

Teaching through Problem Solving, the Three-Part Lesson Facilitators' Guide

Introduction

This facilitator's guide is intended for use by teachers, consultants, and coordinators who are working with a group of teachers who are engaged in job-embedded learning and who are interested in further developing their mathematics practice.

Organization of the Video

The video is organized into 5 sections:

1. Determining Starting Points for Instruction

This section of the video shows how one teacher assesses where his students are and determines what needs to be done so all of his students can access the problem and can experience success.

2. Teaching through Problem Solving: The Three-Part Lesson

The three-part lesson is a framework for teaching through problem solving. This section of the video is organized into three viewing sections: beginning, middle, and end.

3. Continuing the Congress: Pursuing Understanding

Often a three-part lesson takes place over two or three days. On the third day, the teacher continues the conversation with his students and goes deeply into one group's poster. During the discussion, confusions are revealed, leading to greater insight by the students.

4. Classroom Management

Videos are edited with a specific focus in mind. This focus can lead teachers to feel like the class is too perfect or well-behaved. This section of the video highlights a series of classroom management strategies and is designed to support discussion about the importance of creating a positive classroom environment that includes student accountability.

5. Student Voice

Two Grade 9 students share insights into how teaching through problem solving supported their engagement in and understanding of mathematics.

Facilitator's Stance When Viewing and Analyzing Classroom Video

- The video is a gift from this teacher and his students. It takes great courage to invite us into the classroom and we are thankful that Mr. Perry opened his classroom doors so that we can benefit from his journey.
- The video allows us to take a peek inside a classroom and inside the mind of the teacher while he goes about the business of teaching math.
- The video is intended as a platform for supporting in-depth discussion about effective instruction in mathematics.

Big Ideas Embedded in Video

- Teaching is a combination of **'what'** we teach (the mathematics content) and **'how'** we choose to teach that content (the instructional strategies/pedagogy we use to effectively involve students in constructing an understanding of the **'what'** or the content).
- This video addresses the concepts of volume and surface area and the relationship between volume and surface area (the what). The video also outlines high-yield strategies for teaching mathematics: the three-part lesson framework for teaching through problem solving and effective questioning (the how). This facilitator's guide highlights the role of the teacher and the role of the student within the three-part lesson framework.
- Discussions involving assessment for learning and differentiated instruction (DI) also emerge.
- At the heart of this video are two important questions: 'what does it mean to teach mathematics' and 'what does it mean for students to construct knowledge.'

Classroom Environment

- The video captures a Grade 9 Academic mathematics class with an experienced teacher of Grade 9 mathematics in a large secondary school where 30% of students are identified.
- Teaching through problem solving is the foundation of Mr. Perry's classroom instruction.
- Students work in cooperative learning groups all semester, with group members rotating/changing at the beginning of every month.
- The classroom was filmed over three days at the end of May
- On first day of filming, one-third of the class was on field-trips or sporting events, so the teacher restructured the cooperative learning groups.

Inservice Session (Time: 3-4 hours)

Session Goals

Academic Goals

- To develop an in-depth understanding of the mathematics content - volume and surface area - and to explore the relationship between volume and surface area
- To construct an understanding of the developmental progression of concepts related to volume and surface area
- To understand the continuum of expectations from Grades 4 to 10 involving volume and surface area
- To explore high-yield strategies to improve student achievement: the three-part lesson framework for teaching through problem solving, and the use of effective questioning to probe and guide student thinking

Social Goal

- To build strong working relationships among and between members of our mathematics community

Setting Norms for Working Together

- Start and end on time.
- Respond to the signal.
- Each person gets the chance to speak and listen.
- When one person is speaking all turn and face that person.
- Participants direct their discussion to the whole group, not the facilitator.
- Invest in your own learning and the learning of others.
- Contribute to a safe environment that encourages risk taking; be kind.
- Think and act like mathematicians.

Notes

These academic and social goals are suggestions and should be modified to meet the audience with which the facilitator is working. For example, if the group is cross-panel then the social goal could be 'to build cross-panel relationships.'

Notes

It is important for a facilitator to set norms for working together to create a sense of community and a feeling of safety so that all participants have a voice and understand that their perspective is valued.

Norms ensure that one voice does not dominate a discussion, and they support the facilitator in ensuring the discussion is productive and on-task.

