# Module 7

## Representing and Comparing Decimals

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diagnostic</strong></td>
<td>5</td>
</tr>
<tr>
<td>Administer the diagnostic</td>
<td>5</td>
</tr>
<tr>
<td>Using diagnostic results to personalize interventions</td>
<td>5</td>
</tr>
<tr>
<td><strong>Solutions</strong></td>
<td>6</td>
</tr>
<tr>
<td><strong>Using Intervention Materials</strong></td>
<td>7</td>
</tr>
<tr>
<td>Representing Decimal Tenths</td>
<td>8</td>
</tr>
<tr>
<td>Representing Decimal Hundredths</td>
<td>11</td>
</tr>
<tr>
<td>Counting by Tenths or Hundredths</td>
<td>15</td>
</tr>
<tr>
<td>Renaming Decimal Tenths</td>
<td>18</td>
</tr>
<tr>
<td>Comparing Decimal Tenths or Hundredths</td>
<td>21</td>
</tr>
<tr>
<td>Comparing Mixed Decimals</td>
<td>25</td>
</tr>
</tbody>
</table>
REPRESENTING AND COMPARING DECIMALS

Relevant Expectation for Grade 6

• represent, compare, and order … decimal numbers from 0.001 to 1,000,000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);
• demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1,000,000, using a variety of tools and strategies (e.g., use base ten materials to represent the relationship between 1, 0.1, 0.01, and 0.001)

Possible reasons a student might struggle in representing and comparing decimals

Many students struggle working with decimals since some of the intuitions they developed working with whole numbers fail them. Some of the problems include:
• recognizing that even though 100 > 10, 0.01 < 0.1
• lack of familiarity with the names of the tenths and hundredths place value columns
• not realizing that the same digit in a different place value position can represent a different amount
• difficulty counting forward by tenths or hundredths, particularly over transition spots (e.g., from 1.9 to 2.0)
• difficulty renaming numbers with different units (e.g., 0.3 as 30 hundredths and not just 3 tenths)
• believing that the number of digits (irrespective of their place value positions) determines the size of a number
• difficulty comparing decimals written in different units, i.e., with different number of decimal places (e.g., 0.3, 3 tenths, with 0.27, 27 hundredths)
• difficulty relating decimal tenths or hundredths to simple benchmarks (e.g., wholes or tenths)
Administer the diagnostic

If students need help in understanding the directions of the diagnostic, clarify an item's intent.

Using diagnostic results to personalize interventions

Intervention materials are included on each of these topics:

- Representing Decimal Tenths
- Representing Decimal Hundredths
- Counting by Tenths or Hundredths
- Renaming Decimal Tenths
- Comparing Decimal Tenths or Decimal Hundredths
- Comparing Mixed Decimals

You may use all or only part of these sets of materials, based on student performance with the diagnostic.

<table>
<thead>
<tr>
<th>Evaluating Diagnostic Results</th>
<th>Suggested Intervention Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>If students struggle with Questions 1d, e, 3a, c, 6c, 7a, c, 8b, 10a</td>
<td>Use Representing Decimal Tenths</td>
</tr>
<tr>
<td>If students struggle with Questions 1a–c, 3b–d, 4, 5, 6a, b, d, 7b, d, 8a, 10b–d</td>
<td>Use Representing Decimal Hundredths</td>
</tr>
<tr>
<td>If students struggle with Question 2</td>
<td>Use Counting by Tenths or Hundredths</td>
</tr>
<tr>
<td>If students struggle with Question 9 and 13</td>
<td>Use Renaming Decimal Tenths</td>
</tr>
<tr>
<td>If students struggle with Questions 11, 12a, b, c</td>
<td>Use Comparing Decimal Tenths or Hundredths</td>
</tr>
<tr>
<td>If students struggle with Question 12d–g</td>
<td>Use Comparing Mixed Decimals</td>
</tr>
</tbody>
</table>

Solutions

1. a) 0.03  
   b) 0.12  
   c) 0.47  
   d) 0.8  
   e) 4.8  

2. a) 2.0, 2.1, 2.2  
   b) 2.39, 2.40 (or 2.4), 2.41  
   c) 0.99, 1.00 (or 1), 1.01  
   d) 1.09, 1.10 (or 1.1), 1.11  

3. a) one and three tenths  
   b) one and three hundredths
c) one and thirty hundredths
d) thirteen hundredths

4. a) 2 ones + 4 hundredths  
b) 4 ones + 2 hundredths  
c) 24 hundredths  
d) 4 tenths + 2 hundredths

5. 3 in 0.32 is 3 tenths, but 3 in 0.23 is 3 hundredths

6. a) 0.32  
b) 0.02  
c) 0.3 (or 0.30)  
d) 0.86

7. a) 0.1  
b) 0.01  
c) 0.2  
d) 0.34

8. a) e.g., 0.03  
b) e.g., 0.72

9. a) 30  
b) 20, 200  
c) 13, 130

10. a) \(\frac{1}{3}\) of something  
b) \(\frac{1}{4}\) of something  
c) about \(\frac{1}{3}\) of something  
d) most of something

11. a) 4  
b) 2  
c) 1.9 or 19 tenths  
d) 3.5 or 35 tenths

12. a) 0.01, 0.1, 1, 10, 100  
b) 0.03, 0.36, 0.60, 0.63, 0.81  
c) 0.11, 0.2, 0.36, 0.49, 0.7  
d) 1.12, 2.4, 2.8, 3.6, 4.2  
e) 0.12, 1.12, 1.21, 2.1, 12.1  
f) 0.13, 1.03, 3.01, 3.13, 3.31  
g) 1.23, 3.21, 12.3, 13.2, 13.21

