

GAP CLOSING

Comparing and Ordering Whole Numbers

**Junior / Intermediate
Facilitator's Guide**

Module 4

Comparing and Ordering Whole Numbers

| | |
|--|----|
| Diagnostic | 5 |
| Administer the diagnostic | 5 |
| Using diagnostic results to personalize interventions..... | 5 |
| Solutions..... | 5 |
| Using Intervention Materials | 6 |
| Comparing Numbers to 10 000 | 7 |
| Comparing Numbers to 100 000..... | 10 |

COMPARING AND ORDERING WHOLE NUMBERS

Relevant Expectation for Grade 6

- ... compare, and order whole numbers ... to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, ...)
- demonstrate an understanding of place value in whole numbers ... to 1 000 000, using a variety of tools and strategies

Possible reasons why a student might struggle in comparing and ordering larger whole numbers

Many students struggle in comparing large numbers. Some of the problems include:

- not recognizing that it is the most left digit in the number and its place value position that has the most bearing on the size of the number
- an inability to use appropriate benchmarks to estimate the size of numbers
- focusing on individual digits of a number rather than the number as a whole

Additional considerations

It is important that students learn to separate the thousands period from the ones period when writing numbers. When the number is a 4-digit number, that separation is optional. For example, we could write 4212 or 4 212.

Check that students know that $>$ means “greater than” and $<$ means “less than”.

DIAGNOSTIC

Administer the diagnostic

Provide place value charts and counters for students to use. If students need help in understanding the directions of the diagnostic, clarify an item's intent.

Materials

- Place Value Chart (1) (2) templates
- counters

Using diagnostic results to personalize interventions

Intervention materials are included on each of these topics:

- comparing numbers to 10 000
- comparing numbers to 100 000

You may use all or only part of these sets of materials, based on student performance with the diagnostic.

| Evaluating Diagnostic Results | Suggested Intervention Materials |
|---|--|
| If students struggle with Questions 1d–f, 2b, 3b, d, 4b, 5d–f | Use Comparing Numbers to 100 000 |
| If students struggle with Questions 1a–c, 2a, 3a–c, 4a, 5a–c, 6 | Use Comparing Numbers to 10 000 followed by Comparing Numbers to 100 000 |

Begin with the simpler intervention if both appear to be required. Be aware, though, that success with that simpler intervention might minimize the need for the second intervention. Check the relevant diagnostic items again to check.

Solutions

- a) 4503 b) 9238 c) 4003 d) 23 000

e) 15 408 f) 16 248
- a) e.g., 6900 6950 6999 b) e.g., 17 900 18 000 18 001
- a) 8 or 9 b) 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9

c) 7, 8 or 9 d) 7, 8 or 9 e) 3, 4, 5, 6, 7, 8, or 9
- a) e.g., 4000, 3999, 3998 b) e.g., 37 139 37 138 37 137
- a) 4510 b) 1012 c) 2517 d) 31 812

e) 43 199 f) 18 412
- Disagree. For example, 199 is not greater than 8800.

Diagnostic

1. Circle the greater number.

a) 4003 or 4305 b) 923 or 9238

c) 4003 or 3004 d) 20 003 or 23 000

e) 1548 or 15 408 f) 14 248 or 8414

2. List 3 numbers between each pair of numbers.

a) 6899 _____ 7000

b) 17 894 _____ 18 113

3. The first number is greater. List all possibilities for each of the missing digits.

a) 12 > 8547

b) 32 > 3984

c) 3 > 4427

d) 43 > 17 219

e) 36 > 33 298

4. List three numbers that are just a bit less than the first one.

a) 4002 _____

b) 37 140 _____

Diagnostic (continued)

5. Circle the number that is correct.

a) closer to 4000 than to 4000 4033 or 4010

b) closer to 1010 than to 1020 1018 or 1017

c) closer to 2500 than to 2400 2519 or 2517

d) closer to 22 000 than to 21 000 21 025 or 21 012

e) closer to 40 000 than to 30 000 43 199 or 41 002

f) closer to 18 400 than to 18 500 18 412 or 18 477

6. Agree or disagree? A number with two 9's in it is always greater than a number with two 8's in it. Explain your thinking.

