

This module has been designed to support those providing Mathematics Professional learning for K-12 classroom educators.

## Algebraic Reasoning as Functional Thinking Presentation Guide

### Session Description

Generalizing relationships is highlighted in this session. Participants solve problems and examine the connections between multiple representations of the same mathematics. Samples of division specific problems are explored and discussed.

### Importance

Students at all ages can and should be encouraged to reason algebraically including conjecturing and generalizing. The ability to use and make connections between different representations fosters algebraic thinking. “Generalization is the heartbeat of mathematics and appears in many forms. If teachers are unaware of its presence, and are not in the habit of getting students to work at expressing their own generalizations, then mathematical thinking is not taking place” (John Mason, 2004).

### Learning Focus

*Participants will:*

- deepen mathematical content knowledge of algebraic reasoning
- develop awareness of key concepts associated with algebraic reasoning, specifically
  - » functional thinking
  - » multiple representation
  - » generalization
- develop pedagogical knowledge for teaching algebraic reasoning

### Agenda

#### Minds On:

- Guess My Rule Game
- Building and Extending Patterns
- Research – What is Algebraic Reasoning?

#### Action:

- When Do They Meet? (Problem – solving, generalizing and examining multiple representations)
- Making Connections Between Representations

#### Consolidation:

- Division (e.g., primary, junior, intermediate/senior) Specific Problems
- Resources

### Professional Learning Module Contents

- Presentation Guide: Overview, Learning Activities, Questions to Stimulate Conversations (as needed), Aha Moments (possible participants’ insights), Materials, and Adaptations (20 minute, 1.5 hour and 5.5 hour sessions)
- PowerPoint with Script and <<presenter notes>>
- Black Line Masters (BLM)

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
<p style="text-align: center;"><b>Minds On (47 minutes)</b></p> <p><b>Learning Focus (Slide 1- 4) 2 minutes</b></p> <p><b>Guess My Rule Game (Slides 5-6) 15 minutes</b> Participants play a game with the objective of finding the rule that generates the output number for any input number.</p> <p><b>Building and Extending Patterns (Slides 7-12) 25 minutes</b> Participants represent patterns with tiles. Participants view a short video of a student in SK engaged in algebraic reasoning.</p> <p><b>Research - What is Algebraic Reasoning? (Slides 13-15) 5 minutes</b> Participants discuss and share ideas related to what algebraic reasoning is using research quotes from <i>Paying Attention to Algebraic Reasoning</i>.</p>	<ul style="list-style-type: none"> <li>• What effect does input numbers of 0 and 1 have on Guess My Rule Game?</li> <li>• Why is algebraic reasoning important?</li> <li>• At what age can students begin to reason algebraically?</li> <li>• What types of learning experiences do you suppose the student had prior to the video?</li> <li>• How can algebraic reasoning be supported?</li> </ul>	<ul style="list-style-type: none"> <li>• Input 1 gives the multiplier when the rule involves just multiplication</li> <li>• Input 0 gives the constant as output when the rule involves multiplication and addition</li> <li>• Input, Term and Position numbers are the same</li> <li>• When there is no constant, the growing pattern has only a multiplier</li> </ul>	<ul style="list-style-type: none"> <li>• Post-it notes</li> <li>• Post-it notes for labelling terms</li> <li>• Square tiles or mathies colour tiles learning tool (mathies.ca)</li> <li>• <a href="#"><u>Paying Attention to Algebraic Reasoning</u></a> (a few hard copies per table)</li> <li>• Device with internet access</li> </ul>
<p style="text-align: center;"><b>Action (75 minutes)</b></p> <p><b>When Do They Meet? (Slide 16) 15 minutes</b> Participants solve the problem collaboratively (pairs, threes, table groups – depending on the number of participants).</p> <p><b>Let’s Look at Solutions (Slides 17-22) 45 minutes</b> Participants investigate the connections and relationships between a range of solutions through discussion and a gallery walk.</p> <p><b>Making Connections Between Representations (Slides 23-26) 15 minutes</b> Participants consolidate learning about the connections and relationships between and among multiple representations using examples from <i>Paying Attention to Algebraic Reasoning</i>. This task will allow participants to reflect upon their own mathematical reasoning and connect it to others’, including students.</p>	<ul style="list-style-type: none"> <li>• Can a number line or table of values help?</li> <li>• How does seeing the solutions and identifying connections or relationships impact your mathematical thinking?</li> <li>• If you were given a similar problem in the future, would you solve it the same way or differently?</li> </ul>	<ul style="list-style-type: none"> <li>• I never knew that there were so many ways to solve this problem!</li> <li>• Reading and thinking deeply about a few solutions resulted in a very different kind of mathematical thinking than an overview of all of the solutions.</li> <li>• Wow, the different representations show the constant and the multiplier in unique ways.</li> <li>• Wow the multiplier is the rate at which the line increases.</li> </ul>	<ul style="list-style-type: none"> <li>• <i>BLM 1 and 2</i> (1 copy for presenter)</li> <li>• Chart Paper</li> <li>• Markers</li> <li>• Rulers</li> <li>• Concrete or digital relational rods (mathies.ca)</li> <li>• Devices with internet access</li> <li>• Device to photograph participant solutions and embed in slide #16 (optional)</li> </ul>

