# GAP CLOSING

# Volume and Surface Area

Intermediate / Senior
Student Book

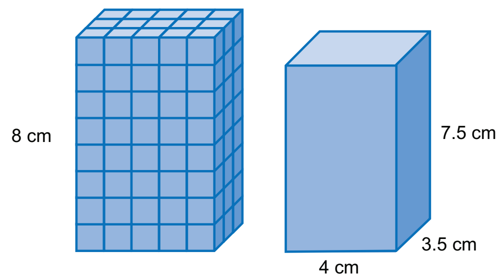
# Volume and Surface Area

# Diagnostic

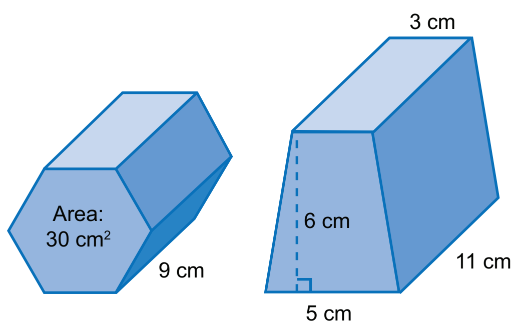
**Note:** If an answer involves the number π, you may leave the answer in terms of π or you may estimate the answer using the value 3.14.

**1.** Which prism has a greater volume? How much greater? Show your work.

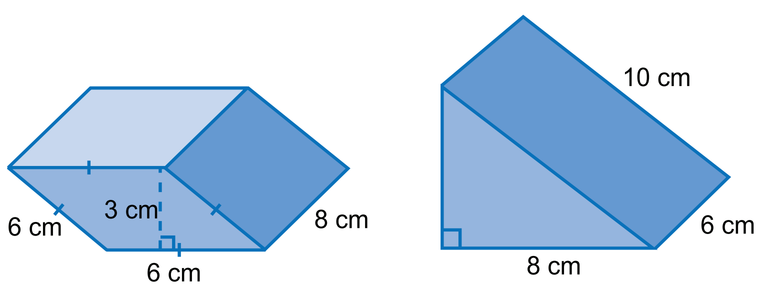
**a)**




**b)**




**c)**



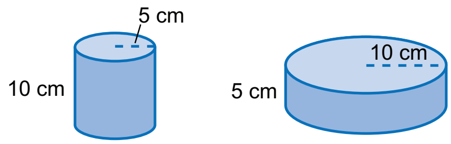

**2.** Give an example of two prisms that have different shaped bases but have the same volume.

**3.** The volume of a prism is 100 cm$^3$. Its height is 4 cm. What else do you know about the prism?

**4.** Which cylinder has a greater volume? How much greater? Show your work.

**a)**

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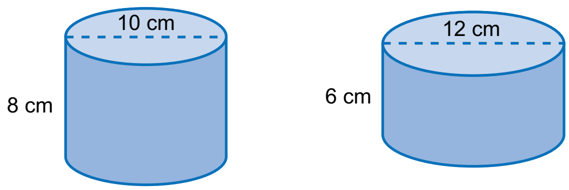
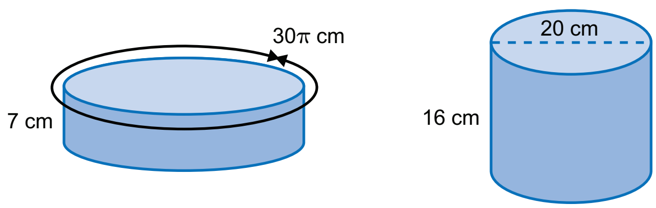
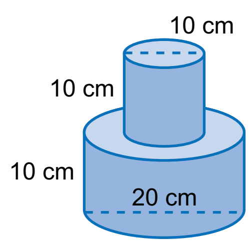



**c)**



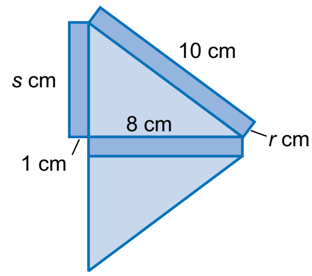

**5.** What is the volume of this container?




**6.** Two cylinders have the same height and same volume. Could they have different base areas? Explain.

**7.** This is a net of a triangular prism.




**a)** What is the value of *r*?

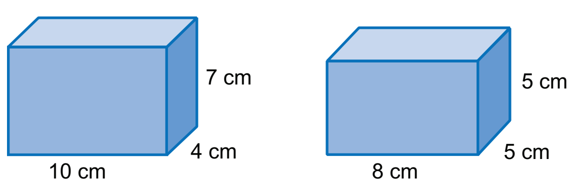
**b)** What is the value of *s*?