13. Three out of 10 columns are shaded so that is 3 tenths and 30 out of 100 squares are shaded and that is 0.30.
USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for working with thousandths and other decimals.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:
• Questions to ask before using the approach
• Using the approach
• Consolidating and reflecting on the approach
### Representing Decimal Tenths

#### Learning Goal
- using the patterns of the place value system to represent tenths.

#### Open Question

<table>
<thead>
<tr>
<th>Questions to Ask Before Using the Open Question</th>
</tr>
</thead>
</table>
| Show a ten frame. Count as you place counters one at a time in each square.  
◊ *One tenth, two tenths, three tenths, … ten tenths.* |
| Show a place value chart. Place one counter in the tenths column. Indicate that you write 0.1 to mean one tenth since you put the 1 in the tenths column in the chart. 
Explain that the 0 means 0 wholes and the decimal point indicates that tenths will be to the right.  
◊ *How do you think I would show 2 tenths?* (Place 2 counters in tenths column.) |
| Have the student place counters until there are 10 counters placed.  
◊ *Why don’t you think you should leave the 10 counters in the tenths column?* (Whenever you have 10 of something on a place value chart, you trade it for one of the next thing.)  
◊ *What would you call it?* (1) |
| ◊ *Does it make sense that it could also be written 1.0?* (e.g., Yes since that is 1 and no tenths.)  
◊ *[Make sure students know we read this as 1 and 0 tenths, i.e., that we read the decimal point as “and.”]*  
◊ *What do you think 1.7 means?* (1 and 7 tenths) |

<table>
<thead>
<tr>
<th>Using the Open Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provide the student with the Fraction Circles and Ten Frames template, Place Value Chart (2) template and counters.</td>
</tr>
<tr>
<td>Encourage the student to draw more than one picture for each situation. Indicate that they can use the circles, ten frames and place value chart, but can draw different things if they wish.</td>
</tr>
<tr>
<td>By viewing or listening to student responses, note if they:</td>
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<tr>
<td>• understand that the place to the right of the decimal point represents 1 part of ten.</td>
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<tr>
<td>• realize that numbers more than 1 can also be expressed as tenths</td>
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<tr>
<td>• notice that 0.1 and 0.9 together make a whole</td>
</tr>
<tr>
<td>• realize that tenths can be used to describe different wholes</td>
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<tr>
<td>Depending on student responses, use your professional judgment to guide specific follow-up.</td>
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</tbody>
</table>

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<thead>
<tr>
<th>Consolidating and Reflecting on the Open Question</th>
</tr>
</thead>
</table>
| ◊ *Why did you use more than one whole for 1.1 and 2.1 but not for 0.1?* (0.1 means 0 wholes and 1 tenth but the others mean 1 or 2 wholes and the tenth.)  
◊ *Why did you use ten frames?* (I needed something divided into 10 equal parts to show tenths.) |
| ◊ *What other decimal would be like 1.1 and 2.1?* (e.g., 3.1)  
◊ *Could it have more than 2 digits?* (yes, e.g., 12.1) |
| ◊ *How else are 0.1 and 0.9 alike?* (They are both less than 1 but more than 0.)  
◊ *What other decimals would share that description?* (0.2, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8)  
◊ *Would any other decimals work?* (not if they are tenths) |
| ◊ *Why is 0.1 closer to 0 but 0.9 closer to 1?* (9 tenths is almost 10 tenths and that is 1 but 1 tenths is just a little more than 0.) |

### Materials
- ten frame
- Fraction Circles and Ten Frames template
- Place Value Chart (2) template
- counters
Solutions

1. e.g.,

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ø</td>
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</tbody>
</table>

Each place value chart has 1 counter in the tenths place but different numbers of counters in the ones place.

Each picture has one ten frame with only one counter in it, but there are different numbers of full ten frames.

Each picture has one circle with ten sections with only one section shaded, but there are different numbers of fully shaded circles.

Each picture has one tally stick by itself, but there are different numbers of groups of ten tallies.

2. e.g.,

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>ø</td>
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</tbody>
</table>

Both place value charts have counters only in the tenths column. But one column has a lot more tenths in it than the other.

I could show 0.1 and 0.9 together in one frame. They are alike since they are both part of a ten frame, but the 0.9 is a lot more of the frame.

I could show 0.1 and 0.9 together in one circle. They are alike since they are both part of the circle with ten parts, but the 0.9 is a lot more of the circle.

I showed 0.1 by the tucked-in thumb and 0.9 with the other fingers. They are alike since they are all fingers on hands, but the 0.9 is a lot more fingers.
Questions to Ask Before Assigning the Think Sheet

On the Place Value Chart (2) template indicate that the dot is a decimal point and is a way to show where the ones column is. The ones column is always to the left of the decimal point.

Put one counter in the tenths place and show how to write 0.1 and read it as one tenth. Repeat placing counters until there are ten counters in the tenths place.
◊ Why should I trade the counters? (When you use place value, you trade 10 of one thing for 1 of the next thing.)

Now put 2 counters in the ones place and 8 counters in the tenths place.
◊ How would you say this? (two and eight tenths)
◊ Is it closer to 2 or to 3? (closer to 3 since it is only 2 more tenths to get to 3 but you'd have to go back 8 tenths to get to 2)
◊ What other decimal might be close to 3? (e.g., 3.1 or 2.9)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand that the place value system works the way they are used to—just like ones are \( \frac{1}{10} \) of tens, tenths are \( \frac{1}{10} \) of ones.