Place Value Chart (1)

| | | | |
|-----------|----------|------|------|
| Thousands | Hundreds | Tens | Ones |
| | | | |
| | | | |

Place Value Chart (2)

| | | | |
|-----------|----------|------|------|
| Thousands | Hundreds | Tens | Ones |
| | | | |
| | | | |

USING INTERVENTION MATERIALS

The purpose of the suggested student work is to help students build a foundation for comparing larger numbers. Later they will extend this work to numbers in hundred thousands, millions, and billions.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

Comparing Numbers to 10 000

Learning Goal

- selecting appropriate strategies to compare and order numbers below 10 000 .

Materials

- Place Value Chart (1) template
- counters
- Number Squares template

Open Question

Questions to Ask Before Using the Open Question

Represent the number 3212 on Place Value Chart (1) by placing counters in the appropriate columns. If necessary, draw students' attention to the names of the place value columns.

- What number am I showing? (3212)
- What number would be a little bit greater? (e.g., 3232)
- How do you know it's greater? (It has the same number of thousands and hundreds but more tens.)
- Suppose you knew that a number was more than 1000 but didn't know all the digits. If I tell you that it's greater than 2003, do you know any of the digits now? (no)
- Are there some digits you're sure are not in the thousands place? (Yes. It can't be 0 or 1 since then it couldn't be more than 2003.)

Using the Open Question

Students can cut out Number Squares and move them around as they work to make the statements true.

Provide Place Value Chart (1) and counters so that they can represent the numbers to more easily compare them.

Make sure students understand that they must use each digit once. Encourage them to begin with digits they are more sure about.

By viewing or listening to student responses, note if they realize that it is the left-most digit that matters most when two numbers are compared, or, if they are the same, the next digit to the right.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

- How many possibilities were there for the first missing digit? What were they? (It had to be 4 or more.)
- How did you know that 4 was a possibility? (Because 142 is more than 12 and the thousands would be the same.)
- How many possibilities were there for the missing digit in the number to the right of 3302? (Only 1 or 2.) Why not 0? (I know that you wouldn't use a 0 at the front of a number.)
- Choose two of your lines. Tell me how you know that your statement is true. (e.g., $4142 > 4129$ since they both have 4 thousands and $142 > 129$; $8100 > 6294$ since 8100 is more than 8000, but 6294 isn't even 7 thousand.)

Comparing Numbers to 10 000

Open Question

Fill in the blanks using each digit 0, 1, 2, ..., 9 once to make all the statements true.

$142 > 412$ $100 > 294$
 $3302 > 94$ $1004 > 102$
 $1003 > 493$ $6231 > 229$

Do it another way:

$142 > 412$ $100 > 294$
 $3302 > 94$ $1004 > 102$
 $1003 > 493$ $6231 > 229$

Do it another way:

$142 > 412$ $100 > 294$
 $3302 > 94$ $1004 > 102$
 $1003 > 493$ $6231 > 229$

Place Value Chart (1)

| | | | |
|-----------|----------|------|------|
| Thousands | Hundreds | Tens | Ones |
| | | | |

| | | | |
|-----------|----------|------|------|
| Thousands | Hundreds | Tens | Ones |
| | | | |

Number Squares

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | | |

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | | |

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | | |

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | | |

Solutions

4142 > 4129 or 7142 > 4129 or 5142 > 4122
 3302 > 2914 3302 > 1984 3302 > 1984
 5003 > 4937 5003 > 4936 7003 > 4936
 8100 > 6294 3100 > 2294 9100 > 4294
 1004 > 1002 1004 > 1002 1004 > 1002
 6231 > 3229 6231 > 4229 6231 > 3229

Think Sheet

Materials

- Place Value Chart (1) template
- counters

Questions to Ask Before Using the Think Sheet

- ◇ Suppose I write down two whole numbers more than 1000. How would you know which one is greater? (It's greater if the first digit is greater.)
- ◇ So is $920 > 1003$? (no)
- ◇ Why not? (1003 is more than 1000 but 920 is not.)
- ◇ What's the biggest hint to you about how big a number is? (How many digits it has.)
- ◇ What if they have the same number of digits? (Then you have to see what the digits are.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand that it helps to either put the digits in Place Value Chart (1) or line them up as if they were on a chart to compare numbers.

Rather than just focusing on a rule, make sure that both methods for comparing numbers described at the bottom of the page are understood.

Assign the tasks.