**c)** What is the surface area (the total area of all of the faces) of the prism?

**8.** The surface area of a cube is 300 $cm^2$. Estimate the side length of the cube to the nearest tenth.

**9.** Which 3-D figure has a greater surface area? How much greater? Show your work.

**a)**

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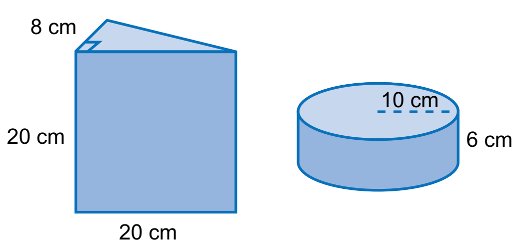


# Volumes of Prisms

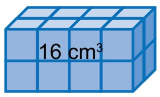
## Learning Goal

- connecting the formula for the volume of any prism to the formula for the volume of a rectangular prism.

**Open Question**

The volume of each prism is indicated.

**a)**

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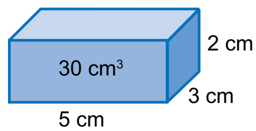
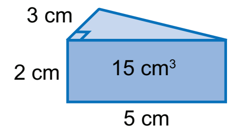



**c)**




- Why does each make sense?

- Choose a volume.

  Create a set of three different prisms, with different types of bases and different heights, each of which has that volume.

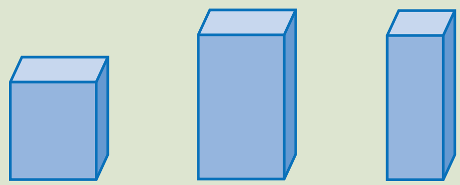
- Repeat once more with a new volume and new types of 3-D figures.

  Make sure to communicate your thinking.

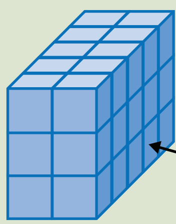
  

## Think Sheet

The **volume** of a 3-D figure tells how much space it takes up. We might also want to know the volume of a 3-D solid to figure out how much it would cost for the material to build it.



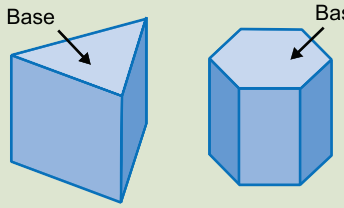

- The centre prism has more volume than the one on the left because the centre prism is taller, with the same "footprint"; the centre prism has more volume than the one on the right because the centre prism has the same height but a bigger "footprint."
- We measure the volume of a rectangular prism by figuring out how many centimetre cubes it would take to build it. This prism has a volume of 30 cubes because there are 3 layers of 10 (5 × 2) cubes:



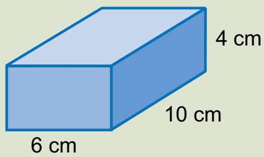

  If the prism were taller, it would have a greater volume. For example, if the base has 10 cubes and there are 6 layers, there would be 6 × 10 = 60 cubes in the volume.

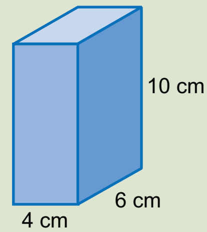
In general, if the area of the base is *B*, and *h* is the height of the prism, use this formula for the volume (V): ***V = Bh*** or ***V = (Area of Base) × height***

- Even if the prism is not rectangular, its volume is the area of its base multiplied by its height. It is important to remember that the base of the prism is the face used to name the prism — the base might be a triangle, a hexagon, an octagon, etc. The base will be a rectangle if the shape looks like a standard box.
- For example:



- We can see that the space each prism takes up depends on how much area the base takes up as well as how high the prism is. So ***V* = *Bh*** or ***V* = (area of base) × height** for these prisms too.
- If the prism is a rectangular prism, any face can be used as the base. Either way, the prism below has a volume of 240 $cm^3$.