Assign the tasks.

By viewing or listening to student responses, note if they can:
• switch between symbolic, verbal, visual, and written forms of decimal tenths
• work with decimal tenths both more than and less than 1
• “round” decimal tenths to the nearest whole or decimal hundredths to the nearest tenth

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why did the wholes need to be divided into tenths to use decimals? (The fractions that decimals show are tenths, not quarters or eighthths or …)
◊ How do you decide whether to put a 0 to the left of the decimal point? (If the number is less than 1.)
◊ What other decimal would be closer to 8 than to 7? (e.g., 7.9)
◊ In Question 6, why do you think that you would need a number greater than 100, but also a digit to the right of the decimal point? (I knew about the digit to the right of the decimal point since I say tenths. I also knew it had to be hundreds since you say hundreds.)

Solutions

1. a) 0.5 and 0.5  b) 0.3 and 0.7  c) 0.6 and 0.4  d) 0.8 and 0.2
2. a) 0.6  b) 0.9  c) 1.5  d) 12.7
3. a) 4.3  b) 5.6  c) 0.8  d) 10.9  e) 1.0
4. a) three and one tenth  b) seven tenths
c) two and eight tenths  d) three tenths
5. a) 4  b) 5  c) 8
6. a) 201.4, 204.1, 402.1, 401.2, 104.2, 102.4  Note: There could also be numbers like 1201.4 or 2.41 if read as 2 and 4 tenths and 1 hundredth.
   b) two hundred one and four tenths, two hundred four and one tenth, four hundred two and one tenth, four hundred one and two tenths, one hundred four and two tenths, one hundred two and four tenths
Representing Decimal Hundredths

Learning Goal

• using the patterns of the place value system to represent hundredths.

Open Question

Questions to Ask Before Using the Open Question

◊ Show the metre stick. How many centimetres make up the metre? (100)
◊ What decimal of a metre could you use to describe how much each centimetre is worth? (0.01)
◊ What does 0.01 mean? (one hundredth)
◊ What decimal would you use to describe 9 cm? (0.09)
◊ What decimal would you use to describe 10 cm? (0.10)
◊ What do you think 2.10 metres means? (2 metres and 10 cm)

Using the Open Question

Encourage students to name lengths that might represent actual distances, e.g., the width of their desk, the length of their arm.

By viewing or listening to student responses, note if they understand that:
• we use two decimal places to indicate hundredths
• we can use decimal hundredths to describes values both more or less than 1
• 0.50 is half
• values like 0.99 are close to 1

Depending on student responses, use your professional judgment to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ How would you write 10 cm as hundredths of a metre? (0.10)
◊ Why would you write 0.10 and not 1.0 or 0.010? (e.g., If you used a place value chart and put 10 counters in the hundredths column, you would trade for 1 counter in the tenths column and have no hundredths left; that is 0.10.)
◊ Why did you think 0.95 is close to a metre? (9 tenths and a little more is close to a whole.)
◊ How many centimetres is half a metre? (50)
◊ How would you write that as a decimal? (0.50)
◊ What other decimals is that near? (e.g., 0.51)
◊ What do you know about a length that is of the form 0.■ m? (It is less than a metre and it can be measured in a whole number of centimetres.)
◊ Why are hundredths little? (It takes 100 of them to make 1 whole.)

Solutions

e.g., 0.11, 0.99, 0.52, 3.42,
1.23 is 1 m and about another quarter of a metre
3.01 is 1 cm more than 3 metres
Think Sheet

Questions to Ask Before Assigning the Think Sheet

On the Place Value Chart (2) template indicate that the dot is a decimal point and is a way to show where the ones column is. The ones column is always to the left of the decimal point.

Put one counter in the hundredths place and show how to write 0.01 and read one hundredth. Repeat placing counters until there are ten counters in the hundredths place.

◊ Why should I trade the counters? (When you use place value, you trade 10 of one thing for 1 of the next thing.)
◊ Why might you read this as 10 hundredths? (It is 10 hundredths.)
◊ How else could you read it? (as 1 tenth and 0 hundredths or just as 1 tenth)

Put 2 counters in the ones place and 8 counters in the hundredths place.

◊ How would you say this? (two and eight hundredths)
◊ Is it closer to 2 or to 3? (closer to 2 since it would take 92 more hundredths to get to 3 but you would only go back 8 hundredths to get to 2)
◊ Is it closer to 2.1 or 2.0? (Closer to 2.1 since it would only take 2 hundredths to get to 2.1 but you would have to go back 8 hundredths to get to 2.0.)
◊ What other decimal hundredth might be close to 2.1? (e.g., 2.09 or 2.12)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand that the place value system works the way they are used to—just like ones are $\frac{1}{10}$ of tens, then hundredths are $\frac{1}{10}$ of tenths (since $\frac{1}{100} = \frac{1}{10}$)

Assign the tasks.