By viewing or listening to student responses, note if they:

- use a variety of strategies to compare large numbers
- realize that estimating can sometimes help us compare numbers but not always
- recognize that it's the leftmost digit of a number that tells most about its size

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

- ◇ What were you sure of when you knew the number was more than 2200 but less than 2300? (That there were 2 thousands and 2 hundreds.)
- ◇ Why were you able to compare $\blacksquare 42$ and $\blacksquare 438$ even though you didn't know all the digits? (I knew that the second number was more than 1000 and the first one was less, no matter what the missing digits were.)
- ◇ How did you know that the number between 1043 and 1421 is close to 1400? (because it's really close to 1421)
- ◇ Why did you say that 5802 is about 6 thousands instead of 5 thousands? (e.g., It would only take 2 more hundreds to be 6 thousand but it's 800 more than 5 thousand.)
- ◇ Why might someone say that the most important part of 3147 is the 3? (e.g., Because thousands are so much more than hundreds or tens or ones.)

Comparing Numbers to 10 000 (continued)

Think Sheet

Sometimes we want to compare numbers greater than 1000.

For example, to decide if a town with a population of 4213 people has more or fewer people than one with 4106 people.

4213 is 4106 since 4213 is a bit more than 4 thousand 2 hundred (or 42 hundred) and 4106 is not as much as 42 hundred. 4200 is 100 more than 4100, but 4106 is only 6 more than 4100.

| Thousands | Hundreds | Tens | Ones |
|-----------|----------|------|------|
| 4 | 2 | 1 | 3 |
| 4 | 1 | 0 | 6 |

Both numbers have the same number of thousands, but 4213 has more hundreds than 4106.

It does not matter that 4106 has more ones than 4 213, what matters is the composition of the greater place values.

We can compare numbers by estimating or comparing both to another number.

- Estimating: 4213 is 4106 since 42 hundred and a little bit more is greater than 41 hundred and a little bit more.
- Comparing both to another number: 7003 > 999 since 7 thousand is more than 1000, but 999 is less than 1000.

- Write a number that fits the rule:
 - greater than 2200 but less than 2300
 - greater than 8000 but less than 8100
 - greater than 1204 but less than 1210

Comparing Numbers to 10 000 (continued)

- Circle the greater number. Fill in each blank with a digit other than 0.
 - 2005 or 998
 - 1815 or 1105
 - 2034 or 1728
 - $\square 34$ or $\square 34$
 - $\square 42$ or $\square 428$
- The distances between numbers on this number line show how far apart they really are. Estimate to name a number that might be at the mark. Explain.
 -
 -
 -
- Use all four digits to create four numbers. Put them in order from least to greatest.

| | | | |
|---|---|---|---|
| 0 | 3 | 1 | 5 |
|---|---|---|---|

Comparing Numbers to 10 000 (continued)

- List a number you might compose both of the given numbers to decide which is greater. Then circle the greater number.
 - 1 354 and 2 700
 - 317 and 8 482
 - 4 023 and 4 500
 - 4 $\square 34$ and $\square 34$
 - $\square 34$ and $\square 37$
- You are comparing a 4-digit number to 3047. You can ask about only one of the 4 digits of the other number but you have to say whether the number is greater or less than 3047. Which digit would you ask for? Why?
- You know that $\square \square 7 > \square \square 46$. The missing digits are 2, 4, 6, 0. List four possibilities for the arrangement of the digits.

Place Value Chart (1)

| Thousands | Hundreds | Tens | Ones |
|-----------|----------|------|------|
| | | | |

| Thousands | Hundreds | Tens | Ones |
|-----------|----------|------|------|
| | | | |

Solutions

1. a) e.g., 2260 b) e.g., 8050 c) e.g., 1208
2. a) 5205 b) 1515 c) 2034 d) 2■21
e) ■438
3. a) e.g., 3040 b) e.g., 8700 c) e.g., 1350

4. e.g., $1035 < 1053 < 3051 < 5310$

5. a) e.g., 2000; circle 2900
b) e.g., 1000; circle 8482
c) e.g., 4050; circle 4060
d) e.g., 1000; circle 4073
e) e.g., 3000; circle 3017

Note: Students might estimate with other numbers of tens, hundreds or thousands since the nearest ten, hundred or thousand was not required.

6. I would ask for the first digit. If the first digit is more than 3, the second number is definitely greater. If it's less than 3, it is definitely less. If it's 3, I can't be sure, but it is probably greater than 3047.
7. 80 and 42, 80 and 24, 84 and 20, 82 and 40

Comparing Numbers to 100 000

Learning Goal

- selecting appropriate strategies to compare and order numbers below 100 000.