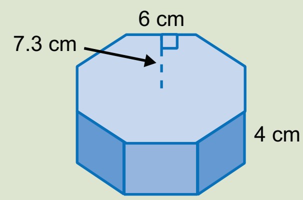


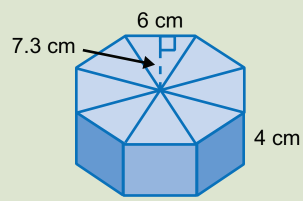

$V = 60\text{ cm}^2 \times 4\text{ cm} = 240\text{ cm}^3$

$V = 24\text{ cm}^2 \times 10\text{ cm} = 240\text{ cm}^3$

- There are times when, to determine the volume, we have to figure out the base's area. For example, suppose we have a regular (all the sides are equal on the base) octagonal prism and we know one side of the base is 6 cm, the distance from the middle of that side to the centre of the base is 7.3 cm, and the height of the prism is 4 cm.




We can think of the octagon as made up of 8 equal triangles, and we know that the base of each is 6 cm and the height of each triangle is 7.3 cm.




Using what we know about area, each triangle area is:
$A = \frac{1}{2} \times 6\text{ cm} \times 7.3\text{ cm} = 21.9\text{ cm}^2$.

So $B$ (area of base of prism) = $8 \times 21.9\text{ cm}^2 = 175.2\text{ cm}^2$.

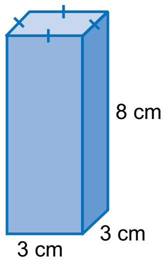
$V$ (volume) = $Bh = 175.2\text{ cm}^2 \times 4\text{ cm} = 700.8\text{ cm}^3$.

**Note:** The $h$ in the formula for the volume is the height of the whole pyramid. Sometimes we need to use a different height to figure out the area of the base, as we did above with the octagon. If there is a triangular base, the $b$ we use in the formula for that triangle is different from the $B$ we use in the formula for the volume.

1. What is the area of the base of each prism?

a)

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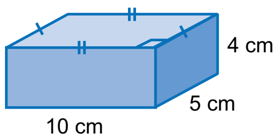
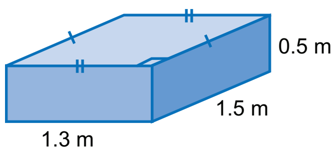



c)




2. Calculate the volume of each prism in Question 1.

a)

b)

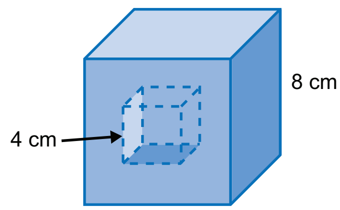
c)

3. Sketch three rectangular prisms, all with a volume of 60 $cm^3$. Label the length, width, and height of each.

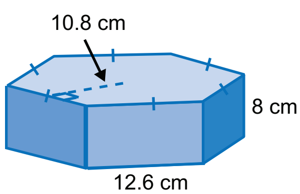
  

4. A long, narrow, but short, rectangular prism has a volume of 50 $cm^3$. What might its dimensions be?

5. A cube has a side length of 8 cm. Inside is a hole, also in the shape of a cube, with a side length of 4 cm. How much volume does the part that is still there have?




6. What is the area of the base of each of these prisms? Show your work.

a)
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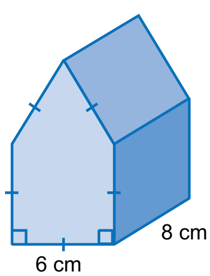
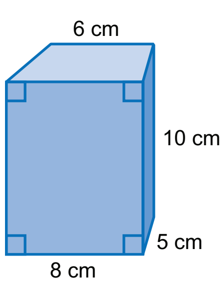
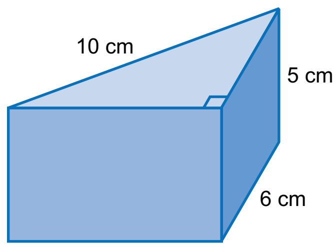