By viewing or listening to student responses, note if they can:

• switch between symbolic, verbal, visual and written forms of decimal hundredths
• work with decimal hundredths both more than and less than 1
• “round” decimal hundredths to the nearest whole or nearest tenth

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why didn’t it matter that the shaded squares were not altogether for question 1c? (You are just counting what is shaded and not where they are.)
◊ Which fractions had 0 to the left of the decimal point in Question 2? (only a and b)
  Why those? (They are less than 1 and if there were a 0, it would mean not even one whole.)
◊ What other hundredth would be close to 8 in Question 5? (e.g., 7.92)
◊ Why was 3.42 closer to 3.4 than 3.5? (It is only two hundredths more than 3.4 but it is 8 hundredths less than 3.5.)
◊ How did you know that the decimal was not 5.24 for Question 7? (You wouldn’t say fifty – you would say five)
◊ Why are hundredths little? (It takes 100 of them to make 1 whole.)
Solutions

1. a) 0.20 (or 0.2) and 0.80 (or 0.8)  
   b) 0.15 and 0.85  
   c) 0.65 and 0.35  
   d) 0.95 and 0.05

2. a) 0.06  
   b) 0.29  
   c) 1.05  
   d) 12.87

3. a) 14.03  
   b) 50.62  
   c) 0.17  
   d) 0.04  
   e) 0.10  
   f) 1.00

4. a) three and one hundredth  
   b) twenty-seven hundredths  
   c) two and eighteen hundredths  
   d) three hundredths

5. a) 4  
   b) 5  
   c) 8

6. e.g., 3.41, 3.42, 3.43

7. a) e.g., 2.54, 4.52, 52.04, 54.02, 452.02  
   b) two and fifty-four hundredths, four and fifty-two hundredths, fifty-two and four hundredths, fifty-four and two hundredths, four hundred fifty-two and two hundredths

8. e.g., Hundredths are smaller than tenths since it takes 100 of them to make the same amount that 10 tenths make. That means that 3 small things are less than 3 bigger things.
Counting by Tenths or Hundredths

Learning Goal

- reasoning about how to count by tenths or hundredths.

Open Question

Questions to Ask Before Using the Open Question

Make sure students are comfortable with reading 0.2 as 2 tenths and 0.02 as 2 hundredths. If they are not, proceed with sections Representing Decimal Tenths and/or Representing Decimal Hundredths first.

Present a portion of a number line where ticks at 2, 4, 6, 8, and 10 are marked. Point to the next tick (where 12 would go).

◊ What three numbers should I put next? Why? (12, 14, 16 since you count by 2s)
◊ Suppose I start counting by $\frac{1}{5}$s using decimals. What are the first three numbers I would say? (0.1, 0.2, 0.3)
◊ Why does that make sense? (They are one tenth, two tenths and three tenths.)
◊ What happens after 0.9? (1.0)

Make sure students realize that we write 1.0 for 10 tenths and not 0.10 [which is the same as 1 tenth]

◊ Suppose you counted by 0.2. What are the first five numbers you would say? (0.2, 0.4, 0.6, 0.8, 1.0)
◊ Are there numbers you would not write down? (Yes, e.g., 0.3.)

Using the Open Question

Provide students with the Number Lines template to help them keep track of their skip counts, if needed.

Make sure students understand that they must list five numbers before 7.2 (or 0.72) and five numbers after if they were skip counting.

If they ask, indicate that they do not have to begin at 0.

By viewing or listening to student responses, note if they understand that:
- they are likely to consistently go up by a certain number of tenths for problem 1 and a certain number of hundredths for problem 2
- there is a distinction between tenths and hundredths

If students know that decimal tenths can also be written as decimal hundredths, they might skip count by 0.01, etc. for problem 1.

Depending on student responses, use your professional judgment to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ Why is it easy to count by 0.1 or 0.01? (It is just like counting by ones but you have to include the decimal point.)
◊ Why did you use decimal tenths (or hundredths) each time? (so that I could end up with 7.2 which is tenths [or 0.72 which is hundredths])
◊ Did you need to use decimal tenths for each number you wrote down? (No, e.g., instead of 7.0, I think I could have written 7.)
◊ What was the most important thing to think about when you counted? (I had to make sure that I kept going up or down by the same number of tenths or hundredths.)
**Solutions**

1. e.g.,
   - Before: 6.7, 6.8, 6.9, 7.0, 7.1  
     After: 7.3, 7.4, 7.5, 7.6  
     by 0.1
   - Before: 6.2, 6.4, 6.6, 6.8, 7.0  
     After: 7.4, 7.6, 7.8, 8.0, 8.2  
     by 0.2
   - Before: 5.7, 6.0, 6.3, 6.6, 6.9  
     After: 7.5, 7.8, 8.1, 8.4, 8.7  
     by 0.3
   - Before: 5.2, 5.6, 6.0, 6.4, 6.8  
     After: 7.6, 8.0, 8.4, 8.8, 9.2  
     by 0.4
   - Before: 4.2, 4.8, 5.4, 6.0, 6.6  
     After: 7.8, 8.4, 9.0, 9.6, 10.2  
     by 0.6
   - Before: 3.2, 4.0, 4.8, 5.6, 6.4  
     After: 8.0, 8.8, 9.6, 10.4, 11.2  
     by 0.8
   - Before: 2.7, 3.6, 4.5, 5.4, 6.3  
     After: 8.1, 9.0, 9.9, 10.8, 11.7  
     by 0.9
   - Before: 1.2, 2.4, 3.6, 4.8, 6.0  
     After: 8.4, 9.6, 10.8, 12.0, 13.2  
     by 1.2

Some students may skip count by 2.4 even though there are not five positive numbers to say first. Other students may skip count not starting at 0, e.g.,