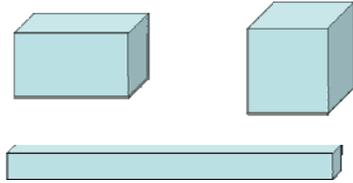
Norms for working together are best constructed together as a community of learners. However, if time does not permit creating them together these norms could be used.

The Optimization Problem (Part 1)

Purpose

To support participants in gaining greater insight into the variety of solutions students will generate when solving a problem.

If you use 64 blocks, which rectangular prism would have the smallest surface area?



Instructions

- Solve the problem in as many ways as you can.
- Spend 2-5 minutes solving the problem individually as your students would solve it.
- Share their solution/solutions with your table group.

The Optimization Problem (Part 2): Developmental Progression of Strategies Involving Volume and Surface Area

Purpose

- To increase participants understanding of the variety of solutions students will generate
- To develop an understanding of the development of conceptual understanding involving volume and surface area

Instructions

- Order your group's solutions developmentally on a ranking ladder.
- Post your ranking ladder for a full group gallery walk.

Notes

Participants work in pairs or in groups of three or four for maximum participation and 'buy in.'

Provide three rectangular prisms (made with 64 blocks each) for each group.

Solving the problem as their students would solve it is a way of acknowledging the insight many teachers have into the struggles their students may have with the problem.

Notes

Participants choose a variety of ways of representing the strategies and generate 6-10 different ways of solving the problem that can be ranked developmentally from least sophisticated to most sophisticated. Groups engage in rich discussion to determine (and reach consensus) about which strategy is higher or lower on the ranking ladder.

Continuum of Concepts

Goals

- To develop an understanding of the continuum of concepts related to volume and surface area, Grades 4-10.
- To identify and synthesize the big ideas contained within the curriculum expectations that address volume and surface area.

Purpose

To support teachers as they construct an understanding about where the gaps in understanding may lie with their students and possible ways to fill gaps.

Instructions

- As a table group analyse, and record the curriculum expectations that address volume and surface area from Grade 4 to Grade 10.
- Organize the expectations according to the big ideas addressed in each grade and create a graphic organizer to illustrate your findings.

Additional Discussion Items

- Identify and discuss any concerns teachers might have about the big ideas and discuss how gaps in understanding might be addressed.
- What beginning concepts are related to volume and surface area?
- What is the difference between volume and capacity?
- At what grade do students start to construct an understanding of the difference between capacity and volume?

Notes

For instruction to work seamlessly within the continuum of learning, we need an understanding of what comes before the grade and what comes in subsequent grades.

Teachers should not simply copy the expectations; rather they should make sense of them in relation to one another.

Determining Starting Points for Instruction

- Read the question from each section aloud so participants are aware of the question they are addressing with this section of the video.
- Discuss each part of the video before proceeding to the next.
- Participants can record their observations and then discuss the role of the teacher and the role of the student in each of the three parts. See Determining Starting Points for Instruction Organizer, p. 12.

Guiding Questions	Notes
<p>Section 1: What are Shawn's goals or end points when planning his lesson? What assumptions does he make?</p>	<p>He is addressing the relationship between volume and surface area, with an end goal of looking at optimization. Assumptions:</p> <ul style="list-style-type: none"> • He assumes students will bring an understanding of volume and surface area. • He assumes students will know the vocabulary/meaning of volume and surface area. • He assumes they will all have access to his beginning diagnostic activity and that demonstration prisms will be enough support.
<p>Section 2: Listen carefully to Kat and Dylan's statements. How does the teacher respond to their statements?</p>	<p>He responds to Kat and Dylan's statements by repeating them, clarifying the statements, and then asking the class to think about what each statement means and decide if they agree or disagree. He also gives each group time to talk and think together. The teacher does not tell Kat and Dylan if they are right or wrong. Why? What is his purpose?</p>
<p>Section 3: Focus on the group of boys the teacher is working with. What do the students know and what are they able to do? Is their conversation surprising to you in any way?</p>	<ul style="list-style-type: none"> • I was surprised by how they struggled with the mathematical language. • I was surprised when the boy said he 'counted' – I thought they would have multiplied. • I was surprised when they said they needed to have the next rectangular prism to touch and count, in order to find out the surface area. They weren't able to generalize what they had learned with the cube. • Clear that all students need the manipulatives in their hands.
<p>Section 4: Record your insights and observations when watching Shawn's reflection.</p>	<p>The teacher realizes that there are gaps in conceptual understanding among his students regarding volume and surface area. What he thinks will be a quick 'accessing prior knowledge' and review, turns out to be an in-depth discussion which brings to the surface confusions that students possess. He is able to adjust the lesson quickly to meet the needs of his students and to make sure the problem he is about to present, is accessible to every one. (puts manipulatives into the hands of every student)</p>