Materials

- Place Value Chart (1) template
- counters
- Number Squares template

Open Question

Questions to Ask Before Using the Open Question

Represent the number 31 212 on Place Value Chart (1) by placing counters in the appropriate columns. If necessary, draw students' attention to the names of the place value columns.

- What number am I showing? (31 212)
- What number would be a little bit greater? (e.g., 31 232)
- How do you know it's greater? (It has the same number of thousands and ten thousands and hundreds but more tens.)
- Suppose I tell you that a number is greater than 21 003. Do you know any of the digits? (no)
- Are there some digits you're sure are not in the ten thousands place? (Yes. It can't be 0 or 1 since then it couldn't be more than 21 003.)

Using the Open Question

Students can cut out Number Squares and move them around as they work to make the statements true.

Provide Place Value Chart (1) and counters so that they can represent the numbers to more easily compare them.

Make sure students understand that they must use each digit only once. Encourage them to begin with digits they are more sure about.

By viewing or listening to student responses, note if they realize that it is the left-most digit that matters most when two numbers are compared, or, if they are the same, the next digit to the right.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

- How many possibilities were there for the first missing digit? What were they? (It had to be 4 or more.)
- How did you know that 4 was a possibility? (Because 342 is more than 27 and the thousands would be the same.)
- How many possibilities were there for the missing digit to the right of 33 002? (Only 1 or 2.)
- Why not 0? (I know that you wouldn't use a 0 at the front of a number.)
- Choose two of your lines. Tell me how you know that your statement is true. (e.g., 50 023 > 49 317 since 50 023 is more than 50 thousand, but 49 317 is less than 50 thousand; 10 034 > 10 021 since both have 10 thousands but 34 is more than 21.)

Comparing Numbers to 100 000

Open Question

Fill in the boxes using each digit 0, 1, 2, ..., 9 once to make all the statements true.

1 342 > 41 27 1003 > 2 954

33 002 > 9 14 10 034 > 1 021

0 02 > 49 317 62 031 > 2 029

Do it another way.

1 342 > 41 27 1 003 > 2 954

33 002 > 9 14 10 034 > 1 021

0 02 > 49 317 62 031 > 2 029

Do it another way.

1 342 > 41 27 1 003 > 2 954

33 002 > 9 14 10 034 > 1 021

0 02 > 49 317 62 031 > 2 029

Place Value Chart (1)

| | | | | | | | |
|--|--|--|--|--|--|--|--|
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

Number Squares

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 | 4 | 5 | 6 | 7 |
| 8 | 9 | 8 | 9 | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 | 4 | 5 | 6 | 7 |
| 8 | 9 | 8 | 9 | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 | 4 | 5 | 6 | 7 |
| 8 | 9 | 8 | 9 | | | | |

Solutions

e.g.,

| | | | | |
|-----------------|----|-----------------|----|-----------------|
| 41 342 > 41 279 | or | 71 342 > 41 274 | or | 41 342 > 41 275 |
| 33 002 > 29 114 | | 33 002 > 19 814 | | 33 002 > 19 614 |
| 50 027 > 49 317 | | 60 029 > 49 317 | | 70 023 > 49 317 |
| 81 003 > 62 954 | | 31 003 > 22 954 | | 91 003 > 82 954 |
| 10 034 > 10 021 | | 10 034 > 10 021 | | 10 034 > 10 021 |
| 62 031 > 32 029 | | 62 031 > 52 029 | | 62 031 > 22 029 |

Think Sheet

Materials

- Place Value Chart (1) template
- counters

Questions to Ask Before Using the Think Sheet

- ◇ Suppose I write down two whole numbers more than 1000. How would you know which one is greater? (e.g., It's greater if the first digit is greater.)
- ◇ So is $9\ 200 > 10\ 003$? (no)
- ◇ Why not? ($10\ 003$ is more than $10\ 000$ but 9200 is not.)
- ◇ What's the biggest hint to you whether one number is greater than another? (if it has more digits.)
- ◇ What if they have the same number of digits? (Then you have to see what they are.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they understand:

- it helps to either put the digits in a place value chart or line them up as if they were on a chart to compare numbers
- both methods for comparing numbers described at the bottom of the page are understood

Assign the tasks.