Before: 4.7, 5.2, 5.7, 6.2, 6.7  
After: 7.7, 8.2, 8.7, 9.2, 9.7  
by 0.5

2. e.g.,
   - Before: 0.67, 0.68, 0.69, 0.70, 0.71  
     After: 0.73, 0.74, 0.75, 0.76  
     by 0.01
   - Before: 0.62, 0.64, 0.66, 0.68, 0.70  
     After: 0.74, 0.76, 0.78, 0.80, 0.82  
     by 0.02
   - Before: 0.57, 0.60, 0.63, 0.66, 0.69  
     After: 0.75, 0.78, 0.81, 0.84, 0.87  
     by 0.03
   - Before: 0.52, 0.56, 0.60, 0.64, 0.68  
     After: 0.76, 0.80, 0.84, 0.88, 0.92  
     by 0.04
   - Before: 0.42, 0.48, 0.54, 0.60, 0.66  
     After: 0.78, 0.84, 0.90, 0.96, 1.02  
     by 0.06
   - Before: 0.32, 0.40, 0.48, 0.56, 0.64  
     After: 0.80, 0.88, 0.96, 1.04, 1.12  
     by 0.08
   - Before: 0.27, 0.36, 0.45, 0.54, 0.63  
     After: 0.81, 0.90, 0.99, 1.08, 1.17  
     by 0.09
   - Before: 1.2, 2.4, 3.6, 4.8, 6.0  
     After: 8.4, 9.6, 10.8, 12.0, 13.2  
     by 1.2

Some students may skip count by 2.4 even though there are not five positive numbers to say first. Other students may skip count not starting at 0, e.g.,

Before: 0.47, 0.52, 0.57, 0.62, 0.67  
After: 0.77, 0.82, 0.87, 0.92, 0.97  
by 0.05
**Think Sheet**

### Questions to Ask Before Assigning the Think Sheet

Make sure students are comfortable with reading 0.2 as 2 tenths and 0.02 as 2 hundredths. If they are not, proceed with sections Representing Decimal Tenths and/or Representing Decimal Hundredths first.

Count orally with the student(s):
- one tenth, two tenths, three tenths, ...
- Why would you say 0.31 if you skip count by decimals?
- Would you ever say 0.31 if you skip counted by 0.2, 0.3, 0.4?

Put a counter into the tenths column each time you say a new number, so students can see that there are the right number of tenths in the tenths column at that point. Have them join in and go as far as twelve tenths.

- How would you write what we just said? (0.1, 0.2, 0.3, 0.4, ... 1.0, 1.1, 1.2)
- Why did you write 1.0 for ten tenths? (e.g., 10 tenths is 1 whole and no tenths. You can see that on the place value chart.)

Put two counters at a time into the tenths place.

- Count the number of tenths as I place the counters and write the decimals for what you say. (0.2, 0.4, 0.6, 0.8, 1.0, 1.2)
- Would you ever say 2.3? How do you know? (I wouldn’t since I only say even number of tenths and 2.3 is an odd number of tenths.)

### Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand the section on skip counting by 0.03. Point out that this time the skip counting is by hundredths and not tenths and that all the skip counting starts at 0.

Assign the tasks.

By viewing or listening to student responses, note if they:
- can skip count by tenths, even over transitions (e.g., going from 3.8 to 4.0)
- can skip count by hundredths, even over transitions (e.g., going from 1.87 to 1.90)
- can predict what numbers and what types of numbers are said when skip counting by decimal tenths or hundredths
- understand how skip counting is used when marking a number line

Dependent on student responses, use your professional judgement to guide further follow-up.

### Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

- Why did you write 1.1 instead of 0.11 for Question 1c? (It has to be 11 tenths and that’s more than 1.0 – 0.11 would only be 11 hundredths and that’s not even close to 1.)
- Why did it take so much longer to get to 3.20 counting by hundredths than to get to 3.2 counting by tenths? (3.2 is 32 tenths, so it is the 32nd number you say. 3.20 is 320 hundredths so it is the 320th number you say.)
- If you count by 0.01 what would be the 32nd number? (0.32)
- Would you ever say 0.31 if you skip counted by 0.2? 0.3? 0.4? (No, not if you start at 0.)
- When might you say 0.31 if you skip count by decimals? (if you start at 0 and skip count by 0.01)
- You said that 0.10 comes after 0.09. What would come after 0.99 if you were skip counting by hundredths? (1.00)
- How do you know? (since 1.00 is 1 whole and that is 100 hundredths)
Solutions

1. a) 3.8, 4.0, 4.2, 4.4, 4.6
   b) 0.60, 0.63, 0.66, 0.69, 0.72
   c) 0.9, 1.1, 1.3, 1.5, 1.7
   d) 1.90, 1.93, 1.96, 1.99, 2.02

2. a) 32 numbers
   b) 320 numbers

3. a) Yes, e.g., 0.44 is 44 hundredths; so if you count by 4 hundredths, it is the 11th number you will say.
   b) Yes, e.g., 1.85 is 185 hundredths; so if you count by 5 hundredths, it is a number you will say.

4. If you have 10 hundredths, you can trade them for 1 tenth and you would have no hundredths left. That is what 0.10 is.

5. e.g.,

I skip counted by 2 hundredths.
I know that 0.48 is 48 hundredths, so before that comes 46 hundredths which is 2 fewer hundredths and 44 hundredths which is 2 fewer than that. Afterwards came 50 and 52 hundredths.
Renaming Decimal Tenths

Learning Goal

• using the patterns of the place value system to rename decimal tenths.

The focus on this lesson is writing tenths as hundredths. In fact, any number of hundredths can be written as tenths, but not necessarily a whole number of tenths. For example, 0.35 can be written as 3.5 tenths. In later grades, students will need to realize this fact, but not at this point.