Notes

"Determining Starting Points for Instruction" has several focusing questions for viewing the first part of the video. It is recommended that the first section of the video be viewed in 4 sections, stopping where indicated:

Section 1: the teacher speaking about the goals for his lesson (stop when the video fades to black)

Section 2: the students' initial thoughts about volume and surface area (stop just after the teacher restates Kat's and Dylan's reasoning)

Section 3: the teacher is working with a group of boys (stop when the video fades to black)

Section 4: the teacher is reflecting upon his lesson

Notes

When the teacher takes the statements that Kit and Dylan propose and presents them for the whole class to discuss, he shows that he honours their ideas. The teacher's actions demonstrate that he feels the students' ideas are worthy for the whole class to discuss and ponder. It takes skill and in-depth understanding by the teacher to create a lesson from the ideas that emerge out of student discussion and reflection. This is powerful and motivating for the students as well.

Teaching Through Problem Solving: the Three-Part Lesson

Purpose

To analyse a high-yield instructional strategy and determine the role of the teacher and the role of the student within the three-part lesson framework.

Instructions

- View the section of the video entitled: Teaching Through Problem Solving: the Three-Part Lesson
- Participants letter themselves as A and B. (lettered heads) Person A records the role of the student while watching the video. Person B records the role of the teacher while watching the video.
- Use evidence from the video to support your thoughts, ideas, and observations and to keep the conversation on track.

Three-Part Lesson: Beginning

- View the beginning and stop the video.
- Participants record their observations (1 minute).
- Participants share their observations with their partner.
- With the full group, establish consensus about roles in the beginning part of the lesson and the importance of the beginning part of the lesson.
- Record on chart paper for reference at the end of the session.

	Student Role	Teacher Roles
Beginning	<ul style="list-style-type: none">• To ask questions about the problem and seek clarification• To engage in private solving time	<ul style="list-style-type: none">• To present the problem• To leave the problem open ended• To ensure the problem is accessible to all students• To provide manipulatives for all students appropriate for solving the problem

Notes

While watching the video, participants record what they observe the student saying and doing or what the teacher is saying and doing. They use evidence from the video to support their thoughts, ideas, and observations. This helps keep the conversation on track.

Additional Guiding Questions:

- What does the teacher say as he introduces the lesson?
- Equally important is what does he not say?
- Why does the teacher give the students private solving time?
 - *Some people think quickly and can begin to solve a problem immediately. Some of us need time to think and ponder before we solve a problem. Private solving time gives all students time to think before being expected to share with a partner or a group.*
 - *Assessment: Private solving time also allows the teacher to circulate and see if anyone is struggling or doesn't have a way in to the problem. He can probe to gain insight into how the student is thinking and can use guiding questions to help the student get started. At no time is it appropriate to 'tell' the student what to do.*

Three-Part Lesson: Middle

- View the middle part of the lesson and stop the video.
- Participants record their observations and share with a partner.
- As a full group, reach consensus about the role of the teacher and student in the middle part of the lesson.

	Student Role	Teacher Role
Middle	<ul style="list-style-type: none">• To actively engage in problem solving• To work with group members so that every one understands the problem and potential solutions• To explain and justify their thinking, and reason out loud• Each group member must be able to articulate how the problem was solved (accountability)	<ul style="list-style-type: none">• To ask probing, guiding questions that support students to further develop their thinking (guide on the side)• To ensure effective group work by clarifying roles and responsibilities, setting guidelines and expectations• To circulate among the groups to ensure all students are engaged and on track

Additional Focusing Questions

Group # 1:

- The first group is engaged in a discussion about volume and surface area.
- What insights do the students gain throughout the discussion?

- How does the teacher guide the group? (He asks probing questions, asks students to paraphrase what another student has said, paraphrases student responses himself, and encourages effective group work by asking students to wait for each other (gives reason) and by assigning students to build the prism.)
- What specific questions does the teacher ask?

Group # 2:

- In what way is the second group struggling? (not working together or discussing possible solutions together)
- How does the teacher support them to work together?
- What does this indicate about his beliefs and understandings?