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
<p><b>Consolidation/Debrief (28 minutes)</b>  <b>Division Specific Problems (Slides 27)</b>  20 minutes  Participants choose, explore and discuss a division specific problem involving algebraic reasoning at their tables.</p> <p><b>Resources (Slides 28-29) 8 minutes</b>  Presenter highlights the corresponding Adobe Presenter for <i>Paying Attention to Algebraic Reasoning</i> as well as other related Ministry of Education resources for personal investigation.  Presenter addresses outstanding parking lot post-its.</p>		<ul style="list-style-type: none"> <li>• The <i>Paying Attention To</i> documents has good stuff in them</li> <li>• Now that I know how to access these resources, I will try them out</li> <li>• I will continue to explore other learning tools from mathies.ca to use with my students</li> <li>• Children enter school with the ability to reason algebraically.</li> </ul>	<ul style="list-style-type: none"> <li>• If participants are not able to link to resources listed in the ppt, have hard copies of the division specific problems (BLM3) available.</li> <li>• Devices with internet access</li> </ul>

**Suggestions if you are offering the session as part of a series:**

- Invite participants to use a problem from slide 27 in a classroom and have them bring student work to a follow-up session for discussion.

**Considerations if you are offering the session through Adobe Connect:**

- Use breakout rooms to solve and discuss the problem *When Will They Meet*

Adaptations	Materials
<p><b>If you have 20 minutes:</b>  <i>Learning Focus:</i></p> <ul style="list-style-type: none"> <li>• Introduction to research based concepts regarding the learning and teaching of Algebraic Reasoning.</li> </ul> <p><i>Activities:</i></p> <ul style="list-style-type: none"> <li>• Participants interact with the Adobe Presenter that supports <i>Paying Attention to Algebraic Reasoning</i>. Start at Slide 8.</li> </ul>	<p><a href="http://www.edugains.ca/resourcesMath/VideoLibrary/AdobePresenter/AP_AlgebraicReasoning/">http://www.edugains.ca/resourcesMath/VideoLibrary/AdobePresenter/AP_AlgebraicReasoning/</a></p>
<p><b>If you have 1.5 hours:</b>  <i>Learning Focus:</i></p> <ul style="list-style-type: none"> <li>• Same as 2.5 hours</li> </ul> <p><i>Activities:</i></p> <ul style="list-style-type: none"> <li>• Omit Solving When do they Meet and Looking at Solutions (slides 14 – 20)</li> </ul>	<p>See 2.5 hours outline</p>
<p><b>If you have 5.5 hours:</b>  <i>Learning Focus:</i></p> <ul style="list-style-type: none"> <li>• Same as 2.5 hours</li> </ul> <p><i>Activities:</i></p> <ul style="list-style-type: none"> <li>• After slide 4, have participants take turns being the robot to play the game. Allow participants to explore MathCLIPS (Cluster 4, Activity 1)</li> <li>• After Slide 6, insert the three slides from slides for five and one half hour session.ppt and complete activities in the notes.</li> <li>• Allow time for participants to do the suggested reading on slide 12 of original power point (pages 3 and 4 of <i>Paying Attention to Algebraic Reasoning</i>)</li> <li>• Allow time for participants to explore mathies and MathCLIPS as suggested in presenter notes (Slide 23 of original powerpoint).</li> </ul>	<p>See 2.5 hours outline</p>