Open Question

Questions to Ask Before Using the Open Question

◊ How many pennies make a dollar? (100)
◊ What fraction of a dollar is each penny? (0.01)
◊ How many dimes make a dollar? (10)
◊ What fraction of a dollar is a dime? (\(\frac{1}{10}\))
◊ How many pennies are 4 dimes worth? (40)
◊ How does that help you write tenths as hundredths? (You can write 4 tenths of a dollar as 0.40)
◊ How many pennies are 14 dimes worth? (You could write it as 140 pennies, so that is 1.40.)
◊ What fraction of a dollar is ten pennies? (\(\frac{10}{100}\) or \(\frac{1}{10}\))
◊ How would you write those as decimals? (0.10 or 0.1)

Using the Open Question

Provide students with the 10 × 10 Grid (1) and pennies.
Assign the task.

By viewing or listening to student responses, note if they understand that:
• any number of tenths can be written as hundredths, but it’s not always as easy to write hundredths as tenths
• a decimal of the form \(\_0\) can easily be written as tenths or hundredths.

Depending on student responses, use your professional judgment to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ Why can you read the number 10 as either 1 ten + 0 ones or as 10 ones? (We use place value and so 10 means 1 ten and 0 ones but you get it from trading 10 ones.)
◊ Why does it make sense that 0.10 could be read as 1 tenth or as 10 hundredths? (There is a 1 in the tenths place and nothing else so that is why it is \(\frac{1}{10}\). But if you had 10 hundredths, you would trade for 1 tenth, so it is also 10 hundredths.)
◊ How could you write 32 tenths as hundredths? (Instead of 3.2, you write 3.20.)
◊ Why can you write 0.30 as a whole number of tenths, but not write 0.32 as a whole number of tenths? (It is more than 3 tenths, but less than 4 tenths, so there is no whole number of tenths.)

Solutions

e.g., 0.40, 0.30, 0.70, 0.90, 1.20
- forty hundredths or four tenths
- thirty hundredths or three tenths
- seventy hundredths or seven tenths
- ninety hundredths or nine tenths
- one hundred twenty hundredths (or 1 and 20 hundredths) or twelve tenths (or 1 and 2 tenths)

The decimal had to have 0 hundredths in it for me to be able to read it as tenths.
Questions to Ask Before Assigning the Think Sheet

◊ How many pennies make a dollar? (100)
◊ Why is a penny a good model for 1/100? (since it is 1/100 of a dollar)
◊ For what fraction is a dime a good model? (1/10 since 10 dimes make a dollar)
◊ Why would 4 dimes be a good model for 0.4? (It is 4 tenths.)
◊ How many pennies are 4 dimes worth? (40)
◊ Why would that be 0.40? (That's 40 hundredths and each penny is 1/100.)

Using the Think Sheet

Read through the introductory box with the students.

Making sure students understand that 1 whole is either 10 tenths or 100 hundredths.

Assign the tasks.

By viewing or listening to student responses, note if they:
• can rename tenths as hundredths
• can rename appropriate hundredths as tenths
• understand that the number of tenths is always fewer than the number of hundredths (1/10 as much) for the same amount
• can explain why tenths can be renamed as hundredths
• can explain which hundredths can be renamed as whole numbers of tenths

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ How did you know, for 1e, that it was tenths on the left and hundredths on the right? (Hundredths are smaller so it takes more of them to make a few tenths.)
◊ When you changed the tenths to hundredths, I noticed you just put a 0 in the hundredths place. Will that always work? (Yes)
◊ Why? (because you are saying there are so many tenths and no extra hundredths)
◊ Why did your picture for Question 3 have 100 sections? (So that I could show 30 hundredths – otherwise, I couldn’t.)
◊ What strategy did you use to decide that the first blank on the last line should be 3.00? (I knew you could read it as ones, tenths or hundredths because there were 0 tenths and 0 hundredths.)

Solutions
1. a) 30  b) 70  c) 5  d) 8
   e) tenths, hundredths
   f) hundredths, tenths
2. a) 1.00  b) 1.20  c) 12.00
3. a) The picture shows that 3 out of 10 columns (or 0.3) is worth 30 out of 100 squares (or 0.30).

b) The picture shows that only 3 and not 30 squares are shaded.

4. a) You can always put a 0 in the hundredths column and it doesn’t change the value. You would read it differently.

b) e.g., If you had 0.46, it is more than 4 tenths, but less than 5 tenths, so there is no whole number of tenths it could be.

5. 0.9 is 9 tenths or 90 hundredths. That is because each 10 hundredths is 1 tenth.

0.78 is 78 hundredths. That’s the same as 7 tenths and 8 hundredths.

3.00 can be read three ways. It can be 300 hundredths, 30 tenths, or 3 ones.
Comparing Decimal Tenths or Hundredths

Learning Goal

• reasoning about how two decimal tenths or two decimal hundredth values can be compared.

It is important not to move quickly to rules for comparing decimals. Students should understand why the whole number parts, then the tenths and then the hundredths are compared.

In this lesson, only decimals with the same numbers of digits after the decimal point are compared.

Open Question

Questions to Ask Before Using the Open Question

Mention that in some sports, scores might be given as decimal tenths.
◊ Suppose one student scores 3.4 and another 6.2. Whose score do you think is greater? (6.2)
◊ Why? (It is more than 6 but 3.4 is less since it’s not even 4.)
◊ Is 3.4 more than 3 or less than 3? (It is more than 3.)
◊ Could a score of 3.4 be greater than 3.4? (yes)
◊ How? (e.g., if it were 3.5 or 3.6, …)
◊ Suppose the scores were given as decimal hundredths. Would a score of 3.42 or a score of 3.50 be greater? (3.50)
◊ Why? (It is 3 and fifty hundredths instead of only 3 and 42 hundredths.)

