Module 5
Multiplying and Dividing

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MULTIPLYING AND DIVIDING
WHOLE NUMBERS

Relevant Learning Expectation for Grade 6

*Use a variety of mental strategies to solve … multiplication, and division problems involving whole numbers (e.g., use the commutative property: 4 × 16 × 5 = 4 × 5 × 16, which gives 20 × 16 = 320; use the distributive property: (500 + 15) ÷ 5 = 500 ÷ 5 + 15 ÷ 5, which gives 100 + 3 = 103)*;

Possible reasons a student might struggle in multiplying and dividing whole numbers

Students may struggle performing either paper-and-pencil or mental multiplication and division involving whole numbers (including multiplying 1-digit by up to 4-digit numbers or 2-digit by 2-digit numbers and the reverse) because they might:
- be uncertain about many multiplication and related division facts
- be unfamiliar with the use of the distributive principle in multiplication and division situations
- not recognize the relationship between potentially related multiplication calculations
- mistakenly apply the distributive principle to division calculations, breaking up the divisor rather than the dividend (e.g., 45 ÷ 9 as 45 ÷ 5 + 45 ÷ 4)
- not recognize the relationship between multiplication and division

Additional considerations

It is the intent of Questions 2 and 8 to determine whether students have facts committed to memory. Students are free to use strategies, but the idea is to discourage paper-and-pencil calculations for these questions.
Administer the diagnostic

You may use all or only part of these sets of materials, based on student performance with the diagnostic.

If students need help in understanding the directions of the diagnostic, clarify an item’s intent.

Using the diagnostic results to personalize interventions

Intervention materials are included for each of these topics:
• Multiplication Fact Strategies
• Multiplying by 1-digit Numbers
• Multiplying Two 2-digit Numbers
• Relating Multiplication to Division
• Relating Division Calculations
• Dividing by 1-digit Numbers

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If students appear to need more than one lesson, begin with the simplest lesson.

Be aware that success with that lesson may minimize the need for some other lessons. You can readminister diagnostic items to decide.

Materials
• base ten blocks (optional)
Solutions
1. a) 1 x 5, or 5  b) 2 x 8, or 16  c) 1 x 9, or 9  d) 3 x 8, or 24
2. a) 35  b) 12  c) 72  d) 28  e) 48  f) 63  g) 32  h) 27  i) 210  j) 360  k) 1500  l) 2400
3. a) 10  b) 6 x 8 (any product that is 48 is correct)  c) 300, 7, 2
4. a) 3 x 48, or 144  b) 2 x 53, or 106  c) 1 x 23, or 23  d) 2 x 58, or 116
5. (Any process to show that 23 is multiplied by both 8 and 40 and the parts are added is appropriate); product is 1104
6. a) e.g., 480  b) e.g., 400  c) e.g., 2400  d) e.g., 4000  e) e.g., 1500  f) e.g., 4800
Note: There are many appropriate estimates for question 6. Rounding is not required. Estimates should not be too far off from values indicated above.
7. a) 7 ÷ 7 or 1  b) 18 ÷ 6 or 3  c) 9 ÷ 9 or 1  d) 16 ÷ 8 or 2
8. a) 7  b) 6  c) 4  d) 7  e) 7  f) 8  g) 9  h) 4  i) 40  j) 20  k) 300  l) 500
9. a) 20  b) 200, 4  c) e.g., 500, 5; 100, 5 (other ways to break up 600 are also reasonable)
10. b, c
11. a) e.g., 30  b) e.g., 60  c) e.g., 40  d) e.g., 300  Note: There are many appropriate estimates. Rounding is not required. Estimates should not be too far off from values indicated.
12. (Any process to show that 328 is shared into 8 equal groups or that groups of 8 are taken from 328 is appropriate.); quotient is 41
The purpose of the suggested work is to help students build a foundation for work in proportional reasoning and integer and fraction computations.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:

• Questions to ask before using the approach
• Using the approach
• Consolidating and reflecting on the approach
Multiplication Fact Strategies

Learning Goal

• reasoning about the relationship between products of single digit numbers.

Open Question

Questions to Ask Before Using the Open Question

To make sure students understand what multiplication is, ask them to use counters to show you what $4 \times 3$ means. Write $4 \times 3$ down. [Read this as 4 times 3 at this point, although subsequently reading $4 \times 3$ as 4 groups of 3 is a good idea. Ideally, students will create 4 groups of 3.]

◊ What does the 4 tell you? (how many groups there are)
◊ What does the 3 tell you? (how many are in each of the groups)
◊ Now show me $3 \times 4$. [If students used 4 groups of 3 before, this time it should be 3 groups of 4. If students used 3 groups of 4 for the first situation (which is not incorrect, although not the Canadian convention), then this time there should be 4 groups of 3.]

Arrange the 3 groups of 4 into an array, if they are not already in an array. Split the array in two.

◊ How does this show that $3 \times 4$ is the same as $3 \times 2$ and $3 \times 2$ added together? (There is a group of $3 \times 2$ and another group of $3 \times 2$.)
◊ How does it show that $3 \times 4$ is double $3 \times 2$? (There is a $3 \times 2$ twice.)

Using the Open Question

Provide counters for students to use.

Work with the students through the introduction to the Open Question, making sure they understand that the fact $4 \times 5 = 20$ is used to make it easier to determine $4 \times 6$ and $5 \times 4$.

Encourage them to choose whichever two facts at the bottom of the page they wish and to list as many related facts as they can.

By viewing or listening to student responses, note if they realize that:

• changing the value of $a$ in $a \times b$ by 1 changes the value of the product by $b$.
• changing the value of $b$ in $a \times b$ by 1 changes the value of the product by $a$.
• doubling $a$ or $b$ results in doubling $ab$
• that any product can be related to simpler products

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe how many relationships students recognize.

