Module 5
Multiplying and Dividing

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1. How much more is the first answer than the second?
   a) $4 \times 5$ than $3 \times 5$
   b) $7 \times 8$ than $5 \times 8$
   c) $6 \times 9$ than $5 \times 9$
   d) $8 \times 8$ than $5 \times 8$

2. Write the answer by figuring it out in your head.
   a) $5 \times 7$    b) $4 \times 3$
   c) $8 \times 9$    d) $4 \times 7$
   e) $6 \times 8$    f) $7 \times 9$
   g) $8 \times 4$    h) $3 \times 9$
   i) $7 \times 30$   j) $6 \times 60$
   k) $5 \times 300$  l) $4 \times 600$
3. Fill in the missing values.
   a) \(5 \times 14 = 5 \times ___\) and \(5 \times 4\)
   
   b) \(6 \times 28 = 6 \times 20\) and \(__ \times ___\)
   
   c) \(7 \times 312 = 7 \times ___\) and \(7 \times 10\) and \(__ \times __\)

4. How much more is the first answer than the second? You may write the answer as a product.
   a) \(23 \times 48\) than \(20 \times 48\)
   
   b) \(42 \times 53\) than \(40 \times 53\)
   
   c) \(70 \times 23\) than \(69 \times 23\)
   
   d) \(90 \times 58\) than \(88 \times 58\)

5. Show how to calculate \(23 \times 48\).
   You may use a diagram or blocks to help you.
6. Estimate each answer.
   a) $8 \times 58$
   b) $9 \times 42$
   c) $3 \times 812$
   d) $5 \times 790$
   e) $51 \times 32$
   f) $62 \times 78$

7. How much more is the first answer than the second? You may write
   the answer as a quotient if you wish.
   a) $35 \div 7$ than $28 \div 7$
   b) $48 \div 6$ than $30 \div 6$
   c) $72 \div 9$ than $63 \div 9$
   d) $56 \div 8$ than $40 \div 8$

8. Write the answer by figuring it out in your head.
   a) $63 \div 9$
   b) $42 \div 7$
   c) $32 \div 8$
   d) $28 \div 4$
   e) $56 \div 8$
   f) $64 \div 8$
   g) $45 \div 5$
   h) $24 \div 6$
   i) $120 \div 3$
   j) $180 \div 9$
   k) $1800 \div 6$
   l) $4500 \div 9$
9. Fill in the missing values.
   a) $420 \div 4 = 400 \div 4$ and _____ $\div 4$

   b) $600 \div 4 = 400 \div 4$ and _____ $\div$ _____

   c) $600 \div 5 =$ _____ $\div$ _____ and _____ $\div$ _____

10. Circle the values that are the same as $400 \div 8$.
   a) $400 \div 4$ added to $400 \div 4$  
   b) $200 \div 8$ added to $200 \div 8$

   c) $400 \div 4$ then divided by 2  
   d) $400 \div 5$ added to $400 \div 3$

11. Estimate each quotient.
   a) $97 \div 3$  
   b) $450 \div 8$

   c) $125 \div 3$  
   d) $2547 \div 8$

12. Show how to calculate $328 \div 8$.
   You may use a diagram or base-ten blocks to help you.
Learning Goal

• reasoning about the relationship between products of single digit numbers.

Open Question

If we know that $4 \times 5 = 20$, we know that 4 groups of 5 things is 20 things.

This should help us figure out that:

$5 \times 5 = 25$ since there is one more group of 5.

$4 \times 6 = 24$ since there is one extra item in each of the 4 groups.

$5 \times 4 = 20$ since an array of $4 \times 5$ is an array of $5 \times 4$ if you turn it around.

Choose two of the facts below. For each of the two, tell as many other facts as you can that would help you figure out the answer. How would each one help you?

$3 \times 6 = 18$  
$4 \times 6 = 24$

$3 \times 4 = 12$  
$5 \times 6 = 30$
Think Sheet

There are strategies we can use to help us remember multiplication facts (e.g., $4 \times 6$ or $8 \times 9$)

A multiplication tells the total in a number of equal groups. For example, the picture for $4 \times 5$ is shown here. The 4 tells how many groups and the 5 tells how many in each group.

