This module has been designed to support those providing
Mathematics Professional learning for K-12 classroom educators.

Absolute and Relative Thinking Presentation Guide

Session Description
This session explores concepts outlined *Paying Attention to Proportional Reasoning (PAPR)*, particularly absolute and relative thinking. Participants solve problems and examine classroom activities that correlate to curriculum expectations in primary, junior and intermediate divisions.

Importance
Proportional reasoning instruction in North America tends to have an over emphasis on procedural solution methods. Proportional reasoning is sometimes perceived as only being the study of ratios, rates, and rational numbers (fractions, decimals and percents), but it actually permeates all strands of mathematics. "Proportional reasoning is difficult to define. It is not something that you either can or cannot do, but is developed over time through reasoning. It is the ability to compare multiplicative relationships between quantities." (Van de Walle, 2006)

Learning Focus
Participants will:
- deepen mathematical content knowledge of proportional reasoning
- develop awareness of the key concepts associated with proportional reasoning
- develop pedagogical knowledge for teaching proportional reasoning
- experience a K-9 perspective of proportional reasoning

Agenda
Minds On:
- Proportional Reasoning – How Do We Use it? Would You Rather Have…?
- Paying Attention to Proportional Reasoning (PAPR) - Overview

Action:
- Multiplicative Thinking
- Unitizing and Spatial Reasoning
- Understanding Quantity Relationships and Change
- Partitioning, Measuring, Unit Rates and Spatial Reasoning

Consolidation:
- Making Our Learning Visible
- Resources

Professional Learning Module Contents
- Presentation Guide: Overview, Learning Activities, Questions to Stimulate Conversations (as needed), Aha Moments (possible participants' insights), Materials, and Adaptations (20 minute, 1.5 hour and 2.5 hour sessions)
- PowerPoint with Script and <<presenter notes>>
- Black Line Masters (BLM)

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
Introduction and Minds On (25 minutes) **Learning Focus, Session Norms and Agenda (Slides 1 - 4)** **2 minutes** **How Do We Use Proportional Reasoning in Our Lives? (Slide 5) 5 minutes** Participants examine where they use proportional reasoning in their lives. **Would you Rather Have…? (Slide 6 - 7) 13 minutes** Participants solve the problem and use solutions to begin construction of anchor charts "Strategies and Representations" & "Modifications and Connections" which will be added to throughout the session. **Paying Attention to Proportional Reasoning Overview (Slides 8 - 10) 5 minutes** Participants examine the concept connections graphic and a quote from *Paying Attention to Proportional Reasoning.*	• What concepts are involved in solving proportional reasoning situations?	• There are many representations and strategies that can be used in the "Would you Rather Have…?" problem (e.g., different units, ways to compare, different measurements, benchmark strategies, personal referents). • Delay teaching rules and algorithms to allow student to "explore" proportional problems.	• *Paying Attention to Proportional Reasoning* (1 per participant) • Anchor chart with headings "Representations & Strategies" and Modifications & Strand Connections • Small quantity of quarters for the Would you Rather Have…? problem

Action (4.5 hours)
Multiplicative Thinking (Slides 11 - 17) 1.5 hours
Participants consider problems that provide opportunities for them to think additively as well as multiplicatively and develop understanding of these concepts.
 	
Through the use of a classroom video, participants explore how students use absolute (additive) and relative (multiplicative) thinking. A second video demonstrates how teachers can support students as they move from additive thinking to multiplicative thinking.
 	
