you have made a pattern? How do you know you’ve repeated your pattern 3 times? Can you say your pattern for me? Can you prove that this is a pattern?
Scaffolding Questions:
I’m not sure this part follows your pattern (underline problem area with your finger). How could you check?
Take a look at _____’s pattern. Can you make your pattern look like that?

Whole Group → Teacher Guided Gallery Walk
Students take a gallery walk looking at the variety of patterns that were created. Teacher may ask some of the following probing Questions:
What do you notice about these patterns?
Are there any similarities?
How do we know these are patterns?
Do you see any patterns that are the same as yours?

Pairs → Apply Knowledge
Students change places with another pair and extend their pattern 2 more times. (Sample 1.6)
Students come together in groups of 4 (original creators and extenders) to analyze the extension of both patterns and revise if needed.

Curriculum Expectations/Observation/Note: Listen to students’ understanding of ‘core’ in a repeating pattern.

Possible manipulatives – stickers, buttons, stir sticks, tiles,… Note: Do not use attribute blocks until later lessons.

Whole Group → Discussion
Ask the class: “What did you need to know to extend the pattern?” Introduce the term ‘core’ to name the unit that repeats. Post on a word wall. Revisit the Whole-Class Frayer model for repeating patterns with students. Together revise the placement of the ‘examples’ and ‘non-examples’ cards where needed. Together analyze the sticky notes in ‘Characteristics’ box to determine agreement and pile similar ones. Form a class definition of a repeating pattern. (Sample 1.4)

Individual → Reflection
Students write in their math journal what the term ‘core’ means in a repeating pattern.

Home Activity or Further Classroom Consolidation
Students look for examples and non-examples of repeating patterns in their home and environment to share in next day’s lesson.
Samples

1.1 Frayer Model - Repeating Patterns

1.2 Repeating Patterns Characteristics

1.3 Frayer Model with Examples & Non-Examples

1.4 Completed Class Frayer Model

1.5 RYB pattern repeated 4 times

1.6 Pattern extended 2 more times
Lesson 2: Labelling Pattern Families

**Math Learning Goals**

Students are learning to:
- represent patterns by labelling using symbols (e.g., letters).
- recognize the underlying structure in a repeating pattern, i.e. there is repetition as identified by the core (unit of repeat).
- sort patterns with the same core into pattern families.
- make sense of different patterns that have the same core.

**Materials**
- variety of manipulatives
- letter cards or stickies
- paper strips
- chart paper
- masking tape
- Frayer Chart- Lesson 1
- BLM 2.1

**Whole Group → Activate Prior Knowledge**

Minds On...
15 min.

**Whole Group → Individual → Exploration**

Action!
30 min.

**Whole Group → Sorting → Discussion**

Consolidate Debrief
30 min.

**MTLC →**

**S?**

**Reflection**

Home Activity

Students take home their section of BLM 2.1 to reflect and share their learning about pattern families. They return their signed slip of paper the next day.
Sample: Pattern Families Anchor Chart
Today we were learning about pattern families in math. Ask me to tell you about what I learned about pattern families.

Signature of person I shared my learning with

Comments:
Lesson 3: Patterns Can Belong to More Than One Family

Math Learning Goals
Students are learning to:

- understand that patterns can be created using 2 or more attributes/properties
- understand that patterns can belong to more than one ‘family’
- represent patterns in different ways (with concrete materials, with letter symbols)
- compare similarities and differences in a variety of patterns
- justify their reasoning about repeating patterns

Materials
- Attribute blocks
- BLM 3.1, 3.2
- Chart paper

75 min

Whole Group ➔ Discussion

Minds On…
15 min

Create an anchor chart to record the attributes and properties from recent patterning lessons (e.g., colour, size, number of sides). Hand out an attribute block to every student.

Say: “Look at your attribute block, then turn to an elbow partner and describe your attribute block by identifying all of its different attributes and properties. “

“Find someone whose shape differs by one attribute.”

“Now find someone whose shape differs by two attributes.”

Listen to students’ conversations. Selects a few students to justify their thinking.

Review attributes and properties.

Ontario Curriculum, 1-8;
Attributes- p. 120
Properties- p.131

Cut BLM 3.1 into cards.

Action!
30 min

Whole Group ➔ Demonstration/Discussion

Gather the students in a circle. Create a pattern on the floor using attribute blocks that can fit into two different pattern families.