For $3 \times 6 = 18$, as an example:

◊ What happens if you increase the 3 to 4? (There would be an extra group of 6, so $4 \times 6 = 24$.)
◊ Why did you change the 6 to 12? (If you have 3 groups of 12, you have twice as much as with 3 groups of 6, so that's 36.)
◊ Why did you change the 3 to 6? (If you have twice as many groups, you have twice as much.)
◊ How is $3 \times 3$ related to $3 \times 6$? (You have half as many in each group, so half as much altogether.)
◊ Is $4 \times 7$ related to $3 \times 6$? (You could, but it would take a couple of steps. I know that $4 \times 6 = 24$ since there is an extra group of 6. Then I know that $4 \times 7 = 28$ since there would be an extra one in each of the four groups.)

Materials

• counters
Solutions

e.g., for $3 \times 6$: $3 \times 7 = 21$ since it’s one more in each of the three groups
4 $\times 6 = 24$ since there is one more group of 6
3 $\times 12 = 36$ since there would be twice as many in each group, so twice as many altogether
6 $\times 6 = 36$ since there are twice as many groups, so twice as many altogether
2 $\times 6 = 12$ since there is one fewer group of 6
5 $\times 6 = 30$ since there are two extra 6s

e.g., for $4 \times 6$: $5 \times 6 = 30$ since there is one more 6
8 $\times 6 = 48$ since there are twice as many groups, so twice as many things altogether
4 $\times 12 = 48$ since there are twice as many in each group, so twice as many things altogether
4 $\times 7 = 28$ since there is one more in each of the 4 groups
2 $\times 6 = 12$ since there would be half as much if there were only half as many groups
4 $\times 3 = 12$ since there would be half as much if there were only half as many things in each group

e.g., for $3 \times 4$: $4 \times 4 = 16$ since there is one more group of 4
3 $\times 5 = 15$ since there is one more in each of the 3 groups
6 $\times 4 = 24$ since there are twice as many groups, so twice as many things altogether
3 $\times 8 = 24$ since there are twice as many in each group, so twice as many things altogether
3 $\times 2 = 6$ since there would be half as much if there were only half as many things in each group

e.g., for $5 \times 6$: $6 \times 6 = 36$ since there is one more group of 6
7 $\times 6 = 42$ since there are two more groups of 6
5 $\times 12 = 60$ since there are twice as many in each group, so twice as many things altogether
10 $\times 6 = 60$ since there are twice as many groups, so twice as many things altogether
5 $\times 7 = 35$ since there is one more in each of the 5 groups
Think Sheet

Questions to Ask Before Assigning the Think Sheet

To make sure students understand what multiplication is, ask them to use counters to show you what \( 4 \times 3 \) means. [Read this as 4 times 3 at this point, although subsequently reading \( 4 \times 3 \) as 4 groups of 3 is a good idea. Ideally, the student will create 4 groups of 3.]

◊ What does the 4 tell you? (how many groups there are)
◊ What does the 3 tell you? (how many are in each of the groups)
◊ Now show me \( 3 \times 4 \). [If the student used 4 groups of 3 before, this time it should be 3 groups of 4. If the student used 3 groups of 4 for the first situation (which is not incorrect, although not the Canadian convention), then this time there should be 4 groups of 3.]

Arrange the 3 groups of 4 into an array, if they are not already in an array. Split the array in two.

◊ How does this show that \( 3 \times 4 \) is the same as \( 3 \times 2 \) and \( 3 \times 2 \) added together? (There is a group of \( 3 \times 2 \) and another group of \( 3 \times 2 \) there.)
◊ How does it show that \( 3 \times 4 \) is double \( 3 \times 2 \)? (There is a \( 3 \times 2 \) twice.)

Using the Think Sheet

Read through the introductory box with the students. Make sure they understand the various strategies shown. You may want to spend a little time making sure students can easily multiply by 2 and 5 first.

Provide counters so that students can represent the relevant multiplications. However, indicate that eventually you want them to be able to use the same kind of thinking without the counters in front of them.

Assign the tasks.

By viewing or listening to student responses, note if they realize that:
- changing the value of \( a \) in \( a \times b \) by 1 changes the value of the product by \( b \).
- changing the value of \( b \) in \( a \times b \) by 1 changes the value of the product by \( a \).
- doubling \( a \) or \( b \) results in doubling \( ab \)
- different multiplications can result in the same product
- that any product can be related to simpler products, usually in several ways

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ How could you have predicted that \( 3 \times 8 \) and \( 6 \times 4 \) would have the same answer? (If you take the 3 groups of 8 and split each in half, you’ll have the same amount, but it will be 6 groups of 4 and that’s what \( 6 \times 4 \) is.)
◊ Why does it make sense that \( 4 \times 3 \) and \( 6 \times 2 \) are both 12? (\( 4 \times 3 \) is the same as \( 3 \times 4 \). \( 6 \times 4 \) would be twice as much as that since there are twice as many groups of 4, but \( 6 \times 2 \) would be back to 12 (half as much as 24), since you only have half as many in each group.)
◊ Can you think of other ways you might multiply by 8? (e.g., You could multiply by 4 and double. You could multiply by 2 three different times. You could multiply by 6 and multiply by 2 and add the parts.)
◊ Why is it useful to have these strategies? (Sometimes you just forget something and it is good to have other ways to help you remember it.)