4 groups of 5

We can use a repeated addition to figure out the total. For example, for $4 \times 5$, you can add four fives: $5 + 5 + 5 + 5$.

We can relate a new multiplication fact to one we already know.

The easiest ones to learn are:

2 × $\Box$: To double a number, or multiply by 2, we add it to itself.
2 × 8 = 8 + 8

5 × $\Box$: To multiply by 5, we can skip count by 5s.
8 × 5 is the 8th number we say: 5, 10, 15, 20, 25, 30, 35, 40, 45

Then we can relate other facts to these facts.

4 × $\Box$: To multiply by 4, we can double a double. That’s because 4 groups of something is twice as much as two groups.
8 × □: To multiply by 8, we can double the result when we multiply by 4. That’s because 8 groups of something is twice as many as 4 groups of something.

3 × □: To multiply by 3, we can add the number to its double. That’s because 3 groups of something is 1 group of it and another 2 groups.

6 × □: To multiply by 6, we can multiply by 3 and double (6 groups is twice as many as 3 groups) or we can multiply by 5 and add another group (5 groups and another group).

7 × □: To multiply by 7, we can multiply by 5 and add the double. For example, 7 × 6 is 5 × 6 and 2 × 6.

9 × □: To multiply by 9, we can multiply by 10 and subtract the number we are multiplying by. That’s because 9 groups of something is 1 less group than 10 groups of it.

1. You know that 3 × 4 = 12. How can that help you figure out each of these facts?
   a) 4 × 3
   b) 4 × 4
   c) 6 × 4
   d) 3 × 8
2. You know that $3 \times 5 = 15$. How can that help you figure out each of these facts?
   a) $4 \times 5$
   b) $6 \times 5$
   c) $3 \times 7$

3. Which multiplication facts result in each product?
   a) 12
   b) 24
   c) 18
   d) 30

4. Why can you always multiply a number by 8 by multiplying it by 5, multiplying it by 2 and then adding those two answers to the number?

5. Describe 2 strategies to figure out each:
   a) $8 \times 9$
   
   b) $7 \times 8$
   
   c) $6 \times 8$
   
   d) $6 \times 7$
Multiplying by 1-digit Numbers

Learning Goal

- representing a product using repeated addition or using an area model.

Open Question

6 × 53 means 6 groups of 53.

Think of it as 6 groups of 50 and 6 groups of 3.

\[
\begin{array}{c}
50 & 3 \\
50 & 3 \\
50 & 3 \\
\end{array}
\]

We multiply a 1-digit number by a 3-digit number. When we are done, there is a 5 in the tens place.

\[
\begin{array}{c}
\times \\
\end{array}
\begin{array}{c}
\end{array}
\begin{array}{c}
= \\
\end{array}
\begin{array}{c}
5 \\
\end{array}
\]

Figure out at least five pairs of numbers we might have multiplied.
Think Sheet

We want to know how many muffins are in 6 boxes of 12 muffins.

We could multiply to figure it out.

\[ 6 \times 12 \text{ is the same as } 6 \times 10 + 6 \times 2. \]

That’s helpful since we probably know that \( 6 \times 10 \) is 6 tens, or 60.

We just add \( 6 \times 2 = 12 \) to 60 to get 72 muffins.

We could use the same idea if there were 6 boxes of 22 muffins.

We could think of \( 6 \times 10 + 6 \times 10 + 6 \times 2 \), or we could use a model to see that there are two sets of \( 6 \times 10 \) (or \( 6 \times 20 \)) and 6 sets of 2:

The total value is 132.

We can use the same idea to multiply \( 6 \times 144 \) when we want to know how many people can sit in 6 parts of a gym, if each part has 144 chairs.

Since \( 144 = 100 + 40 + 4 \), then \( 6 \times 144 \) is \( 6 \times 100 + 6 \times 40 + 6 \times 4 \).
We can use diagrams, models, or other strategies to figure out the parts that we can put together to get the total.