Probing Questions:

Is this a pattern? How do you know?
What pattern family does this pattern belong to?
How might we show that this pattern belongs to both pattern families depending on what attributes or properties we look at?

Show the pattern rule cards from BLM 3.1. Place the AB and Colour pattern rule cards and the ABC and Shape pattern rule cards under the pattern to show how this pattern can be viewed in two different ways.

Pairs ➔ Investigation

Create similar ability partners to create a pattern that can belong to two pattern families. Partners choose four cards appropriate to their readiness level- two Pattern Family Names and two Pattern Rules. They choose manipulatives and work together to create a pattern that follows both rules and label appropriately.

Curriculum Expectations/Photo/Note: Take photos of student patterns. Note level of student understanding of pattern families; where scaffolding and challenge is needed.

Select three specific partnerships to share their patterns, looking for accuracy and varying levels of complexity.

Consolidate
Debrief
30 min

Whole Group ➔ Teacher Guided Walk

Gather students to the first selected work; the simplest pattern created.

Ask students if they agree or disagree that the pattern matches the pattern family and rule cards. Students turn to an elbow to a partner and share their thinking about the pattern and cards. Remind them that the pattern has to belong to two ‘families’ and .match all the cards.

Repeat this process for two additional selected student patterns, increasing the level of complexity each time.

Whole Group ➔ Discussion

Lead a discussion to articulate the important pattern family learning from today; i.e. A pattern can belong to more than one family depending on which attributes are the focus. Patterns can look different but belong to the same family. Post the class summary on a strip of paper.

MTLC ➔

Reflection
Concept
Practice

Further Classroom Consolidation

In their math journal students create a Venn diagram to compare shapes based on attributes. Over the next week students continue to consolidate their understanding about attributes and properties using the pattern family name and rule cards to create more complex patterns i.e. they belong to more than one family (See samples). Students investigate: Is it possible to create a pattern that belongs to more than two families?

See sample.

Student play games for further practice:
- BLM 3.2
- Attribute Trains from the National Library of Virtual Manipulatives.
Samples: Two Pattern Family Rules

AB pattern: Shape - Square, Rectangle

ABC pattern: Colour - Blue, Red, Yellow
Sample: Venn Diagram for Attributes
BLM 3.1: Two Pattern Family Rules

<table>
<thead>
<tr>
<th>Pattern Family Names</th>
<th>Pattern Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>Shape</td>
</tr>
<tr>
<td>ABC</td>
<td>Direction</td>
</tr>
<tr>
<td>ABCD</td>
<td>Thickness</td>
</tr>
<tr>
<td>AAB</td>
<td>Size</td>
</tr>
<tr>
<td>ABB</td>
<td>Colour</td>
</tr>
<tr>
<td>ABBA</td>
<td>Shape</td>
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<tr>
<td>AABB</td>
<td>Direction</td>
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<tr>
<td>AABC</td>
<td>Thickness</td>
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<td>ABAC</td>
<td>Size</td>
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<td>ABCB</td>
<td>Colour</td>
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<td>ABAC</td>
<td>Size</td>
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<tr>
<td>ABCB</td>
<td>Colour</td>
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</tbody>
</table>
**BLM 3.2: Changing Attributes Game**
Adapted from Nelson Mathematics Grade 3 by Kestell/Super/Kelleher, © 2004. Used with permission.

**Number of players:** 2 - 4

**Materials:** Attribute blocks
                Game board- 10x10 grid

**Step 1** Each player takes an equal amount of attribute blocks.

**Step 2** The first player begins by putting an attribute block anywhere on the game board.

**Step 3** Players take turns placing a block beside, above, or below any block on the board. The new block must have only one attribute (i.e. colour, size, shape, thickness) that is different from a neighbouring block. Players who can not “go” lose their turn.

**Step 4** Play until a player runs out of blocks. If no one can “go”, the player with the least number of blocks left wins.

**Variation:** Instead of having just one attribute change, try with two or even three attributes changing.
Lesson 4: Growing Patterns

**Math Learning Goals**
Students are learning to:
- articulate the similarities and differences between repeating and growing patterns.
- analyze and name the pattern rule in growing patterns.
- use position numbers in growing patterns to indicate each term.
- communicate their understanding of a growing pattern.

**Whole Class/Pairs → Discussion**
Read a story or rhyme that has a ‘growing by one’ pattern such as ‘There Was an Old Lady Who Swallowed a Fly’. Read the first few pages to introduce students to the growing pattern in the story. Ask the students what they notice. They may say something like, “It’s a pattern.”; “It is getting bigger each time.”, etc.