Materials

◊ counters
Solutions

1. a) 12 since there are the same amount in 4 rows of 3 as 3 columns of 4
   b) 16 since there is one more group of 4
   c) 24 since there are twice as many groups of 4
   d) 24 since there are twice as many things in each group

2. a) 20 since there is one more group of 5
   b) 30 since there are twice as many groups of 5
   c) 21 since there are 2 more things in each group and since there are 3 groups, that’s 6 more altogether

3. a) 2 × 6, 3 × 4, 4 × 3, 6 × 2
   b) 3 × 8, 4 × 6, 6 × 4, 8 × 3
   c) 2 × 9, 3 × 6, 6 × 3, 9 × 2
   d) 5 × 6, 6 × 5 [Some students may also consider 3 × 10, 10 × 3, 2 × 15, 15 × 2.]

4. 8 groups of something is like 5 groups of it and 2 more groups of it and 1 more group of it since 5 + 2 + 1 = 8

5. a) e.g., figure out 4 × 9 and then double it OR figure out 8 × 10 and subtract 8
   b) e.g., figure out 7 × 4 and double it OR figure out 5 × 8 and 2 × 8 and add them
   c) e.g., figure out 3 × 8 and double it OR figure out 6 × 4 and double it
   d) e.g., figure out 3 × 7 and double it OR figure out 6 × 5 and 6 × 2 and add them
Multiplying by 1-digit Numbers

Learning Goal
• representing fractions greater than 1 using an area as a whole.

Open Question

Questions to Ask Before Using the Open Question

Begin by asking
◊ How would you calculate 6 × 20 and 6 × 200. (6 sets of 2 tens is 12 tens, so that is 120. 6 sets of 2 hundreds is 12 hundreds, so that’s 1200.)
◊ Why did you really only need to know 6 × 2 to figure these out? (It was always 6 × 2 I was doing, but the two was sometimes tens and sometimes hundreds.)

Write the expression 4 × 26 and ask
◊ How many items are in 4 groups of 26? Let students proceed to explain their own ways. Have counters and base ten blocks readily available to use but do not explicitly suggest their use. (I know 4 quarters is a dollar, so it’s 100 plus an extra four cents – 104)
◊ If you had thought of 26 as 10 and 10 and 6, could that have helped too? (Yes, 4 groups of 10 is 40 and 4 groups of 6 is 24, so there is 40 + 40 + 24.)
◊ How could you model that with base ten blocks? (You could put down 2 tens blocks and 6 ones blocks four times)
◊ What if you had been multiplying 4 × 126? How would the product change? (It would be 400 more since there are 4 extra 100s.)
◊ How might you have predicted that the ones digit in the product would be 4? (4 × 20 ends in a 0, so only the 4 × 6 part gives you something in the ones place. Since 4 × 6 is 24, it’s a 4.)
◊ How would you predict the tens place? (You would need to consider the 2 from the 6 × 4 part as well as the 4 × 2 tens.)

Using the Open Question

Provide base ten blocks for students to use.

Work with them through the introduction to the Open Question, making sure they understand that the result must be a 4-digit number (so that, for example, 4 × 212 is not a possible situation).

If a student is still struggling with 3-digit number multiplication, the problem can be changed to a 1-digit × 2-digit multiplication.

They will, undoubtedly, do a lot of practice with multiplication in order to come up with their solutions.

By viewing or listening to student responses, note if they recognize that:
• it is useful to break up the 3-digit number into parts to perform the multiplication
• the multiplication involving the ones place as well as the multiplication involving the tens place are the only ones that contribute to the fact that there is a 5 in the tens place in the product
• the 5 in the tens place could be part of 5, 15, 25, 35, or 45

Depending on student responses, use your professional judgement to guide specific follow-up.
Consolidating and Reflecting on the Open Question

◊ How did you know that you wouldn’t multiply by 1? (I wouldn’t get a 4-digit number.)
◊ How did you know that if you multiplied by 2, there should be a fairly large hundreds digit? (To get a 4-digit number, the hundreds should be 5 or more.)
◊ How do you know that $2 \times 877$ makes sense? (Because when you multiply $2 \times 7$ tens, you get 14 tens. When you multiply $2 \times 7$, you get an extra ten, so there are 15 tens altogether. You would trade 10 tens for 100 and be left with 5 tens in the tens place.)
◊ How did you figure out what numbers to try? (I knew that $2 \times 25 = 50$, so I used 2 with different numbers that ended in 25. I also knew that $5 \times 10 = 50$, so I used 5 with different numbers that ended in 10. I knew that if I multiplied $3 \times$ something ending in 51 I would get more answers. Then I just tried other combinations.)

Solutions

e.g., $2 \times 625 = 1250$
$5 \times 410 = 2050$
$3 \times 651 = 1953$
Questions to Ask Before Assigning the Think Sheet

Create an array that is 4 × 15. Place a pencil to divide it into two sections: a 4 × 10 section and a 4 × 5 section.

◊ How do you know that there are 4 × 15 counters here? (There are 4 rows of 15.)
◊ How does my pencil show that it’s actually the same as 4 × 10 and 4 × 5? (Because there are 4 rows of 10 at the left and 4 rows of 5 at the right.)

Now make a rectangle by lining up 4 identical rows of one horizontal tens block and 5 ones blocks.

◊ How does this show 4 × 15? (There are 4 rows of 15.)
◊ How does it show 4 × 10 and 4 × 5? (There are 4 tens blocks and 4 rows of 5 one-blocks.)

◊ How could you use the blocks to show 4 × 32? (I could make 4 rows, each with 3 tens blocks and 2 ones blocks.)
◊ How many tens would there be? How many ones? (12 tens and 8 ones)
◊ How could you have predicted that? (There are 4 sets of 3 tens, so that’s 12 tens. There are 4 sets of 2 ones, so that’s 8 ones.)