We can estimate products by estimating the numbers being multiplied.

For example, $5 \times 352$ is about $5 \times 3$ hundred. That is 15 hundred (1500).

$5 \times 679$ is about $5 \times 7$ hundred. That is 35 hundred (3500).

1. a) How many pencils are in 4 boxes with 18 pencils in each box?

   b) How many cupcakes are in 8 boxes with 24 cupcakes in each box?

   c) How many days are in 19 weeks?

   d) How many sides are in 49 triangles?

2. List two or three products you can add together to figure each product:
   a) $4 \times 29$

   b) $7 \times 56$

   c) $8 \times 212$

   d) $9 \times 573$
3. Estimate each product:
   a) $8 \times 212$
   b) $7 \times 589$
   c) $3 \times 421$
   d) $6 \times 818$

4. Lucy said that to figure out $9 \times 88$, she starts by figuring out $10 \times 88$. How will that help her figure out $9 \times 88$?

5. Zachary said that to figure out $6 \times 95$, you could add $(2 \times 95) + (2 \times 95) + (2 \times 95)$? Do you agree or not? Explain.

6. Put the digits in the boxes in the blanks in at least 3 different ways. Predict which answer will be greater. Test your prediction.

\[
\begin{array}{cccc}
4 & 6 & 3 & 8 \\
\times & & & \\
\times & & & \\
\times & & & \\
\end{array}
\]
Learning Goal

• representing a product using an area model or relating to a simpler product.

Open Question

16 × 23 means 16 groups of 23.

Think of it as 10 groups of 23 and 6 groups of 23.

That’s the same as 10 groups of 20 and 10 groups of 3 and 6 groups of 3 and 6 groups of 20.

• Multiply each of the four pairs of numbers:

  37 × 39

  38 × 38

  19 × 76

  33 × 41

Compare each pair of numbers with each other pair and tell one way the products (answers) are alike.
Think Sheet

We want to know how many muffins there are in 16 boxes of 24 muffins.

There are many ways to think about the problem.

- We could figure out how many muffins are in 8 boxes and double it, since 16 is double 8.
  
  \[8 \times 24 = 192\] and \[2 \times 192 = 384\] muffins.
  
  Since \(16 \times 24\) is close to \(20 \times 20 = 400\), this makes sense.

- We could figure out how many muffins are in 10 boxes and how many muffins are in 6 boxes and add them.
  
  We know that \(10 \times 24 = 240\).
  
  We also know that \(6 \times 24 = 6 \times 20 + 6 \times 4\), or 144.
  
  The total is 384 muffins.

- We could also model a rectangle that is 16 by 24 and find its area. That’s because there are 16 rows of 24.

Notice that the two blue sections are each \(10 \times 10\). We could call them 100s.

Notice that the 40 green squares are actually 4 columns of 10.

Altogether, there is \(200 + 40 + 120 + 24 = 384\).
1. Match the model with the product it shows.
   a) $12 \times 35$ _____
   b) $24 \times 27$ _____
   c) $53 \times 48$ _____
   d) $18 \times 83$ _____

E: [Diagram]

F: [Diagram]

G: [Diagram]

H: [Diagram]
2. Match the products with their descriptions.
   a) \(14 \times 73\)  \hspace{1cm} E: \(10 \times 30 + 10 \times 7 + 4 \times 30 + 4 \times 7\)
   b) \(41 \times 37\)  \hspace{1cm} F: \(10 \times 30 + 10 \times 4 + 7 \times 30 + 7 \times 4\)
   c) \(14 \times 37\)  \hspace{1cm} G: \(10 \times 70 + 10 \times 3 + 4 \times 70 + 4 \times 3\)
   d) \(17 \times 34\)  \hspace{1cm} H: \(40 \times 30 + 40 \times 7 + 1 \times 30 + 1 \times 7\)

3. Which of these are close to 2000? Circle all that make sense.
   a) \(60 \times 55\)
   b) \(70 \times 29\)
   c) \(97 \times 35\)
   d) \(49 \times 37\)
   e) \(33 \times 15\)

4. Figure out the total number of:
   a) pencils in 14 boxes with 15 pencils in each box.
   b) cupcakes in 24 boxes with 24 cupcakes in each box.
   c) students in 13 classes with 25 students in each class.
   d) photos on 32 pages with 16 photos on a page.
5. Lucy said that to figure out $39 \times 88$, she starts by figuring out $40 \times 88$.
   How will that help her figure out $39 \times 88$?