Using colour tiles ask the students how they could be used to show what they are noticing. With student assistance build the first three terms of the pattern showing a vertical step growth. See sample: term 1-1 tile, term 2-2 tiles, term 3-3 tiles.

Ask how showing the pattern in this way is helpful. Students might say, “It shows the pattern getting bigger. It shows the pattern is going up by one each time. It helps us to predict what comes next.” Ask the students to predict what will happen next on our chart. Read the next page to see if prediction was correct. Repeat for a few pages/terms to allow the students to see the growth in the pattern.

Ask the students: “How did you know how many would come next? What will our pattern rule be? Will it always be true? How do you know? ‘Turn and Talk’ with your elbow partner.” Students will use informal language at this time. e.g., ‘a plus 1’ pattern, a ‘grows by 1’ pattern.

Ask the students how this is the same/different from previous patterns they’ve seen.

**Whole Group → Demonstration**
Introduce and post the growing pattern shown on the board. Introduce the terminology ‘building mats’ and ‘position number’ as you place sticky notes with the numbers 1, 2, and 3 under each building mat.

**Clock Partners → Investigation**
Tell students: “Work with your ‘six o’clock partner. Using manipulatives of your choice, copy the pattern from the board onto building mats to create the pattern. Use your sticky notes to label each position or step in the pattern.

With your partner analyze and discuss the pattern. Determine a pattern rule. Use your rule to build the next two positions of this pattern on two new building mats.

**Consolidate Debrief → Teacher Guided Gallery Walk**
Select a few, varied, student work samples to discuss.

Probing Questions: What do you notice about the patterns? What is the same? What is different? Which arrangements made it easier to see a pattern rule?

Lead a discussion to compare repeating and growing patterns. Namely, in repeating patterns the focus is on the attributes/properties used to make the core, which repeats. Whereas in growing patterns the focus is on the repeated action of adding the same amount each time. With the students begin creating a Frayer Model chart on Growing Patterns. Post examples and characteristics from today’s activities. Add more details in subsequent lessons.

**Home Activity or Further Classroom Consolidation**
In their math journal students show an example of a repeating pattern and an example of a growing pattern. They create a Venn diagram to compare the similarities and differences between them. Students have many opportunities to build and expand growing patterns at centres. Include patterns that involve spatial representations. (See samples.)
**Samples**
Growing by 1 Patterns

Both arrangements below show the growing pattern of 5, 10, 15, but their arrangements are different. Which arrangement makes the pattern easier to see?

More examples of growing patterns involving spatial representations:
Lesson 5: Two Ways to Analyze and Name Growing Patterns

**Math Learning Goals**
Students will learn to:
- represent patterns using a ratio table.
- examine growing patterns in more than one way—additively and multiplicatively.
- communicate about pattern rules.
- justify the pattern rule.

**Materials**
- chart paper
- manipulatives
- BLM 5.1
- BLM 5.2 (teacher notes)
- paper and sticky notes
- Frayer Model—Lesson 4

**Pairs ➔ Discussion**
Students are gathered at the meeting area. Use tiles to create a doubling pattern on chart paper. On sticky notes write the term numbers and post below each term. (See sample.)

Elbow partners analyze and discuss how the pattern is growing and what might come next in this growing pattern.

Place the numbers into a ratio table labelled position number and number of objects. (See sample.) Ask students to find as many patterns in the table as they can. (See possibilities given.)

When a student identifies the ‘adding 2’ pattern between successive terms, highlight that this is called a ‘Growing Pattern’. When a student notices the doubling pattern between the position number and its number of objects, introduce the name for this as the ‘Position Pattern’ (Elizabeth Warren).

**Small Groups ➔ Investigation**
Distribute BLM 5.1 to students. They choose one of the patterns to investigate and form small groups working on the same pattern. Using manipulatives of their choice, students build the first three terms of their selected pattern using building mats to create each term value and sticky notes to create position cards labelled 1, 2, 3.

Tell the students after they have built their first three terms on their building mats to analyze the pattern and make a ‘far’ prediction for terms 8 and 16”.

Probing/Scaffolding Questions:
- What pattern have you found? Have you figured out the growing pattern rule?
- Does finding the growing pattern help you to make the next term? Make a ‘far’ prediction?
- What is the position pattern rule? (If needed, point to a position number and its number of objects.)
- Does finding the position pattern rule help you to make the next term? Make a ‘far’ prediction?
- Curriculum Expectations/Observation/Note: Listen to and probe the students’ thinking about the strategies they use to determine the values in their table. (e.g. using manipulatives always, sometimes using the numbers—thinking additively and/or multiplicatively.)