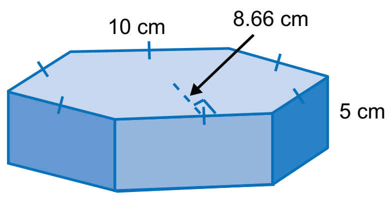

c)


**d)**




**e)**




**7.** Calculate the volume of each prism in Question 6.

**a)**

**b)**

**c)**

**d)**

**e)**

**8.** Another prism is just like the trapezoid-based prism in Question 6c) with one change. Tell how the new volume compares to the old one:

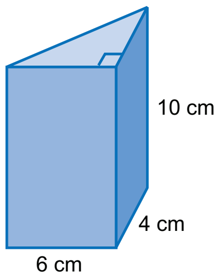
**a)** The base is the same, but the height of the prism is 20 cm (double the original)

**b)** The height is the same, but the sides of the base are all double the original side lengths.

**c)** The height is half the original prism's height, but the side lengths of the base are double the original side lengths.

**9.** Describe the dimensions of (or sketch with labels) another triangular prism with different measurements but the same volume as this one:




**10.** Cain said that if the base of a prism has area 20 $cm^2$, then the volume could be 40 $cm^3$ or 80 $cm^3$, but could not be 90 $cm^3$. Do you agree or disagree? Explain.

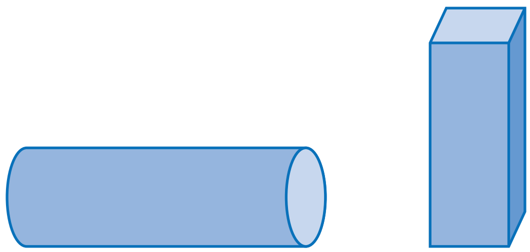
# Volumes of Cylinders

## Learning Goal

- connecting the formula for the volume of a cylinder to the formula for the volume of a prism.

### Open Question

A very tall cylinder has the same volume as a rectangular prism.



- Choose three possible volumes. For each choice, describe the dimensions of both a cylinder and a rectangular prism with that volume. Show your reasoning.

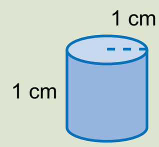
**Note:** If an answer involves the number $\pi$, you may leave the answer in terms of $\pi$ or may estimate using the value 3.14.

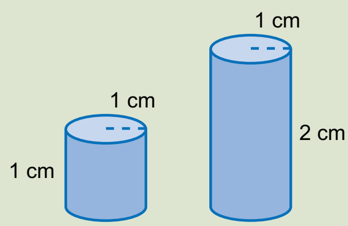
## Think Sheet

A cylinder is a lot like a prism, only with a round base, so it makes sense that to calculate the volume of a cylinder, we think the same way as we do about a prism.

For example, a cylinder with a base of radius 1 cm and a height of 1 cm would have $V$ (volume) = $\pi$ since the area of the base $B = \pi \times (1\text{ cm})^2$. If we multiply by the height, which is 1 cm, we get $V = \pi\text{ cm}^2 \times 1\text{ cm} = \pi\text{ cm}^3$.



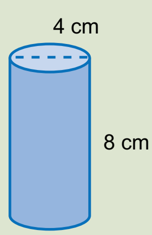

If the cylinder was twice as tall, it would be like stacking two of the original cylinders on top of each other, so the new cylinder's volume would be twice as much as the original volume.




In general, the volume of a cylinder with base radius of $r$ and height $h$ is: $V = \pi r^2 h$.

Notice that we need two values, $r$ and $h$, to calculate the volume of a cylinder. But we can calculate the volume if we have other measures instead of the radius and height.

For example, if we know the diameter of the base, instead of the radius, we could divide that distance by 2 to get the radius and then use the formula.




$d = 4$ cm, so $r = 2$ cm, so $V = \pi(2\text{ cm})^2 \times 8\text{ cm} = 32\pi\text{ cm}^3$ (or $100.48\text{ cm}^3$).

Even if we know the circumference of the base, we can figure out the radius using the formula $C = 2\pi r$ and proceed to calculate the volume.