Put out 3 groups of 1 hundred block, 3 tens blocks, and 4 ones blocks.

◊ What multiplication am I showing here? (3 × 134)

Using the Think Sheet

Read through the introductory box with the students.

Encourage students to follow along by using their own blocks to see what is happening on the page.

Spend time discussing the last section about estimating. Make sure students understand that when you estimate, there are many appropriate answers, but you just want something reasonable. Point out, for example, that someone might have estimated 5 × 679 as 5 × 7 hundred. That is 35 hundred (3500).

Assign the tasks.

By viewing or listening to student responses, note if they realize that:

• it is useful to break up the 2-digit or 3-digit number into parts to perform the multiplication
• an estimation might be based on using only the front digit of each number or other nearby benchmarks
• sometimes you might add and other times you might subtract from a related multiplication (e.g., Question 4) to simplify a multiplication calculation

Depending on student responses, use your professional judgement to guide further follow-up.
## Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ **What are some different ways you could have figured out** $4 \times 29$? (I chose to think of 29 as 25 + 4. I’d multiply each part by 4 and add them. I could have multiplied $4 \times 20$ and $4 \times 9$ and added them too.)

◊ **Which method do you prefer?** (I like 25 + 4 since it’s really easy to multiply 25 by 4.)

◊ **How is multiplying by a 3-digit number like multiplying by a 2-digit number?** (It’s really the same idea except that there’s one extra piece to include.)

◊ **What do you think is the most important to think about when you estimate?** (That the answer doesn’t have to be exact, but it should be close.)

◊ **Why did you predict that** $8 \times 346$ **would be more than** $6 \times 348$? (because the first one is about 2400 and the second one is about 2000.) Would it be easy to predict whether $8 \times 346$ or $6 \times 438$ would be more? (No since both are close to 2400.)

### Solutions

1. a) 72  
   b) 192  
   c) 133  
   d) 147

2. a) e.g., $2 \times 29 + 2 \times 29$ OR $4 \times 20 + 4 \times 9$  
   b) e.g., $7 \times 50 + 7 \times 6$  
   c) e.g., $8 \times 200 + 8 \times 10 + 8 \times 2$  
   d) e.g., $9 \times 500 + 9 \times 70 + 9 \times 3$

3. a) e.g., 1600  
   b) e.g., 4000  
   c) e.g., 1200  
   d) e.g., 5000

4. She could subtract 1 group of 88 and she’d have the right amount.

5. Agree, since 6 groups of 95 is the same as 2 groups and 2 groups and 2 groups

6. e.g., $4 \times 836$, $8 \times 346$ and $3 \times 468$. I predict $8 \times 346$ is greatest.  
   It is since $4 \times 836 = 3344$, $8 \times 346 = 2768$, $3 \times 468 = 1404$
Multiplying Two 2-Digit Numbers

Learning Goal

- representing a product using an area model or relating to a simpler product.

Open Question

Questions to Ask Before Using the Open Question

Suggest to students that a school has 12 classes, each with 28 students.

◊ How would you figure out how many students are in the school? (e.g., I would multiply 6 × 28 and then double it.)

◊ That’s a good plan. Is there any other way you could do it? (Yes, I think you could figure out 4 × 28 and then triple it.)

Put out 12 groups of 2 tens blocks and 8 ones blocks.

◊ What multiplication am I showing here? (12 × 28)

◊ Show me how to figure out how much it is. You can trade whenever you need to. (336)

Using the Open Question

Provide base ten blocks for students to use.

Work with them through the introduction to the open question. Have them make the 16 × 23 array shown, replacing each group of 10 tens blocks with a hundred block and each group of 10 adjacent ones blocks with a ten-block.

Help them notice that the rectangle they formed has a length of 23 and a width of 16.

Have students calculate each of the four products requested and respond to the request to compare them.

By viewing or listening to student responses, note if they:

- can multiply two 2-digit numbers efficiently or whether they are attracted to less efficient approaches
- recognize that it may be helpful to break up the product required into four partial products
- see that a 2-digit × 2-digit multiplication can be modelled by determining the area of an appropriate rectangle
- recognize that halving one factor and doubling another results in the same product

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ How could you have predicted that 37 × 39 would be close to 1600? (It’s close to 40 × 40 and that’s 1600.)

◊ How did you figure out 37 × 39? (e.g., I made a rectangle that was 39 wide and 37 deep. I filled it with blocks)

◊ I noticed that you modelled 38 × 38 by making 38 groups of 38. That was a good idea, but would there have been a quicker way to do it? (I could have made a rectangle 38 by 38 and started by filling it with hundred blocks. That would have been faster.)

◊ How might you have predicted that 19 × 76 and 38 × 38 would have the same product? (I know that 19 is half of 38, so 19 × 38 is half of 38 × 38. But 19 × 76 is twice as much as 19 × 38, so I’m back to the original product.)
Solutions

e.g., 37 × 39 and 38 × 38 are only 1 apart (1443 and 1444)
   37 × 39 and 19 × 76 are only 1 apart
   37 × 39 and 33 × 41 both end in 3
   38 × 38 and 19 × 76 have the same product
   38 × 38 and 33 × 41 are both in the 1 thousands
   19 × 76 and 33 × 41 both have at least one digit repeated
Questions to Ask Before Assigning the Think Sheet

Suggest to students that a school has 12 classes, each with 28 students.
◊ How would you figure out how many students are in the school? (e.g., I would multiply 6 × 28 and then double it.)
◊ That’s a good plan. Is there any other way you could do it? (Yes, I think you could figure out 4 × 28 and then triple it.)

