Session 1: Teaching through Mathematical Processes

GAINS

120 min	**Math Learning Goals** • Reflect and share about yourself as a teacher. • Assess prior knowledge of the Mathematical Processes. • Make connections between solving problems and the Mathematical Processes. • Connect the process "name" to its mathematical meaning. Rationale	**Materials** • BLM 1.1–1.3 • journals • coloured card stock • Math Processes S1.ppt
Minds On… A for L	**Whole Group → Reflection Journal** Participants create their own Reflection Journal booklet to record their reflections for the six sessions. **Individual/Whole Group → Reflection** Participants record their responses to the ice breaker activity: My Math-Self in their journals and share responses with whole group. **Individual → Anticipation Guide** Participants complete the Anticipation Guide: The Mathematical Processes (BLM 1.1). **Curriculum Expectations/Observation/Mental Note:** Observe participants' understanding of the Mathematical Processes as they complete the anticipation guide.	Ice Breaker: **M**y name is… **A** favourite math learning memory is… **T**eaching to me means… **H**elp in my professional growth…
Action!	**Whole Group → Set a Context** Introduce the relationship between problem solving and the Mathematical Processes. Provide an overview of the Mathematical Processes using a gears analogy. Make connections to the previous mathematics curriculum document to note how this Mathematical Process formed part of the expectations for the strands. **Expert Groups → Learn the Processes** Use numbered heads or number cards to assign participants into heterogeneous groups of approximately 7 (minimum of 5). Provide each person in the home group with one of the process cards from (BLM 1.2). Participants form expert groups for each process. Group members read, discuss, and summarize the statement: The most important thing about *process name* is _____. **Home Group → Save the Last Word** Participants return to their home groups. In turn, experts hold up their card for all to read the Mathematical Process expectation. The expert reads his/her side. One by one each group member makes a comment about that process, with the expert making the final statement and answering questions from others.	Print each of the seven Mathematics Process expectations and its context on coloured card stock (BLM 1.2). These cards are used again in Session 4. If using groups of 5, leave out Problem Solving and Communications, as they contain the other processes.
Consolidate Debrief DI	**Pairs → Matching Game** Participants cut out the phrases in each row (BLM 1.3a) and paste the phrase with the matching Mathematical Process (BLM 1.3b). Participants check their answers (BLM 1.3c). **Individual → Reflection** Participants write a journal response: • Two things I hope to gain from this workshop. • Expertise that I bring to this workshop. Differentiate content based on participant readiness in order to work in zone of proximal development.	
Reflection	**Home Activity or Further Classroom Consolidation** Explain the 'gears' metaphor of the Mathematical Processes in your Reflection Journal.	

BLM 1.1: Anticipation Guide: The Mathematical Processes

Check your prior knowledge of the Mathematical Processes by answering Yes or No in the left hand column for each statement. At the end of the session, respond using the right hand column.

Before		Statement about the Mathematical Processes	After	
Yes	No	The Mathematical Processes are new in the revised curriculum (2004).	Yes	No
Yes	No	There are seven Mathematical Process expectations.	Yes	No
Yes	No	The Mathematical Processes are specific expectations.	Yes	No
Yes	No	The Mathematical Processes must be evaluated.	Yes	No
Yes	No	The Mathematical Processes are part of all mathematics curriculum Grades 1–12.	Yes	No
Yes	No	The Mathematical Processes are addressed through Problem Solving.	Yes	No

Problem Solving

Throughout the course, students will:

- develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding

Problem Solving

Problem solving is central to learning mathematics. It forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction. It is considered an essential process through which students are able to achieve the expectations in mathematics, and is an integral part of the mathematics curriculum in Ontario, for the following reasons. Problem solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident mathematicians;
- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;
- offers excellent opportunities for assessing students' understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas;
- promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics;
- helps students find enjoyment in mathematics;
- increases opportunities for the use of critical-thinking skills (e.g., estimating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, evaluating results, and making judgements).

Not all mathematics instruction, however, can take place in a problem-solving context.

Certain aspects of mathematics must be explicitly taught. Conventions, including the use of mathematical symbols and terms, are one such aspect, and they should be introduced to students as needed, to enable them to use the symbolic language of mathematics.

Reasoning and Proving

Throughout the course, students will:

- **develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments**

Reasoning and Proving

An emphasis on reasoning helps students make sense of mathematics. Classroom instruction in mathematics should always foster critical thinking – that is, an organized, analytical, well reasoned approach to learning mathematical concepts and processes and to solving problems.