If $C = 30$ cm, then $2\pi r\ cm = 30$ cm, so $\pi r = 15$ cm and $r = \frac{15}{\pi}$ cm (or 4.78 cm).

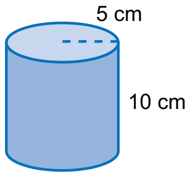
We can now use that value in the regular formula.

**Note:** If an answer involves the number π, you may leave the answer in terms of π or you may estimate the answer using the value 3.14.

**1.** What is the volume of each cylinder?

**a)**

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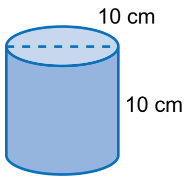


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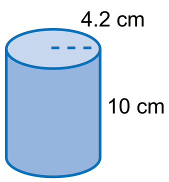


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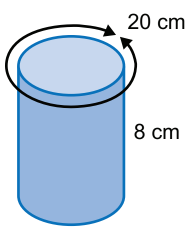
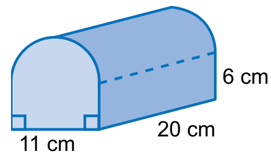



**2.** Why did it make sense that the volume in Question 1c was less than the volume in Question 1a?

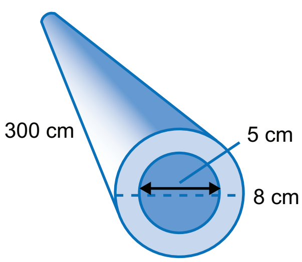
**3.** The volumes of three different cylinders are each about 300π $cm^3$ (or 942 $cm^3$). What could the dimensions be?

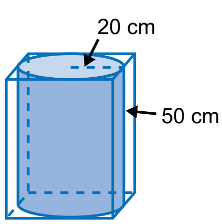
**4.** What is the volume of this loaf of bread, assuming that the top part of the end of the loaf is a semi-circle? Show your work.




**5.** How much material is required to make a pipe that is 300 cm long whose outer diameter is 8 cm with an inner diameter of 5 cm? Show your thinking.




**6.** What is the volume of the smallest square-based prism that would hold this cylinder?



**7.** You roll a piece of loose-leaf paper (22 cm × 29 cm) the long way to make an open-ended cylinder. You roll an identical piece of paper the other way to make a different open-ended cylinder that is shorter. Do the cylinders hold exactly the same amount? Explain.

**8.** A cube and a cylinder have volumes of 64 $cm^3$. What could be the dimensions of each figure?

**9.** A cylinder has a diameter that is the same as the side length of a square-based prism. Their volumes are almost the same. How do their heights compare? Show your work.

**10.** One cylinder has double the volume of another. Does the one with double volume have to be taller? Explain.

# Surface Areas of Prisms and Cylinders

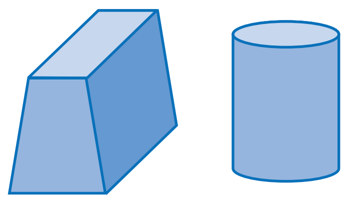
## Learning Goal

- composing and decomposing shapes to simplify surface area calculation.

**Open Question**

The **surface area** of a 3-D figure is the total of the area of all of its surfaces.

We might use it to figure out how much material it would take to exactly cover a box or can.



- Choose a number of square centimetres between 100 and 1000.
- Use that number to create five different figures with that surface area. Show your thinking.

  **1.** A prism with a square base

  **2.** A prism with a non-square rectangular base

  **3.** A prism with a trapezoid base

  **4.** A prism with a hexagon or octagon base

  **5.** A cylinder

**Note:** If an answer involves the number $\pi$, you may leave the answer in terms of $\pi$ or you may estimate using the value 3.14.

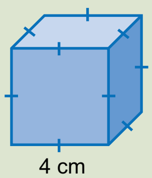
## Think Sheet

The **surface area** of a 3-D figure is the sum of all of the areas of its faces. We might use the surface area to know how much wrapping paper is needed to wrap a box or perhaps how much aluminum is needed to make a can.

Surface area is measured in square units, like square centimetres, square metres, etc.

When we know something about the relationship between the faces of the prism there are fewer calculations to determine the surface area than in other situations.