6. Tell why $38 \times 50$ has the same answer as $76 \times 25$.

7. Put the digits in the boxes in the blanks in at least 3 different ways. Predict which answer will be the greatest. Test your prediction.

\[
\begin{array}{cccc}
4 & 6 & 3 & 8 \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\end{array}
\]
Learning Goal

• connecting any division question to a related multiplication.

Open Question

The question $42 \div 6 = \boxed{}$ is another way of writing $6 \times \boxed{} = 42$.

We want to know what number to multiply 6 by to get 42.

Each of these pairs describe A and B:

1. A is between 40 and 60 and B is 6
2. A is between 30 and 50 and B is 7
3. A is between 300 and 400 and B is 5
4. A is between 200 and 300 and B is 4

• Divide A by B.

• Tell what has to be true about the quotient.

• Explain how you know.

• Choose two other values for A and B and answer the same three questions above.
We divide when we want to find out how many groups of a certain size are in a number.

There are 7 days in a week. If we want to know how many full weeks there are in 35 days, we divide 35 by 7.

We can figure it out different ways:

• We can keep subtracting 7s to see how many 7s are in 35.
  \[35 - 7 - 7 - 7 - 7 - 7 = 0.\]
  We could subtract five 7s, so \(35 \div 7 = 5\) weeks.

• We could figure out what to multiply 7 by to get to 35, in other words \(7 \times \square = 35\).
  Think of multiplication facts. Since \(7 \times 5 = 35\), then \(35 \div 7 = 5\) weeks.

We also divide to share equally.

• Suppose 7 people share $35. We can keep subtracting 7s until we get to 0.
  That would tell us how many times each person can get $1.

• We can also figure out what to multiply 7 by to get 35.
  If the 7 people had shared $350 (35 $10 bills), the answer would be 10 times as much. Since \(7 \times 5 = 35\), \(7 \times 50 = 350\).

1. Describe two ways to solve each problem.
   
   a) How many hands are there, if you can see 45 fingers and thumbs?
   
   b) How many full weeks are there in 56 days?
   
   c) How many packages of 6 apples do you buy, if you want 54 apples?
2. What multiplication facts would help you figure each out?
   a) $36 \div 4$
   b) $32 \div 8$
   c) $630 \div 9$
   d) $490 \div 7$

3. You divide 2 numbers and the quotient (answer in division) is 8. List 5 possible pairs that you might have divided.

4. You divide 2 numbers and the quotient is 40. List 5 possible pairs that you might have divided.

5. You figure out $480 \div 6$. How do you know that the quotient for $540 \div 2$ must be a greater number?
Relating Division Calculations

Learning Goal

• reasoning about the relationship between quotients.

Open Question

If you know that $20 \div 4 = 5$, you know that there are 5 groups of 4 in 20.

There are also 4 groups of 5 in 20.

This should help you figure out that:

• $24 \div 4 = 6$ since there is one more group of 4.

• $16 \div 4 = 4$ since there is one fewer group of 4.

• $40 \div 4 = 5 + 5$ since there are 5 groups of 4 in each 20 but there are two 20s.
• $200 \div 4 = 50$ since 200 is 20 tens, so there are 4 groups of 5 tens (or 50) in 200.

Choose two of the division questions below.

For each division question, tell 3 or 4 division questions that are related and explain how they are related.

$63 \div 9 = 7$  $28 \div 7 = 4$

$36 \div 9 = 4$  $56 \div 8 = 7$
We can figure out the answer to a division question by using another division question with an answer we know.

For example, suppose we want to know how many paperclips each person gets, if 4 people share 64 paperclips equally.