Put out 12 groups of 2 tens blocks and 8 ones blocks.
◊ What multiplication am I showing here? (12 × 28)
◊ Show me how to figure out how much it is. You can trade whenever you need to. (336)

Using the Think Sheet

Read through the introductory box with the students.

Encourage students to follow along by using their own blocks to see what is happening on the page. Read what happens by saying: We showed 16 × 24 as multiplying 10 × 24 and 6 × 24 and adding the pieces. But we did those two calculations in steps too—we showed 16 × 24 by adding 10 twenties to 10 fours and we showed 6 × 24 by adding 6 twenties and 6 fours. Make sure students relate each of those partial products to the appropriate section of the rectangle.

Discuss why estimation is always a good idea to see if a calculated answer makes sense.

Assign the tasks.

By viewing or listening to student responses, note if they:
• can multiply two 2-digit numbers efficiently or whether they are attracted to less efficient approaches
• recognize how to break up the product required into four partial products
• see that a 2-digit × 2-digit multiplication can be modelled by determining the area of an appropriate rectangle
• can estimate the product of two 2-digit numbers
• recognize that sometimes there are shortcuts for multiplying by relating one calculation to another simpler calculation
• recognize that you can halve one factor and double another without changing the product

Depending on student responses, use your professional judgement to guide further follow-up.
Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What did you focus on in the models in Question 1 to help you decide the product it matched? (e.g., At first I just checked to see if the number of ones was the product of the ones places, but the last two were the same. For those I looked for one rectangle that was almost a square and the other that was long and thin.)
◊ Where was the 24 and where was the 27 in the rectangle you chose for b? (The 24 was how deep it was and the 27 was how wide it was.)
◊ So where is the product? (It's the area.)
◊ Which of the products did you match to 10 seventies plus 10 threes plus 4 seventies plus 4 threes in Question 2? (14 × 73)
◊ Why does that make sense? (Because 14 groups of 73 is 10 groups of 73 and 4 groups of 73. And to get groups of 73, you can break it into groups of 70 and groups of 3.)
◊ You didn’t choose 60 × 55 as being close to 2000 in Question 3? Why not? (It's actually more than 3000 since 60 × 50 is 30 hundreds and that's 3000.)
◊ What other two products can you think of that would have the same relationship to each other as 38 × 50 and 76 × 25? Why? (e.g., It could be 40 × 50 and 80 × 25 since both times I double the first number and take half of the second one.)
◊ What strategies did you learn that would help you figure out 48 × 52? (e.g., I could make a rectangle that is 48 wide and 52 deep and see what blocks fill it. I could figure out 40 × 50 and 40 × 2 and 8 × 50 and 8 × 2 and add them.)
◊ Why did you predict that 83 × 64 would be more than 38 × 46? (e.g., The rectangle would be a lot wider and deeper.)

Solutions

1. a) H
   b) E
   c) F
   d) G

2. a) G
   b) H
   c) E
   d) F

3. b, d

4. a) 210
   b) 576
   c) 325
   d) 512

5. She could subtract 1 group of 88 from 40 groups to have 39 groups.

6. e.g., If you have 38 groups of 50, you could split each group in 2 and you’d have 76 groups of 25.

7. e.g., 36 × 48, 86 × 43, 63 × 48
   I predict 86 × 43 is greatest since it’s almost 90 × 40, and the others are a lot less.
   I checked.
   36 × 48 = 1728
   86 × 43 = 3698
   63 × 48 = 3024
Relating Multiplication to Division

Learning Goal

- connecting any division question to a related multiplication.

Open Question

Questions to Ask Before Using the Open Question

◊ What does 20 ÷ 4 mean? (It means 5.)
◊ But where does the 5 come from? (You can share 20 into 4 groups and each group has 5 things in it.)
◊ Let's do that. Show 4 groups of 5. I see 4 × 5 = 20 here. How did that happen if we were solving 20 ÷ 4? (When you multiply you have a lot of equal groups making up a total. When you divide, you have the total and know how many groups and then you see how many are in each group.)
◊ How could you use multiplication to help you solve 30 ÷ 6? (You figure out what to multiply 6 by to get 30.)
◊ Suppose I divide a number between 20 and 24 by 3. What do you know about the quotient? [Note: You may have to define quotient as the result of a division.] (It's more than 6 and less than 8.)
◊ How do you know? (since 18 ÷ 3 = 6 and 24 ÷ 3 = 8)

Using the Open Question

Provide counters and base ten blocks

Work with the students through the introduction to the Open Question.

Have students respond to each of the four provided parts. Remind them that they will need to create two of their own.

By viewing or listening to student responses, note if they:

• use multiplication facts to help them relate multiplications to divisions
• understand why there is a range of possible responses when there is a range in dividends

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ How did you know that a number between 30 and 50 divided by 7 is more than 4? (I know that 7 × 4 = 28 and 30 is more.)
◊ How did you use multiplication facts to figure out the possible results when A was between 200 and 300 and you divided by 4? (I thought that 4 × 5 is 20, so 4 × 50 is 200. I also thought that 4 × 7 is 28, so 4 × 70 is 280.)
◊ Suppose you divided 420 by 8. How would you estimate the result? (I know that 8 × 5 = 40, so 8 × 50 = 400. The quotient has to be close to 50.)