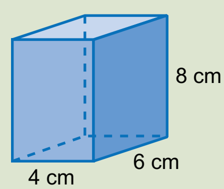
For example, suppose there is a cube with a side length of 4 cm.




Each face has an area of 4 cm × 4 cm = 16 $cm^2$, so instead of adding six numbers, we can multiply one number by 6.

The total area is 6 × 16 $cm^2$ = 96 $cm^2$.

But if the prism is a rectangular prism that was not a cube, the six faces would not be identical. There would be three pairs of identical faces:



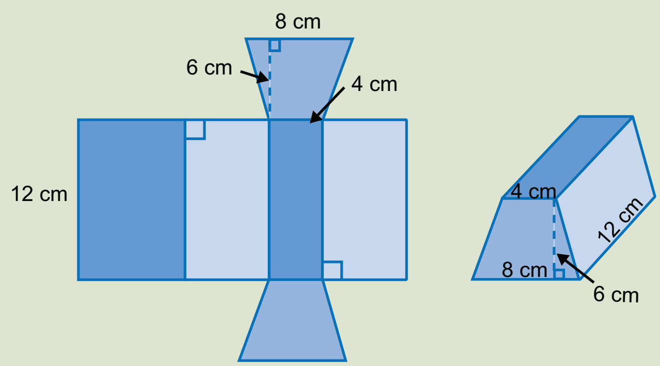

Two of the faces (front and back) have areas of 4 cm × 8 cm.

Two of the faces (sides) have areas of 8 cm × 6 cm.

Two of the faces (top and bottom) have area of 4 cm × 6 cm.

So the total surface area is 2 × 32 + 2 × 48 + 2 × 24 $cm^2$ = 208 $cm^2$.

One way to visualize the total surface area is to use a net of the 3-D figure — a flattened-down representation of it. For example, the net of a trapezoidal-based prism might look like this:



We would determine the area of each shape in the net and add those values.

For the trapezoid-based prism, the height of each rectangle is 12 cm. The width varies, depending on the side to which each rectangle attaches. One width is 8 cm, one width is 4 cm, and two widths are the lengths of the slanted sides of the trapezoid.

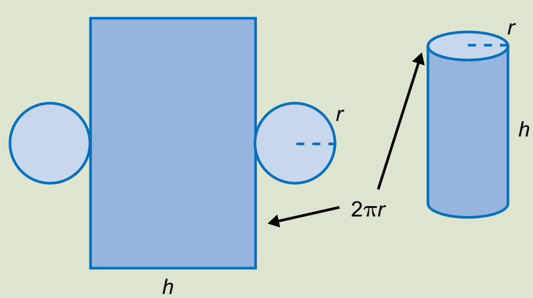
To determine those values, we could use the Pythagorean theorem, realizing that one leg of a right triangle is 6 cm and the other leg is half of 8 cm – 4 cm. The hypotenuse (the side we need) is $\sqrt{36 + 4}$ cm = 6.32 cm.

Each trapezoid has an area of $\frac{1}{2}$(8 cm + 4 cm) × 6 cm, or 36 cm$^2$.

The total area is
2 × 36 cm$^2$ + 12 cm × 8 cm + 12 cm × 4 cm + 12 cm × 6.32 cm + 12 cm × 6.32 cm.
That is 367.68 cm$^2$.

We could have written 2 × 36 cm$^2$ + 12 cm (4 cm + 6.32 cm + 8 cm + 6.32 cm). The expression in brackets is the perimeter of the trapezoid base of the prism. So another formula for the surface area of a prism is *SA* = 2*B* + *Ph* where *B* is the area of the base, *P* is the perimeter of the base and *h* is the height of the prism.

- We can calculate the surface area of a cylinder in a similar way. We could create a net showing its two circular bases and its lateral surface (the tall round part) and calculate the area of each part of the net.




The rectangle is the result of cutting straight down the height of the cylinder.

Notice that one dimension of the rectangle is the height of the cylinder but the other dimension is the circumference of the base of the cylinder; this happens to ensure that the lateral surface wraps all the way around the top and bottom circular bases.

That means the area of the rectangle is $2\pi rh$, where $r$ is the radius of the base and $h$ is the height of the cylinder.

The full surface area (SA) is twice the area of the base (for a cylinder with a top and bottom) and the area of the rectangle cut from its lateral surface.