• One strategy is to break up what we are sharing into parts:

  We could think: I know that $40 \div 4 = 10$. If 4 people share 40 paper clips, they each get 10. Since $64 = 40 + 24$, we will share 40 paper clips and then the other 24. $40 \div 4 = 10$ and $24 \div 4 = 6$, so $64 \div 4 = 10 + 6 = 16$.

• Another strategy is to multiply or divide both numbers by the same amount.

  For example, $48 \div 8 = 24 \div 4$. We can divide both numbers by 2. That’s because if 8 people share 48, then 4 (half as many as 8) of them share 24 (half of the whole 48).

We can always try to relate the new question to questions that we find easier.
1. How much more is the first quotient than the second one? Tell why.
   a) \( 48 \div 4 \) than \( 44 \div 4 \)
   b) \( 30 \div 5 \) than \( 25 \div 5 \)
   c) \( 56 \div 8 \) than \( 40 \div 8 \)
   d) \( 78 \div 6 \) than \( 60 \div 6 \)

2. How would you break up each amount to make it easy to work with the groups?
   a) 55 to make groups of 5
   b) 75 to make groups of 3
   c) 96 to make groups of 6
   d) 108 to make groups of 4

3. How would you break up each amount to make it easier to share?
   a) 68 into 4 groups
   b) 92 into 4 groups
   c) 84 into 7 groups
   d) 93 into 3 groups
4. Alyssa says that you can figure out $72 \div 6$ by taking half of 72 and dividing that half by 3. So she figures out $72 \div 2$ and then divides the answer by 3. Do you agree with Alyssa? Explain.

5. Why is it easier to compare $48 \div 8$ with $40 \div 8$ than to compare $48 \div 8$ with $48 \div 3$?

6. Describe three different strategies you could use to calculate $96 \div 4$. 
Dividing by 1-digit Numbers

Learning Goal

• representing a quotient using sharing and grouping models.

Open Question

Select 5 different numbers between 50 and 500.

Divide each of them by at least three different whole numbers between 1 and 10. Try to get as many different remainders as you can.

• What did you divide by?

• What are your remainders?
Dividing by 1-digit Numbers

(Continued)

Think Sheet

We have to put 86 sandwiches into bags of 2 sandwiches each.
We want to know how many bags we need.

• To figure out $86 \div 2$,
  we might think: I’ll put the first 40 into 20 bags of 2.
  Then I’ll put the next 40 into 20 bags of 2.
  Then I’ll put the last 6 into 3 bags of 2.
  I used $20 + 20 + 3 = 43$ bags.

• We could also think: $86 = 80 + 6$
  So $86 \div 2 = (80 \div 2) + (6 \div 2)$
  \[= 40 + 3\], or 43

Suppose 2 people were sharing $87.

• We could model 8 tens and 7 ones using base-ten rods and small cubes.

• We could make 2 equal piles of 4 tens and 3 ones, with a remainder of 1.

So $87 \div 2 = $43 R1 ($R$ means remainder).
Each share is $43 with $1 left. Since it is money, we could trade the $1 for quarters and each person would get 2 quarters or 50 cents.
Each person’s share is $43.50.

1. Complete each question with a single digit.
   a) ___5 ÷ 3 is about 20.
   b) ___44 ÷ 7 is about 30.
   
   c) ___71 ÷ 9 is about 40.
   d) ___02 ÷ 5 is about 80.
2. Calculate each quotient. Tell why your answer seems reasonable.
   a) \(95 \div 5\)
   
   b) \(79 \div 6\)
   
   c) \(120 \div 8\)
   
   d) \(153 \div 6\)

3. In what ways can you seat 96 people in equal rows of chairs?

4. You know that \(96 \div 3 = 32\).
   a) How do you know that \(98 \div 3 = 32 \text{ R}2\)?
   
   b) How do you know that \(99 \div 3 = 33\) (and not \(32 \text{ R}3\))?

5. Use the digits 4, 1, and 2 to make four different 3-digit numbers (e.g., 412).
   • Divide each number by 3.
• What do you notice about the remainders?

• Does the same thing happen if you use a different set of three digits to start with?