Solutions

e.g., The quotient is more than 6 but not more than 10 since 6 × 6 is only 36 and 6 × 10 is 60.
The quotient is more than 4 but not (much) more than 7 since 7 × 4 = 28 and 7 × 7 = 49.
The quotient is between 60 and 80. That's because 5 × 60 = 300 and 5 × 80 = 400.
The quotient is between 50 and 180 since 4 × 50 = 200 and 4 × 80 = 320.

e.g., A is between 50 and 60 and B is 8. (The quotient is between 6 and 8).
A is between 730 and 800 and B is 9. (The quotient is between 80 and 90 since 9 × 80 = 720 and 9 × 90 = 810).
Questions to Ask Before Assigning the Think Sheet

◊ How many groups of 4 can I make if our class has 28 students? (7)
◊ How do you know? (I just keep using up 4s until I get up to 28.)
◊ Is there another way to know? (I guess since 7 \times 4 = 28 there would be 7 groups of 4.)
◊ In what sorts of real-life situations do we divide? (when you are sharing or when you are making equal groups)
◊ Suppose I had 20 books and I move 5 of them at a time. How many moves will it take? (4)
◊ How do you know? (I would go from 20 to 15 to 10 to 5 to 0 and that’s four moves.)
◊ How is that like doing this calculation: 20 – 5 – 5 – 5 – 5? (I started with 20 and just kept taking 5 until I got to 0.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure that they realize that division applies both to counting how many equal groups and to sharing. Point out that in both cases, there are equal groups, which is true for multiplication as well as division.

Make sure that they understand that repeated subtraction is an alternative to relating to a known multiplication fact to help them divide. Provide counters so students can see the repeated subtraction being performed.

Finally, ensure that they see the relationship between situations such as $35 \div 7$ and $350 \div 7$ either by relating to multiplication or thinking of $350$ as $35$ tens.

Assign the tasks.

By viewing or listening to student responses, note if they:
- recognize that one way to divide is to repeatedly subtract
- recognize that multiplication and division are intrinsically related, both describing a situation where equal groups are created
- realize that division can be used to describe both sharing situations and ones involving the creation of equal groups

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why is there always more than one way to do a division? (You can do it as a multiplication or a subtraction or just do it as a division with counters.)
◊ What divisions might knowing that $4 \times 9 = 36$ help you do? (e.g., $36 \div 4$ or $360 \div 9$)
◊ When you just know the quotient, why can’t you be sure what numbers were divided? (e.g., You get the same quotient lots of times. For example $16 \div 2$ is the same as $24 \div 3$.)
◊ What do you know? (You know how many times as much as the second number the first number is.)
Solutions

1. a) 9, e.g., since $5 \times ? = 45$ has to be 9; also $45 - 9 - 9 - 9 - 9 = 0$, so I took away five 9s.
   b) 8, e.g., $56 = 28 + 28$ and $28 \div 7 = 4$, so the answer is $4 + 4$; also if $7 \times ? = 56$, it must be 8.
   c) 9, e.g., $54 = 30 + 24$. $30 \div 6 = 5$ and $24 \div 6 = 4$ and $5 + 4 = 9$, also $6 \times 9 = 54$

2. a) $4 \times 9 = 36$ b) $8 \times 4 = 32$ c) $9 \times 7 = 63$ d) $7 \times 7 = 49$

3. e.g., $40 \div 5$, $16 \div 2$, $24 \div 3$, $32 \div 4$, $48 \div 6$

4. e.g., $360 \div 9$, $280 \div 7$, $200 \div 5$, $160 \div 4$, $120 \div 3$

5. e.g., If 6 people share 480 things, $480 \div 6$ tells the share each person gets. But for $540 \div 2$, there are more things and fewer people sharing them, so each person gets more.
Relating Division Calculations

Learning Goal

• reasoning about the relationship between quotients.

Open Question

Questions to Ask Before Using the Open Question

◊ What does $15 \div 3$ mean? (You have to figure out how many in each group if you are making 3 groups.)
◊ Let’s do that. How many were in each group? (5)
◊ Suppose you had started with 18 counters. Do you know how many would be in each group? (6)
◊ How do you know? (There were 3 extra, so that’s one more in each of the 3 groups.)
◊ What division is this? ($18 \div 3$)
◊ Suppose you had started with only 12 counters and had wanted to make 3 groups. How many would be in each group? (4 since there is one fewer in each of the groups)
◊ What if you had started with twice as many counters and still wanted 3 groups. How many would be in each group? (twice as many)
◊ Here are 15 tens blocks. You want to make 3 groups. How many would be in each group? (5 tens blocks)
◊ What division could you write? ($150 \div 3 = 50$)

Using the Open Question

Provide counters and base ten blocks

Work with the students through the introduction to the Open Question.

Make sure they understand that the goal is to see how knowing one fact actually makes it easier for you to figure out other ones.

Ask students to choose two of the calculations on the page to work with. Some students may choose to do more.

By viewing or listening to student responses, note if they:
• use the meaning of division to see how different division calculations are related
• can relate divisions involving multiples of ten to those involving simpler values

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ Why did you double the 28 but not the 7? (e.g., I knew that if I doubled the 28, there would be twice as many in each group. I could have doubled the 7, I guess. Then there would be half as many in each group since there would be twice as many groups.)
◊ How did you know that $45 \div 9 = 5$ from knowing $36 \div 9 = 4$? (There would be one more group of 9 since 45 is 9 more than 36.)
◊ Could you figure out what $2800 \div 7$ is? (Yes, it’s 28 hundreds $\div 7$, so it’s 4 hundred.)
◊ Why would $56 \div 4$ be twice as much? (Because there are only 4 groups instead of 8, so you have to combine things from two groups at a time.)