So $SA = 2B + Ch$, where $B$ is the area of the base, $C$ is the circumference of the base and $h$ is the height of the cylinder. Since the circumference is really a perimeter, it is the same formula as for a prism.

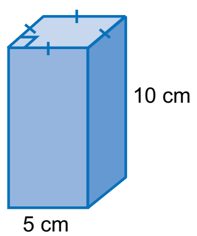
Two other versions of the formula for the surface area of a cylinder are based on being more specific about the area of the base and circumference:

$SA = 2\pi r^2 + 2\pi rh$ or $SA = 2\pi r(r + h)$.

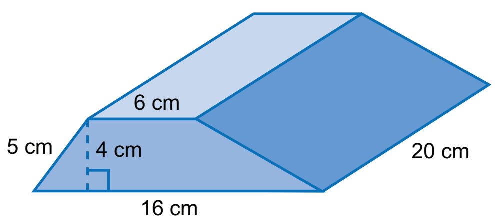
**Note**: If an answer involves the number $\pi$, you may leave the answer in terms of $\pi$ or you may estimate the answer using the value 3.14.

**1.** What is the area of the base of each of these prisms or cylinders? Show your work.

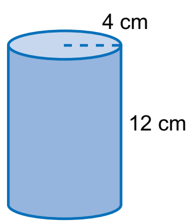
**a)**



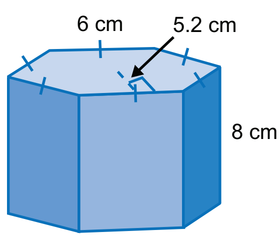

**b)**



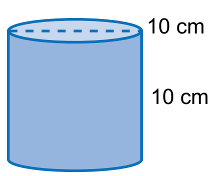

**c)**




**d)**




**e)**



**2.** Tell how many separate areas you must figure out in order to determine the surface area of each prism or cylinder in Question 1. Explain your thinking.

**3.** Calculate the surface areas of each prism or cylinder in Question 1.

**4.** Decide whether each statement is sometimes, always or never true when calculating the surface area of a prism with a 4-sided based. Explain.

A: All the faces could have the same area.

B: At least two faces have to have the same area.

C: There could be exactly four faces with the same area.

**5.** The calculations for the surface areas of different prisms or cylinders are described. Sketch or tell about the dimensions and faces of the prism.

**a)** $6 \times 25\ \text{cm}^2$

**b)** $2 \times 12\ \text{cm}^2 + 2 \times 15\ \text{cm}^2 + 2 \times 20\ \text{cm}^2$

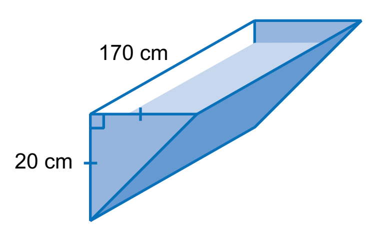
**c)** $2 \times \frac{3 \times 4}{2}\ \text{cm}^2 + 3 \times 10\ \text{cm}^2 + 4 \times 10\ \text{cm}^2 + 5 \times 10\ \text{cm}^2$

**d)** $18\pi\ \text{cm}^2 + 60\pi\ \text{cm}^2$

**6.** Could a tall prism have a smaller surface area than a shorter one? Explain.

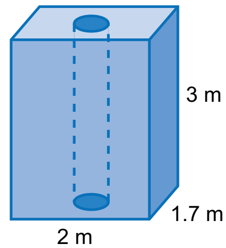
**7.** How many square centimetres of metal would be needed to make this water trough? Remember there is no top.

Show your thinking.



**8.** A hole in the shape of a cylinder is drilled all the way through from top to bottom inside this wooden rectangular prism. The radius of the hole is 0.45 m.




**a)** What would be the surface area of the prism if there were no hole?

**b)** What is the surface area of the cylinder inside not including the top and bottom?

**c)** What is the surface area of the block of wood remaining? Explain.

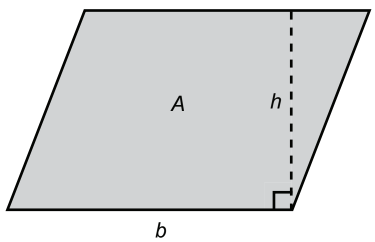
**9.** A rectangular prism is very, very long, but not very high and not very wide. Which number do you think will be greater — the number of square centimetres in its surface area or the number of cubic centimetres in its volume? Explain your thinking.