As students work look for 3-4 groups to share their strategies for making far predictions in the consolidation part of the lesson.

**Whole Group ➔ Discussion**
On chart paper or document camera, groups share their strategies for making ‘far’ predictions. Use probing/scaffolding questions given in the Action part of lesson to encourage students to notice that determining the position rule is an efficient way to make ‘far’ predictions.

With the class determine what new information to add about analyzing patterns to the ‘Characteristics’ section of the Growing Patterns Frayer Model chart.

**Differentiated Exploration**
Ongoing centres with growing patterns/ratio table opportunities should take place before Lesson 6.

Most students will notice the recursive (additive) pattern—looking to the right in the ratio table. Look for students who may notice that there is a multiplicative relationship between the position number and the number of objects.

To encourage the relational thinking point to a position number and its number of objects.

Highlight that there are two ways to analyze and name pattern rules—the ‘growing’ pattern (additive) and the ‘position pattern’ (multiplicative).

To learn more:
http://www.emis.de/proceedings/PME30/5/377.pdf

A Position Pattern Rules encourage Algebraic Thinking

**Additional activities:**
CLIPS—Representations of Linear Growing Patterns
Sample - Doubling Growing Pattern with Ratio Table

Some possible patterns:

Adding 2:

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of tiles</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Doubling within the table (position number):

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of tiles</td>
<td>$\frac{1}{2}$</td>
<td>$1 \times 2$</td>
<td>$2 \times 2$</td>
<td>$4 \times 2$</td>
</tr>
</tbody>
</table>

Doubling between terms and their values:

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># of tiles</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td></td>
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</tbody>
</table>

Additive or Recursive Thinking

Multiplicative Thinking

Multiplicative Thinking
### Ratio Tables

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>8</th>
<th>16</th>
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</thead>
<tbody>
<tr>
<td>Number of objects</td>
<td>5</td>
<td>10</td>
<td>15</td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>8</th>
<th>16</th>
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</thead>
<tbody>
<tr>
<td>Number of objects</td>
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<td>20</td>
<td>30</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of objects</td>
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<td>8</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>8</th>
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<tbody>
<tr>
<td>Number of objects</td>
<td>8</td>
<td>16</td>
<td>24</td>
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</tbody>
</table>
**Lesson 6: Creating and Representing Growing Patterns**

**Math Learning Goals**
Students are learning to
- extend a growing pattern.
- identify growing pattern rules.
- represent growing patterns using ratio tables.
- create a growing pattern.
- justify their reasoning for near and far predictions.

**Materials**
- chart paper,
- document camera or
- Interactive White Board
- manipulatives
- paper
- sticky notes

### 90 min

#### Minds On...  
**Investigation**
Display the following ratio table on chart paper, document camera or interactive white board. Tell students the elements in the table come from a growing pattern. In small groups students determine the missing elements.

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>90</td>
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</tr>
</tbody>
</table>

#### Whole Class → Discussion
Once groups are finished elicit solutions for the first blank in the table. Use thumbs up/down to check for agreement from the group.

Probing questions:
- What strategy did you use to find the missing value?
- How do you know you have the correct number?
- Did someone use a different strategy?

Continue this process for the remaining blanks in the table.

#### 20 min

#### Partners → Problem Solving
Students work in similar ability pairs to create their own growing pattern. They select their manipulatives and create the first three terms of their growing pattern on building mats. (See samples.) They must include labels for the position numbers and their position pattern rule.

Students create a ratio table showing the first 3 terms, room for additional entries and a ‘far’ prediction. They prepare to potentially share their work in a Math Congress.

Curriculum Expectations/Interview/Note: Individually ask students to justify their position rule and their far prediction.

Select work that will be presented in the Math Congress.

#### 40 min

#### Whole Class → Congress
Selected partners present their math findings to their peers.

Focus question: What strategies are helpful in making ‘far’ predictions?

#### 30 min

#### Reflection

**Further Individual Consolidation**
Students write in their Math Journal what they have learned about growing patterns and making ‘far’ predictions.
Samples: Growing Pattern

Student A

Notice that both of the above patterns show a growing pattern of 5, 10, 15, but they look different. What would you say to Student B?

Student C

What would you say to Student C?

Student D

What would you say to Student D?