Using the Open Question

Provide students with the Place Value Chart (1) template and counters to represent their scores. Some students might want to use base ten blocks.

Assign the tasks.

By viewing or listening to student responses, note if they understand that:
• the whole number value is the first clue as to which of two decimals is greater
• the tenths matter more than the hundredths in deciding which of two decimals is greater

Depending on student responses, use your professional judgment to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ Which two scores are close together? (e.g., 6.7 and 6.8)
◊ How do you know they are close together? (They are both between 6 and 7 and the difference between them is only one tenth.)
◊ Which one is greater? (6.8)
◊ How do you know? (There are the same number of ones, but more tenths.)
◊ How did you order your second set of scores? (I picked a score under 6 to be least and a score over 7 to be most. Then for the three scores between them, I just figured out which had the least number of hundredths, which the most and which was in the middle.)
◊ What advice would you give someone who was trying to order a set of decimals? (I would tell them to look at the whole number part first, then the tenths and then the hundredths if there were any.)

Materials

• Place Value Chart (1) template
• counters
• base ten flats, rods and units
Solutions

1. e.g., Andrew 4.2
   Scott 4.3
   Maya 5.8
   Ian 9.3
   Brian 5.4

I know that 4.2 and 4.3 are close together since they are only 0.1 apart.

4.2, 4.3, 5.4, 5.8, 9.3

I know that numbers between 4 and 5 (like 4.2 and 4.3) are less than numbers that are more than 5.

I know that 4 and 2 tenths is less than 4 and 3 tenths since there are the same number of ones both times, but more tenths.

I know that 5.4 < 5.8 since there are the same number of ones, but more tenths for 5.8.

I know a number more than 9 is more than a number that is between 5 and 6.

2. e.g., Ethan 6.23
   Paula 5.12
   Mia 6.84
   Helena 7.34
   Chris 6.15

5.12, 6.15, 6.23, 6.84, 7.34

I know that numbers between 6 and 7 are more than 5.12 since it is only \( \frac{12}{100} \) and more than 5 and 6 is \( \frac{100}{100} \) more than 5. I know that 7.34 is greatest since it is more than 7 but 6.23, 6.84 and 6.15 are all less than 7.

I know that \( \frac{23}{100} < \frac{84}{100} \) and that \( \frac{23}{100} > \frac{15}{100} \).

That means that 6.15 < 6.23 < 6.84.
Questions to Ask Before Assigning the Think Sheet

◊ Suppose this flat block is called a 1 (not a 10 like we used to call it.)
◊ What value describes the rod? (1/10)
◊ What value describes the small cube? (1/100)
◊ So how would you model 2.31? (with 2 flats, 3 rods and 1 small cube)
◊ How do you know this is more than 2.12? (that would be 2 flats, but there would only be 1 rod and 2 small cubes and that's less than 3 rods and 1 small cube)
◊ How would you show the 0.31 part of 2.12 on the grid? (as 12 squares)
◊ Which is greater? (the 0.31)
◊ Why does that mean that 2.31 > 2.12? (They are both a little more than 2 but 0.31 has more extra than 2.12.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure that they understand that the same principles are used to compare two decimal tenths or two decimal hundredths.

Assign the tasks.

By viewing or listening to student responses, note if they:
• can compare two decimals in the same units verbally, concretely or pictorially, or written symbolically
• have a sense of what whole number or decimal tenth is near a particular decimal hundredth
• realize that the whole number part of a decimal value is the most important thing to consider when comparing it to another value; only afterwards do tenths, or then hundredths, matter

Dependent on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Was it easier to compare the decimals using the grid, the blocks, or words? (e.g., for me it was the grid)
◊ Why? (I could just see what took up more space without having to think about what the numbers were.)
◊ Why might it be useful to be able to rename 4.1 as 41 tenths and 4.2 as 42 tenths to compare them? (You know that 42 of something is more than 41 of that thing.)
◊ Why is 3.8 closer to 4 but 3.2 closer to 3? (3 and 8 tenths is only 2 tenths less than 4 but it is 8 tenths more than 3; 3.2 is only 2 tenths away from 3.)
◊ When do you need to know all the digits to compare two decimals and when don’t you? (I need to know all the digits to compare 3.42 and 3.41 but I don’t to compare 3.3 to 2.3.)
◊ What advice would you give someone who was trying to order a set of decimals? (I would tell them to look at the ones first, then the tenths and then the hundredths if there were any.)
Solutions

1. a) 12 hundredths
   b) 93 hundredths
   c) 7 tenths
   d) 12 tenths

2. a) 0.7 and 0.4; 0.7 is greater
   b) 0.1 and 0.5; 0.5 is greater
   c) 0.23 and 0.19; 0.23 is greater

3. a) 1.1 and 0.9; 1.1 is greater
   b) 1.9 and 1.5; 1.9 is greater
   c) 0.33 and 0.44; 0.44 is greater
   d) 0.21 and 0.19; 0.21 is greater

4. a) 0, 0.3
   b) 1, 0.9
   c) 1, 1.4
   d) 2, 1.8

5. e.g., Because 2.
   is either 2.9 or less and it’s not as much as 3, but 3.
   is at least 3

6. e.g., 99.88 > 66.64 > 3.32 > 1.00
   e.g., 91.42 > 88.30 > 63.08 > 9.63 > 6.00
Comparing Mixed Decimals

Learning Goal

• reasoning about how decimal tenths can be compared to decimal hundredths.

