This module has been designed to support those providing
Mathematics Professional learning for K-12 classroom educators.

Algebraic Reasoning as Equality, Representations and Proof Presentation Guide

Session Description
Equality, representations and proof are highlighted in this module. Participants solve problems and examine the connections between and among these topics. Problems that can be used in primary, junior and intermediate divisions are explored and discussed.

Importance
Students at all ages can and should be encouraged to reason algebraically including conjecturing and generalizing. The ability to use and make connections between different representations fosters algebraic thinking. "Generalization is the heartbeat of mathematics and appears in many forms. If teachers are unaware of its presence, and are not in the habit of getting students to work at expressing their own generalizations, then mathematical thinking is not taking place" (John Mason, 2004).

Learning Focus
Participants will:
- deepen mathematical content knowledge of algebraic reasoning
- develop awareness of key concepts associated with algebraic reasoning, specifically
  » equality as a relationship between quantities
  » representations
  » proof
- develop pedagogical knowledge for teaching algebraic reasoning

Agenda
Minds On:
- What is Algebraic Reasoning?

Action:
- Exploring Equality – A relationship between quantities
- Exploring Representation – Double number line and symbols
- Exploring Proof – Generalizing mathematical properties and relationships

Consolidation:
- Consolidating the Use of Symbols - "Solve for ?" problem
- Resources

Professional Learning Module Contents
- Presentation Guide: Overview, Learning Activities, Questions to Stimulate Conversations (as needed), Aha Moments (possible participants' insights), Materials, and Adaptations (20 minute, 1.5 hour and 5.5 hour sessions)
- PowerPoint with Script and <<presenter notes>>

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
Introduction and Minds On (17 minutes) **Introduction (Slides 1 to 4) 2 minutes** **What is Algebraic Reasoning (Slides 5-7) 15 minutes** Participants reflect on a definition of Algebraic Reasoning and brainstorm grade appropriate mathematical generalizations for students.	• Why is algebraic reasoning important? • At what age can students begin to reason algebraically? • How can algebraic reasoning be supported?	• I never considered having young learners generalize mathematical properties. • I have new ideas for generalizations to support my students.	• Post-it notes • *Paying Attention to Algebraic Reasoning* (a few hard copies per table)
Action (110 minutes) **Equality (Slides 8-19) 25 minutes** • Participants predict student responses regarding equality and the equal sign (slides 8-16) • Participants explore ways to engage students in exploring equality as a relationship between quantities (slides 17-19) **Representation (Slide 20-33) 45 minutes** • Participants solve a problem collaboratively (pairs, threes, tables – depending on the number of participants) (slides 20-21) • Participants explore solutions using double number lines as well as symbolic representations (slides 22-24) • Participants solve a second problem and apply learning of double number lines and symbolic representation (slides 25-33) **Proof (Slides 34-42) 30 minutes** • Participants use a problem as well as a video clip to investigate what it means to "prove" a conjecture (slides 34-40) • Participants engage in the process of proving mathematical properties (slides 41-42) **Using Symbols, Including Letters as Variables (Slides 43-45) 10 minutes** • Participants contrast the functions of variables in algebraic representations	• How has your understanding of equality changed? • How might this impact your instruction? • How can a double number line help students to reason algebraically? • How does seeing the solutions and identifying connections or relationships impact your mathematical thinking? • If you were given a similar problem to Sunny's Jumps, would you solve it the same way or differently? Explain. • What are mathematical generalizations that are appropriate for your grade level? • Why is it important to differentiate between the functions of variables (changing quantities and unknown)?	• I never realized that only showing equations like 5+2 = 7 could create misconceptions around the meaning of the equal sign? • Wow, I never realized how a double number line allows algebra to be represented spatially. (e.g., In Sunny's Jumps, It shows that 4 jumps from each attempt can be ignored. • I think some of my students who are visual learners will benefit from the double number line representation. • I never thought about using concrete materials to prove generalizations.	• Chart Paper • Lined paper • Markers • Rulers • Interlocking Cubes • mathies.ca • Devices with internet access

Learning Activities	Questions to Stimulate Conversation	Aha Moments	Materials
Consolidation/Debrief (23 minutes) **Solve for? (Slides 46 - 48) 20 minutes** • Participants consolidate learning about the use of symbols by solving a problem – "Solve for?" **Resources (Slides 49 - 50) 3 minutes** • Presenter highlights the corresponding Adobe Presenter for Paying Attention to Algebraic Reasoning as well as other related Ministry of Education resources for personal investigation. • Presenter addresses outstanding parking lot post-its	• How does seeing the solutions and identifying connections or relationships impact your mathematical thinking? • How could this type of problem support learners who are struggling with algebra? • How could this type of problem support learners who have not yet been formally introduced to algebra?	• I believe that seeing (Horse + Butterfly) as an object will help with later concepts in algebra. • The Paying Attention To documents have good stuff in them. • Now that I know how to access these resources, I will try them out. • I will continue to explore other learning tools from mathies. ca to use with my students • Children enter school with the ability to reason algebraically.	• Devices with internet access • (optional)

Suggestions if you are offering the session as part of a series:
Invite participants to use the problems on slides 21 and 25 in a classroom and have them bring student work to a follow-up session for discussion.

Considerations if you are offering the session on-line:
Use breakout rooms to solve and discuss the problems Sunny's Jumps (slides 21-33)

Adaptations	Materials
If you have 20 minutes: *Learning Focus:* • Introduction to research based concepts regarding the learning and teaching of Algebraic Reasoning. *Activities:* • Participants interact with the Adobe Presenter that supports Paying Attention to Algebraic Reasoning. Start at Slide 8.	*http://www.edugains.ca/ resourcesMath/VideoLibrary/ AdobePresenter/AP_ AlgebraicReasoning/*
If you have 1.5 hours: *Learning Focus:* • Same as 2.5 hours *Activities:* • Omit Solving (slides 25 -32 & 46 - 48)	See 2.5 hours outline
If you have 5.5 hours: *Learning Focus:* • Same as 2.5 hours *Activities:* • Combine this module with Algebraic Reasoning as Functional Thinking module	See 2.5 hours outline