As students investigate and make conjectures about mathematical concepts and relationships, they learn to employ *inductive reasoning*, making generalizations based on specific findings from their investigations. Students also learn to use counter-examples to disprove conjectures. Students can use *deductive reasoning* to assess the validity of conjectures and to formulate proofs.

Reflecting

Throughout the course, students will:

- **demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions)**

Reflecting

Good problem solvers regularly and consciously reflect on and monitor their own thought processes. By doing so, they are able to recognize when the technique they are using is not fruitful, and to make a conscious decision to switch to a different strategy, rethink the problem, search for related content knowledge that may be helpful, and so forth. Students' problem solving skills are enhanced when they reflect on alternative ways to perform a task even if they have successfully completed it. Reflecting on the reasonableness of an answer by considering the original question or problem is another way in which students can improve their ability to make sense of problems.

Selecting Tools and Computational Strategies

Throughout the course, students will:

- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems

Selecting Tools and Computational Strategies

Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

Calculators, Computers, Communications Technology. Various types of technology are useful in learning and doing mathematics. Students can use calculators and computers to extend their capacity to investigate and analyse mathematical concepts and to reduce the time they might otherwise spend on purely mechanical activities.

Students can use calculators and computers to perform operations, make graphs, manipulate algebraic expressions, and organize and display data that are lengthier or more complex than those addressed in curriculum expectations suited to a paper-and-pencil approach. Students can also use calculators and computers in various ways to investigate number and graphing patterns, geometric relationships, and different representations; to simulate situations; and to extend problem solving. When students use calculators and computers in mathematics, they need to know when it is appropriate to apply their mental computation, reasoning, and estimation skills to predict results and check answers.

The computer and the calculator must be seen as important problem-solving tools to be used for many purposes. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the particular applications that may be helpful to them as they search for their own solutions to problems.

Students may not be familiar with the use of some of the technologies suggested in the curriculum. When this is the case, it is important that teachers introduce their use in ways that build students' confidence and contribute to their understanding of the concepts being investigated.

Students also need to understand the situations in which the new technology would be an appropriate choice of tool. Students' use of the tools should not be laborious or restricted to inputting and learning algorithmic steps. For example, when using spreadsheets and statistical software (e.g., Fathom), teachers could supply students with prepared data sets, and when using dynamic geometry software (e.g., The Geometer's Sketchpad), they could use pre-made sketches so that students' work with the software would be focused on manipulation of the data or the sketch, not on the inputting of data or the designing of the sketch.

Computer programs can help students to collect, organize, and sort the data they gather, and to write, edit, and present reports on their findings. Whenever appropriate, students should be encouraged to select and use the communications technology that would best support and communicate their learning. Students, working individually or in groups, can use computers, CD-ROM technology, and/or Internet websites to gain access to Statistics Canada, mathematics organizations, and other valuable sources of mathematical information around the world.

Manipulatives. Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas. Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others.

Using manipulatives to construct representations helps students to:

• see patterns and relationships; • make connections between the concrete and the abstract; • test, revise, and confirm their reasoning;

• remember how they solved a problem; • communicate their reasoning to others.

Computational Strategies. Problem solving often requires students to select an appropriate computational strategy. They may need to apply the standard algorithm or to use technology for computation. They may also need to select strategies related to mental computation and estimation. Developing the ability to perform mental computation and to estimate is consequently an important aspect of student learning in mathematics.

Mental computation involves calculations done in the mind, with little or no use of paper and pencil. Students who have developed the ability to calculate mentally can select from and use a variety of procedures that take advantage of their knowledge and understanding of numbers, the operations, and their properties. Used effectively, mental computation can encourage students to think more deeply about numbers and number relationships.

Knowing how to estimate, and knowing when it is useful to estimate and when it is necessary to have an exact answer, are important mathematical skills. Estimation is a useful tool for judging the reasonableness of a solution and for guiding students in their use of calculators. The ability to estimate depends on a well-developed sense of number and an understanding of place value. It can be a complex skill that requires decomposing numbers, compensating for errors, and perhaps even restructuring the problem. Estimation should not be taught as an isolated skill or a set of isolated rules and techniques. Knowing about calculations that are easy to perform and developing fluency in performing basic operations contribute to successful estimation.

Connecting

Throughout the course, students will:

- **make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports)**

Connecting

Experiences that allow students to make connections – to see, for example, how concepts and skills from one strand of mathematics are related to those from another – will help them to grasp general mathematical principles. As they continue to make such connections, students begin to see that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another. Seeing the relationships among procedures and concepts also helps deepen students' mathematical understanding.

In addition, making connections between the mathematics they study and its applications in their everyday lives helps students see the usefulness and relevance of mathematics beyond the classroom.