Information is added to the group's anchor chart.
	• How do you determine when to use additive or multiplicative thinking?	• A situation sometimes can be described both additively and multiplicatively. The context determines which is appropriate.
		• Some students need support moving from additive reasoning to multiplicative reasoning.
			• BLM 1- 4, 6 (1 per participant)
			• BLM 5 (1 copy for presenter)
			• Stickers
			• Scissors
			• Attribute blocks
			• Colour tiles
			• Relational Rods
			• Learning tools by mathies.ca
			  » Money
			  » Colour Tiles
			  » Relational Rods
			  » Set

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
Action (Continued) **Unitizing and Spatial Reasoning (Slides 18 - 23) 1 hour** Participants solve division appropriate proportional reasoning problems that focus on unitizing and spatial reasoning concepts. The problems reveal how representations and strategies support students in developing multiplicative reasoning. Participants are encouraged to suggest modifications to problems to meet the needs of a variety of learners and how proportional reasoning connects well with other strands. Information is added to the group's anchor chart. BLM#2 can be used for additional problems if the need arises. **Understanding Quantity Relationships and Change (Slides 24 - 28)** **1.25 hours** Participants solve a proportional reasoning problem that involves how quantities relate and change. Participants examine samples of student work for evidence of proportional thinking through the lens of the concepts diagram found in Paying Attention to Proportional Reasoning. Participants add to the group anchor chart. **Understanding Quantity Relationships and Change (Slides 24 - 28) 1 hour** Participants solve division specific problem that demonstrates partitioning, measuring, unit rates and spatial reasoning. The problems will reveal how representations and strategies support students in developing multiplicative reasoning. Participants will see how to modify problems in order to meet the needs of a variety of learners, and how proportional reasoning connects well with other strands.	• How does spatial reasoning help students understand proportional • How does examining how the quantities in a relationship change help identify if it involves a proportional situation? • How can you use two shapes to create a new unit help solve the primary problem? • How does transformational geometry help solve the primary problem? • How can we use non-standard units to solve the Super Bear problem?	• Ratio tables are a powerful strategy for representing proportional relationships. • Unitizing is the basis for multiplicative thinking. • Multiplicative relationships have quantities that co-vary (i.e., if one quantities doubles then the other also doubles) • All linear growing patterns without a constant are proportional • When tiling with shapes, it helps to create a rectangular shape from multiple copies of the original. The original shape will only tile the entire grid if the new rectangular shape is proportional to the grid.	

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
Consolidation/Debrief (35 minutes) **Making Our Learning Visible** **(Slides 37 and 38) 30 minutes** Participants, in groups, refine and organize recorded information from the anchor charts. Participants complete an exit ticket to provide assessment as learning data. **Resources (Slide 39) 5 minutes** Share the Paying Attention to Proportional Reasoning Adobe Presenter on EduGAINS and learning tools on mathies.ca	• What other representations (mind map, concept map, other graphic organizer, list with subtitles) can be used?	• The *Paying Attention To* documents has good stuff in them • Now that I know how to access these resources, I will try them out • I will continue to explore other learning tools from mathies. ca to use with my students • Children enter school with the ability to reason algebraically.	• Completed anchor charts • Chart paper & markers • BLM 7 (1 per participant)

Suggestions if you are offering the session as part of a series:
- Before the session direct participants to watch Paying Attention to Proportional Reasoning K-12 Adobe Presenter. If you are offering a two-day session, consider addressing fractions using the Ways We Use Fractions Professional Learning Module as the presentation for Session 2. After Session 1, direct participants to return with student responses for further discussion and analysis on Session 2.

Considerations if you are offering the session through Adobe Connect:
- Use a poll to gather information for the question "Would you rather.."
- Create two pods for Making Our Thinking Visible for participants to add to throughout the session

Adaptations	Materials
If you have 20 minutes: *Learning Focus:* • Same as the 5.5 hour session but with less depth. *Activities:* • Minds On	See 5.5 hours outline
If you have 1.5 hours: *Learning Focus:* • Same as the 5.5 hour session but with less depth. *Activities:* • Minds On and Action (Unitizing and Spatial Reasoning)	See 5.5 hours outline
If you have 2.5 hours: *Learning Focus:* • Same as the 5.5 hour session but with less depth. *Activities:* • Minds On and Action – (Multiplicative Thinking & either Unitizing and Spatial Reasoning or Partitioning, Measuring, Unit Rates and Spatial Reasoning)	See 5.5 hours outline