Note: Take care about asking students to “add” decimals to ensure there are the same number of decimal places. We are not really adding, so that language might confuse students.

Open Question

Questions to Ask Before Using the Open Question

◊ How many centimetres are in a metre? (100)
◊ So if I jumped 2.32 m, how far did I jump in centimetres? (232 cm)
◊ What if I jumped 2.3 m; how far did I jump in centimetres? (230 cm)
◊ How do you know? (I know that 0.3 = 0.30, so that is 30 cm.)
◊ Which is more: 2.3 m or 2.32 m? Why? (2.32 m, since it is 232 cm and not just 230 cm)
◊ Is there another explanation too? (Yes. They are both 2 m and 30 cm, but one has an extra 2 cm.)

Using the Open Question

You might have students attempt a jump to see what numbers might be reasonable to use for jumps (They are likely to jump between 1 m and 2 m).

Provide base ten blocks, and/or place value templates and counters, if needed.

Assign the tasks.

By viewing or listening to student responses, note if they understand that:
• the whole number value is the first clue as to which of two decimals is greater
• the tenths matter more than the hundredths in deciding which of two decimals is greater

Depending on student responses, use your professional judgment to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ Which of your scores are close together? (e.g., 4.2 and 4.21)
◊ How do you know? (one is only 0.01 more than the other one since 4.2 is actually 4.20)
◊ How is ordering decimals that include some tenths and some hundredths different from ordering decimals all of one type? (You have to remember not to quickly just look at the digits. For example, someone might think 1.6 is less than 1.47 since 16 < 147 but it is not.)
◊ What advice would you give someone who was trying to order a set of decimals? (I would tell them to look at the whole number part first, then the tenths and then the hundredths if there were any.)

Solutions

e.g., 1.2, 1.32, 1.1, 0.9, 1.5, 1.47, 1.28, 1.39, 1.45, 1.62

1.1 and 1.2 are pretty close together, but 1.45 and 1.47 are even closer together - only 2/100 apart.
In order: 0.9, 1.1, 1.2, 1.28, 1.32, 1.39, 1.45, 1.47, 1.5, 1.62

You could write them all as centimetres:

90 cm, 110 cm, 120 cm, 128 cm, 132 cm, 139 cm, 145 cm, 147 cm, 150 cm, 162 cm
**Questions to Ask Before Assigning the Think Sheet**

◊ Suppose this flat block is called a 1. What would you call the rod? (0.1)
◊ What would you call the small cube? (0.01)
◊ How could you use the blocks to show that 0.3 > 0.21? (0.3 is 3 rods and that is more than 2 rods and 1 cube)
◊ Does it matter that 3 < 21? (No, since the 3 is tenths and not hundredths but the 21 is hundredths.)
◊ Why might someone think that 0.3 > 0.21 since 3 > 2? (e.g., They are thinking that 0.21 is really 0.210 and then it’s obvious that 0.3 is more than 0.210.)

**Using the Think Sheet**

Read through the introductory box with the students. Make sure that they are comfortable with the various approaches to comparing that are described.

Assign the tasks.

By viewing or listening to student responses, note if they:
• recognize that the whole number value is the first clue as to which of two decimals is greater
• recognize that the tenths matter more than the hundredths in deciding which of two decimals is greater
• know how to relate a decimal hundredth to nearby decimal tenths

Dependent on student responses, use your professional judgement to guide further follow-up.

**Consolidating and Reflecting: Questions to Ask After Using the Think Sheet**

◊ Was it easier to compare decimals using base ten blocks, or when written as decimals? (e.g., I think it is easier with base ten blocks since I can just see what is more.)
◊ How does your picture in Question 3 show why 0.5 > 0.12? (It shows that 0.5 is really 0.50 and then it’s obvious that 0.50 > 0.12.)
◊ Can you always rename decimal tenths to compare them to decimal hundredths? (Yes, e.g., to compare 0.3 to 0.23, I could rename 0.3 as 0.30)
◊ Can you use the number of digits in a decimal number to decide if it’s greater? (no)
◊ Why not? (e.g., One might be tenths and one might be hundredths. For example, 0.4 is more than 0.23 even though 0.23 has more digits.)
◊ Is this any different than with whole numbers? (Yes, since there more digits means a bigger number.)

**Solutions**

1. a) 3 tenths
   b) 2 tenths
   c) 85 hundredths
   d) 42 hundredths

2. a) 0.23 and 0.3; 0.3 is greater
   b) 0.17 and 0.2; 0.2 is greater
   c) 0.33 and 0.2; 0.33 is greater
   d) 0.6 and 0.19; 0.6 is greater
3. a) e.g.,

![Shaded grids for 0.5 and 0.85](image)

There is more shaded for 0.5.

b) e.g.,

![Shaded grids for 0.85, 0.4, and 0.13](image)

There is more shaded for 0.85 than for 0.4 (which is also 0.40) and that is more than 0.13.

4. a) 0.3
   b) 1.0
   c) 1.5
   d) 0.5

5. The first number is more than 3 and the second one is not even up to 3

6. a) 0.23, 0.5, 0.89, 1.4
   b) It would be 23 cm, 50 cm, 89 cm and 140 cm and 23 < 50 < 89 < 140

7. e.g., 5.42 > 0.31, 5.24 > 0.31, 4.52 > 0.31, 4.25 > 0.31, 4.25 > 0.13