Representing

Throughout the course, students will:

- **create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems**

Representing

In elementary school mathematics, students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols.

In secondary school mathematics, representing mathematical ideas and modelling situations generally takes the form of numeric, geometric, graphical, algebraic, pictorial, and concrete representation, as well as representation using dynamic software.

Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret mathematical, physical, and social phenomena. When students are able to represent concepts in various ways, they develop flexibility in their thinking about those concepts. They are not inclined to perceive any single representation as "the math"; rather, they understand that it is just one of many representations that help them understand a concept.

Communicating

Throughout the course, students will:

- **communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions**

Communicating

Communication is the process of expressing mathematical ideas and understandings orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words.

Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and to clarify ideas, relationships, and mathematical arguments.

The development of mathematical language and symbolism fosters students' communication skills. Teachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate. For example, teachers can:

- model proper use of symbols, vocabulary, and notations in oral and written form;
- expect correct use of mathematical symbols and conventions in student work;
- ensure that students are exposed to and use new mathematical vocabulary as it is introduced (e.g., by means of a word wall; by providing opportunities to read, question, and discuss);
- provide feedback to students on their use of terminology and conventions;
- ask clarifying and extending questions and encourage students to ask themselves similar kinds of questions;
- ask students open-ended questions relating to specific topics or information;
- model ways in which various kinds of questions can be answered.

Effective classroom communication requires a supportive and respectful environment that makes all members of the class comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher.

The ability to provide effective explanations, and the understanding and application of correct mathematical notation in the development and presentation of mathematical ideas and solutions, are key aspects of effective communication in mathematics.

BLM 1.3a: Mathematical Processes Matching Activity

Reasoning and Proving	Reflecting	Selecting Tools and Computational Strategies	Connecting	Representing
Role of Students				
Understand that various representations can be used to appropriately represent the same situation.	Make connections between new and prior knowledge to make sense of what they are learning.	Combine given information with intuition to make a reasoned guess when prompted.	Consider the reasonableness of their answer.	Use manipulatives and/or technology to develop understanding of new concepts, for communicating, or for performing certain tasks.
Instructional Strategies				
Introduce new concepts using concrete materials.	Ask questions that require students to hypothesize and make conjectures, e.g., What if…?	Encourage students to ask themselves "What-if" questions.	Model different computational strategies, and explain why you choose to use them.	Integrate strands, explicitly demonstrating and reinforcing connections.
Sample Questions				
What estimation strategy did you use? Was your result sufficiently accurate for the question?	How could you represent this idea algebraically? Graphically?	When could this mathematical concept or procedure be used in daily life?	How does this compare to….?	In what cases might our conclusion not be true?
Sample Feedback				
How can you relate your understanding of… to this problem?	In what other ways can you represent this problem?	Present your solution, showing all the steps so someone else will understand your thinking.	I can follow your thinking up to here. How can you help me understand your next ideas?	Share your solution with someone who has used a different tool, and discuss the merits of each.

BLM 1.3b: Mathematical Processes Matching Activity

Reasoning and Proving	Reflecting	Selecting Tools and Computational Strategies	Connecting	Representing
Role of Students				
Instructional Strategies				
Sample Questions				
Sample Feedback				

BLM 1.3c: Mathematical Processes Matching Activity Answers

Reasoning and Proving	Reflecting	Selecting Tools and Computational Strategies	Connecting	Representing
Role of Students				
Combine given information with intuition to make a reasoned guess when prompted.	Consider the reasonableness of their answer.	Use manipulatives and/or technology to develop understanding of new concepts, for communicating, or for performing certain tasks.	Make connections between new and prior knowledge to make sense of what they are learning.	Understand that various representations can be used to appropriately represent the same situation.
Instructional Strategies				
Ask questions that require students to hypothesize and make conjectures, e.g., What if…?	Encourage students to ask themselves "What-if" questions.	Model different computational strategies, and explain why you choose to use them.	Integrate strands, explicitly demonstrating and reinforcing connections.	Introduce new concepts using concrete materials.
Sample Questions				
In what cases might our conclusion not be true?	How does this compare to….?	What estimation strategy did you use? Was your result sufficiently accurate for the question?	When could this mathematical concept or procedure be used in daily life?	How could you represent this idea algebraically? Graphically?
Sample Feedback				
Present your solution, showing all the steps so someone else will understand your thinking.	I can follow your thinking up to here. How can you help me understand your next ideas?	Share your solution with someone who has used a different tool, and discuss the merits of each.	How can you relate your understanding of… to this problem?	In what other ways can you represent this problem?