# Formula Sheet

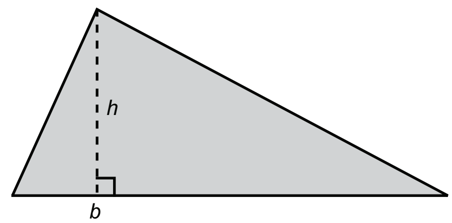
## Area of a parallelogram

$A = bh$



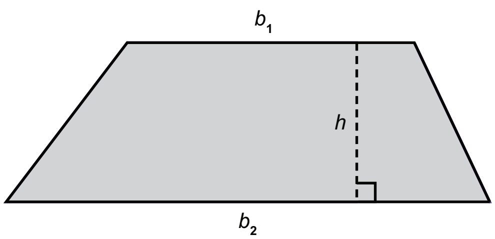

## Area of a triangle

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$



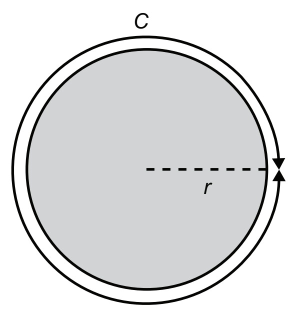

## Area of a trapezoid

$A = \frac{1}{2}(b_1 + b_2)h$ or $A = (b_1 + b_2)h \div 2$



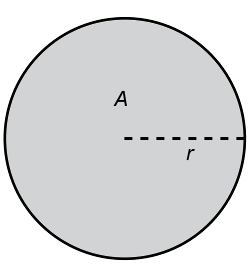

## Circumference of a circle

$C = 2\pi r$ or $C = \pi d$




## Area of a circle

$A = \pi r^2$

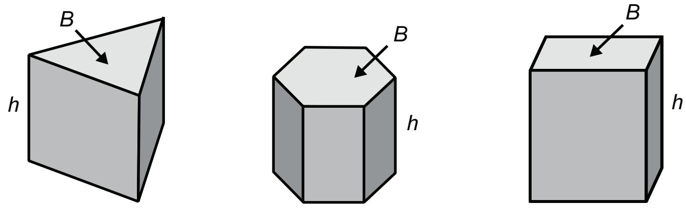


# Formula Sheet cont.

### Volume of any prism

$V = Bh$ where $B$ is the area of the base and $h$ is the height of the prism

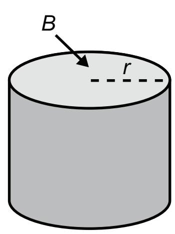



### Volume of a cylinder

$V = Bh$ where $B$ is the area of the base and $h$ is the height of the cylinder

OR

$V = \pi r^2 h$ where $r$ is the radius of the base and $h$ is the height of the cylinder

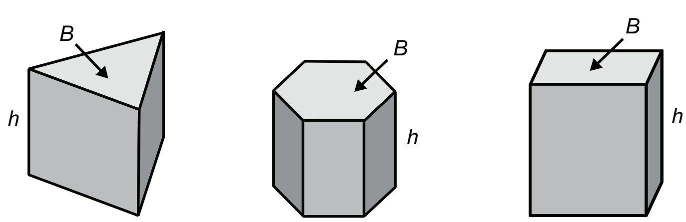



### Surface area of any prism

$SA = 2B +$ *areas of all rectangles* where $B$ is the area of the base

OR

$SA = 2B + Ph$ where $B$ is the area of the base, $P$ is the perimeter of the base and $h$ is the height of the prism



# Formula Sheet cont.

**Surface area of a cylinder**

$SA = 2B + Ch$ where $B$ is the area of the base, $C$ is the circumference of the base and $h$ is the height of the cylinder

OR

$SA = 2\pi r^2 h + 2\pi rh$ where $r$ is the radius of the base and $h$ is the height of the cylinder
OR

$SA = 2\pi r(r + h)$ where $r$ is the radius of the base and $h$ is the height of thecylinder