By viewing or listening to student responses, note if they:

- use a variety of strategies to compare large numbers
- realize that estimating can sometimes help us compare numbers but not always
- round numbers (although without using that language)
- recognize that it's the leftmost digit of a number that tells most about its size

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

- ◇ What were you sure of when you knew a number was more than $22\ 000$ but less than $23\ 000$? (That there were 2 ten thousands and 2 thousands.)
- ◇ Why were you able to compare $\blacksquare 942$ and $\blacksquare 4138$ even though you didn't know all the digits? (I knew that the second number was more than $10\ 000$ and the first one was less, no matter what the missing digits were.)
- ◇ How did you know that the number between $82\ 915$ and $90\ 043$ is more than $85\ 000$?
(It's closer to $90\ 000$ than to $83\ 000$.)
- ◇ Why did you say that $32\ 900$ is about 33 thousand instead of 32 thousand? (e.g., it would only take 1 more hundred to be 33 thousand but it's 900 more than 32 thousand.)
- ◇ Why might someone say that the most important part of $32\ 147$ is the 3? (e.g., Because ten thousands are so much more than thousands or hundreds or tens or ones.)

Comparing Numbers to 100 000 Continued

Think Sheet

If we compare two numbers, such as 35 210 and 34 153, we see that $35\ 210 > 34\ 153$ since 35 210 is 1000 more than 34 153 and 34 153 is not as much as 35 thousand.

Both numbers have the same number of ten thousands, but 35 210 has more thousands than 34 153.

| Ten Thousands | Thousands | Hundreds | Tens | Ones |
|---------------|-----------|----------|------|------|
| 3 | 5 | 2 | 1 | 0 |
| 3 | 4 | 1 | 5 | 3 |

It does not matter that 34 153 has more tens than 35 210, what matters is the comparison of the greater place values.

We can compare numbers by estimating, or comparing both to another number.

- Estimating: $35\ 210 > 31\ 000$ since 41 thousand and a little bit more is greater than 31 thousand and a little bit more.
- Comparing both to another number: $27\ 000 < 9999$ since 27 thousand is more than 10 000, but 9999 is less than 10 000.

- Write a number that fits the rule.
 - greater than 22 000 but less than 23 000
 - greater than 80 030 but less than 80 100
 - greater than 12 004 but less than 12 010

Comparing Numbers to 100 000 Continued

- Circle the greater number. Fill in each blank with a digit other than 0.
 - 52 105 or 954
 - 15 015 or 11 055
 - 20 034 or 17 289
 - $\square 254$ or $\square 2718$
 - $\square 42$ or $\square 158$
- The distances between numbers on these number lines show how far apart they really are. Estimate to name a number that might be at the mark. Explain.
 -
 -
 -
- Use all five digits to create four numbers. Put them in order from least to greatest.

| | | | | |
|---|---|---|---|---|
| 0 | 3 | 1 | 5 | 9 |
|---|---|---|---|---|

Comparing Numbers to 100 000 Continued

- List a number you might compare both of the given numbers to to decide which is greater. Then circle the greater number.
 - 41 354 and 32 900 _____
 - 51 417 and 8 878 _____
 - 14 003 and 18 720 _____
 - $\square 473$ and $\square 245$ _____
 - $\square 194$ and $\square 117$ _____
- You are comparing a 5-digit number to 33 047. You can ask about only one of the 5 digits of the other number but you have to say whether the number is greater or less than 33 047. Which digit would you ask for? Why?
 - You know that $\square 317 > \square 148$. The other digits are 0, 2, 5, and 4. List four possibilities for the arrangement of the digits.

Place Value Chart (1)

| Thousands | | | | | Hundreds | | | | | Tens | | | | | Ones | | | | |
|-----------|--|--|--|--|----------|--|--|--|--|------|--|--|--|--|------|--|--|--|--|
| | | | | | | | | | | | | | | | | | | | |

Solutions

1. a) e.g., 22 500 b) e.g., 80 080 c) e.g., 12 008
2. a) 52 105 b) 15 015 c) 20 034 d) 2 218
e) 4 138
3. a) e.g., 30 400 b) e.g., 87 000 c) e.g., 13 500
4. e.g., 13 059, 13 950, 31 950, 95 130
5. a) e.g., 40 000; circle 41 356
b) e.g., 50 000; circle 51 417
c) e.g., 15 000; circle 18 920
d) e.g., 10 000; circle 1 245
e) e.g., 30 000; circle 3 117
6. I would ask about the first digit. If the first digit is more than 3, the second number is definitely greater. If it's less than 3, it is definitely less. If it's 3, I can't be sure, but it is probably greater than 30 047.
7. e.g., 60 on the left and 25 on the right
62 on the left and 50 on the right
50 on the left and 26 on the right
60 on the left and 52 on the right