Materials

• counters
• base ten blocks
Solutions

e.g., 63 ÷ 9: 630 ÷ 9 = 70 since 630 is 63 tens and 63 tens ÷ 9 = 7 tens
   72 ÷ 9 = 8 since there is one extra group of 9
   54 ÷ 9 = 6 since there is one fewer group of 9

e.g., 28 ÷ 7: 14 ÷ 7 = 2 since, if there is half as much, there are half as many groups
   56 ÷ 7 = 8 since, if there is twice as much, there are twice as many groups
   35 ÷ 7 = 5 since, if there is 7 more, there is 1 more group of 7
   280 ÷ 7 = 40 since 280 = 28 tens and 28 tens ÷ 4 = 7 tens

e.g., 36 ÷ 9: 18 ÷ 9 = 2 since, if there is half as much, there are half as many groups
   72 ÷ 9 = 8 since, if there is twice as much, there are twice as many groups
   360 ÷ 9 = 40 since 360 = 36 tens and 36 tens ÷ 9 = 4 tens
   45 ÷ 9 = 5 since, if there is 9 more, there is 1 more group of 9

e.g., 56 ÷ 8: 56 ÷ 7 = 8 since, if there are 7 groups of 8 in 56, there are also 8 groups of 7
   112 ÷ 8 = 14 since, if there is twice as much, there are twice as many groups
   560 ÷ 8 = 70 since 560 = 56 tens and 56 tens ÷ 8 = 7 tens
   28 ÷ 4 = 7 since, if there is half as much stuff, but half as many people share, they get the same amount
Questions to Ask Before Assigning the Think Sheet

Put out 42 counters, 21 red ones and 21 blue ones. Tell students you want to put the counters into 3 separate groups.
◊ Would it help me to put just the red ones and then just the blue ones each into 3 groups? (Yes, since then I could combine each red group with a blue group and the whole 42 counters would be in 3 groups.)
◊ What is the division we just did? (We did 42 ÷ 3 by doing 21 ÷ 3 and 21 ÷ 3 and adding.)
◊ How could you use what you know about 20 ÷ 4 to help you figure out 32 ÷ 4? (You could think of 32 as 20 and 12. You could share the 20 into 4 groups and then the 12 into 4 groups and then put them together.)
◊ Show me how you’d model 10 ÷ 2. (I made 2 piles and there are 5 in each pile.)
◊ Suppose I put out another 2 piles of 5. Would it change how many are in a pile? (No)
◊ What division am I doing now? (20 ÷ 4)
◊ What do you notice about the numbers in the two divisions? (The numbers in 20 ÷ 4 are the doubles of the numbers in 10 ÷ 2.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure that they understand both of the suggested strategies – both breaking up the dividend and changing both dividend and divisor by multiplying or dividing.

Most students will find the second strategy more complex. You may have to show another example, e.g., Since 36 ÷ 2 = 18, that means there are 18 groups of 2 in 36. So if you had twice as much to work with, but made groups of 4 (72 ÷ 4), you would have the same number of groups. (You would just double the size of each group.) It is important that students realize that they cannot add or subtract the same amount to or from both numbers. For example, 12 ÷ 3 is not the same as 10 ÷ 1.

Assign the tasks.

By viewing or listening to student responses, note if they:
• relate two quotients when the divisor is the same
• determine the simpler calculations to relate to
• recognize that divisions might be performed “in parts”, e.g., dividing by 6 can be accomplished by dividing by 2 and then 3 (since 2 × 3 = 6) or by 3 and then 2
• realize that changing both dividend and divisor makes it harder to compare quotients than only changing one of them

Depending on student responses, use your professional judgement to guide further follow-up.

Materials
• counters
Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ How does knowing $40 \div 8$ help you with $56 \div 8$ in Question 1? (There is an extra 16, so that's an extra 2 groups of 8.)

◊ Why is it smarter to break up 75 as $60 \div 15$ than as $70 + 5$ to make groups of 3 in Question 2? (Because you can make groups of 3 with 60 and 15 but there are leftovers with 70 and 5)

◊ How does knowing your multiplication facts help you with deciding how to break things up? (I break up the number using products of the number I am dividing by with other numbers.)

◊ Why does it make sense to divide by 4 by dividing by 2 twice? (Because you want to know how many are in a group if you make 4 groups, so you could start by making 2 groups and then splitting them. That's like dividing by 2 twice.)

◊ Can every division involving a bigger number to share be figured out by using smaller numbers? (I think so. You just break it up into smaller pieces. For example, for $66 \div 3$, just use $30 + 30 + 6$ for 66.)

Solutions

1. a) 1 more since there is 1 extra group of 4  
b) 1 more since there is 1 extra group of 5  
c) 2 more since there are 2 extra groups of 8  
d) 3 more since there are 3 extra groups of 6

2. a) e.g., $50 + 5$  
b) e.g., $60 \div 15$  
c) e.g., $60 + 36$  
d) e.g., $80 + 20 + 8$

3. a) e.g., $40 + 20 + 8$  
b) e.g., $80 + 12$  
c) e.g., $70 + 14$  
d) e.g., $90 + 3$

4. Yes since, if 6 people are sharing 72, then 3 of them are sharing half of 72 and the other 3 are sharing the other half. If you figure out the share for the 3 in the half, it's the same as the share for all 6.

5. e.g., 48 has 1 more group of 8, so you just add 1 to the answer of $40 \div 8$. But figuring out how many groups of 3 are in 48 doesn't really help you see how many groups of 8 there are.

6. e.g., Break up 96 into $80 + 16$ and find out how many groups of 4 are in each of those parts and add them.  
Divide 96 by 2 to figure out how much half of it is and then divide that amount by 2 to share it among 2 people. Break up 96 into $40 + 40 + 8 + 8$. Find out how many groups of 4 are in each of those parts and add them.
Dividing by 1-digit Numbers

Learning Goal

- representing a quotient using sharing and grouping models.