Group # 3:

- Why does the teacher ask three students to explain their solution? Why not ask just one student?
- Explain how the group solved the problem, include evidence they provided.

Three-Part Lesson: End

- View the end part of the lesson and stop the video.
- Participants record their observations and share with a partner.
- In a full group, reach consensus about the importance of the third part of the lesson.

	Student Role	Teacher Role
End	<ul style="list-style-type: none"> • To be prepared to present their solution to the class • To explain and justify their thinking • To answer questions posed by classmates 	<ul style="list-style-type: none"> • To clearly articulate the expectations for the congress • To ensure all students/groups are accountable and prepared to present their solution • To choose carefully and with a purpose, the solutions that will be brought to full group • To facilitate the class discussion • To probe for clarification

Additional Questions

- Why does the teacher tell his class that every group must be prepared to present?
- (To ensure that all groups feel accountable and understand that any group might be presenting.)

- What do you think the teacher's reasons are for choosing these two groups to present?
 - *The first group has represented their thinking in several ways – in a chart, with drawings and with student generated algorithms. It is possible that other groups only represented their thinking in one of the three ways demonstrated in this solution and the teacher might have wanted to show connections.*
 - *The first group used 64 blocks and most of the class used 64 blocks to solve the problem. So the first solution is more easily understood by the whole class.*
 - *The second solution was done with 27 blocks, so this was a good solution to have students present next. This solution leads to an understanding of the question 'does the cube always have the smallest surface area or does it only work with 64 blocks'?*

Overall Guiding Questions

- What do Mr. Perry's actions and questions reflect about his beliefs and understandings about how kids learn?
- Give examples from the video to support your ideas:
 - *Groupings and lots of discussion show that he believes students need to talk and learn together*
 - *Questions constantly – does not tell them the answer – shows that he believes students must construct an understanding themselves, that if he jumps in and gives the answer he effectively stops them from thinking.*

Continuing the Congress: Pursuing Understanding

In this section of the video, the teacher continues the Congress with his students and goes deeply into one group's poster. During the discussion confusions are revealed, leading to greater insight by the students.

This lesson is a continuation of the conversation or congress that began on Friday.

One young man struggles at certain points as he presents his group's thinking and reasoning.

Notes

Most teachers will not catch the error, just let the video run and discuss it after the young woman has identified the error and is convincing her group where they went wrong and what they should have done.

The group has made an error on their poster when creating the student-generated algorithm.

- How does the teacher facilitate the discussion that reveals the error?
 - *(The teacher never draws attention to the error; rather he lets the students discover it themselves and then poses the question to the whole class.)*
- Who recognizes there is a problem with the algorithm?
 - *(One of the students from that group realizes they have the correct answer, but that the dimensions within the algorithm are not correct. This leads to a discussion of the dimensions of the rectangular prism (the tissue box shaped prism) and in turn, leads to a discussion of how to calculate surface area – a review or summary of the problem-solving discussion. Some students are solid in their understanding and some are still constructing an understanding.)*

Student Voice

Research indicates that teaching through problem solving develops conceptual understanding in students and fosters a positive disposition towards mathematics. In this section of the video, we hear about the impact that teaching through problem solving has had on two Grade 9 students.

Additional Questions

How can we create the conditions within our classrooms to engage all our learners in deep mathematical discussions; and in essence, to think and act like mathematicians?

Notes

At one point in the video, the girls state that their teacher 'never' used the text book. This teacher uses a variety of resources when planning lessons, and does not rely heavily or exclusively on the text book.

Organizer: Teaching Through Problem Solving

Determining Starting Points for Instruction

Guiding Questions	Observations/Notes
<p>What are Shawn's goals or end points when planning his lesson?</p> <p>What assumptions does he make?</p>	
<p>Listen carefully to Kat and Dylan's statements. How does the teacher respond to their statements?</p>	
<p>Focus on the group of boys with whom the teacher is working.</p> <p>What do the students know and what are they able to do?</p> <p>Is their conversation surprising to you in any way? Why or why not?</p>	
<p>Record your insights and observations when watching Shawn's reflection.</p>	

Organizer: Teaching Through Problem Solving

Student Voice

Guiding Questions	Observations/Notes
Record all the ideas the Grade 9 girls share about why they like their Grade 9 class.	
Record any challenges they faced in the class. How did they respond to these challenges?	
What insights have you gained from hearing the perspective of the Grade 9 students?	

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