Open Question

Questions to Ask Before Using the Open Question

◊ What does 50 ÷ 5 mean? (It means that you want to know how many groups of 5 are in 50)
◊ How do you figure it out? (e.g., I just know that 5 × 10 = 50, so the answer is 10.)
◊ What about 155 ÷ 5? (You have 100 + 50 + 5. You could share the 100 by making 5 groups of 20 and the 50 by making 5 groups of 10. Then you combine to make 5 groups of 20 + 10 + 1.)

Using the Open Question

Provide base ten blocks for students to use.
Make sure students understand that they must perform the calculations and then list other divisions for the list.
By viewing or listening to student responses, note if they:

• use suitable strategies to divide 3-digit numbers by 1-digit numbers
• relate multiplication to division to create the examples to add to the list

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

◊ What different remainders did you get when you divided 101 by different numbers? (e.g., I got a remainder of 1 when I divided by 2, 4, and 5, but I got a remainder of 2 when I divided by 3 and I got a remainder of 3 when I divided by 7.)
◊ Why was it easy to figure out the remainder when you divided by 2? (It was 0 if the number was even and 1 if it was odd.)
◊ Did you get many or few remainders when you divided 311 by different numbers? (e.g., with 311, I got remainders of 1 when I divided by 2 and 5; 2 when I divided by 3; 3 when I divided by 4; 5 when I divided by 6; 6 when I divided by 7; 7 when I divided by 8; and 5 when I divided by 9—that’s a lot of remainders!)
◊ What’s the most number of remainders you could get? (8 since you won’t get a remainder if you divide by 1 and there could maybe be different remainders for dividing by 2, 3, 4, 5, 6, 7, 8 and 9.)

Solutions

74 60 101
74 ÷ 2 Remainder = 0 60 ÷ 2 Remainder = 0 101 ÷ 2 Remainder = 1
74 ÷ 4 Remainder = 2 60 ÷ 3 Remainder = 0 101 ÷ 3 Remainder = 2
74 ÷ 5 Remainder = 4 60 ÷ 7 Remainder = 4 101 ÷ 7 Remainder = 3
60 ÷ 8 Remainder = 4

201 311
201 ÷ 2 Remainder = 1 311 ÷ 2 Remainder = 1
201 ÷ 3 Remainder = 0 311 ÷ 3 Remainder = 2
201 ÷ 7 Remainder = 5 311 ÷ 4 Remainder = 3

Materials

• base ten blocks
Questions to Ask Before Assigning the Think Sheet

Put out 60 counters. Ask students
◊ How many groups of 4 can you make. (15)
◊ How could you figure out if you didn’t have the counters? (I could think about 60 as 20 and 20 and 20. There are 5 groups of 4 in each 20.)

Now show 72 using 7 tens blocks and 2 ones blocks.
◊ Suppose this represented 72 hockey cards that two boys were sharing. How many would each get? (36)
◊ How do you know? (They would each get 3 tens and then there would be 12 more to share, so they would each get 6.)
◊ How could you have solved that without the blocks? (the same way—you could think of 72 as 60 + 12 and divide each part by 2)

Using the Think Sheet

Read through the introductory box with the students.

Make sure that students understand that division can represent sharing or counting how many groups and both of those models are used.

Discuss, in particular, the notion of the remainder—that sometimes when you share and make equal groups, there might be leftovers.

Assign the tasks.

By viewing or listening to student responses, note if they:
• can estimate when dividing
• can divide a 2-digit or 3-digit number by a 1-digit number
• recognize that different factors can produce the same product
• have an understanding of what a remainder means when dividing

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ How did you know that it had to be 371 ÷ 9 that is around 40 in Question 1? (because 9 × 40 is 360)
◊ How did you calculate 79 ÷ 6 in Question 2? (I wrote 79 as 60 + 18 + 1. Then I divided each part by 6.)
◊ How is arranging 96 people in equal rows a division problem? (Equal rows are equal groups so you’re finding out how many equal groups and how many in each group. Both of those are about dividing.)
◊ Why is the remainder when you divide by 3 never 3? (because if you’re making groups of 3, you’d just make another group—there would not be a remainder)

Materials
• counters
• base ten blocks
Solutions

1. a) 6
   b) 2
   c) 3
   d) 4

2. a) 19, e.g., That answer seems reasonable since 100 ÷ 5 is the number of nickels in a loonie and that's 20.
   b) 13 R1, e.g., That answer seems reasonable since 60 ÷ 6 = 10 and since 79 is 19 more than 60, there should be about 3 more groups of 6.
   c) 15, e.g., That answer seems reasonable since 80 ÷ 8 = 10 groups and there's an extra 40 in 120. 40 is half of 80, so there are 5 extra groups and that makes 15.
   d) 25 R3, e.g., That answer seems reasonable since 6 ÷ 25 is like 6 quarters and that's $1.50 and $1.53 is close.

3. 48 rows of 2 chairs, 32 rows of 3 chairs, 24 rows of 4 chairs, 16 rows of 6 chairs, 12 rows of 8 chairs, 8 rows of 12 chairs, 6 rows of 16 chairs, 4 rows of 24 chairs, 3 rows of 32 chairs or 2 rows of 38 chairs

4. a) If you can make 32 groups of 3 with 96, with 98, there are 2 left over.
   b) Since 99 is 3 more than 96, you can make one extra group of 3 compared to 96, so that's 32 + 1 = 33.

5. 412 ÷ 3 = 137 R1; 214 ÷ 3 = 71 R1; 124 ÷ 3 = 41 R1; The remainders are all the same.
   I tried 5, 1, and 5. My three numbers were 551, 155 and 515.
   551 ÷ 3 = 183 R2; 155 ÷ 3 = 51 R2; 515 ÷ 3 = 171 R2
   It happened again.