GAP CLOSING

Volume and Surface Area

Intermediate / Senior
Facilitator's Guide

Volume and Surface Area

The Facilitator's Guide for this entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/9-3DMeasurement(Volume)_FG_IS.pdf. The Student Book for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/9-3DMeasurement(Volume)_SB_IS.pdf.

VOLUME AND SURFACE AREA

Relevant Expectations for Grade 9

MPM1D

Measurement and Geometry

- develop, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere
- determine, through investigation, the relationship for calculating the surface area of a pyramid
- solve problems involving the surfaces areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures

MPM1P

Measurement and Geometry

- develop, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere
- solve problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres

Possible reasons why a student may struggle with volume and surface area

Students may struggle with volumes and surface areas of prisms and cylinders.

Some of the problems include:

- difficulty applying the area formulas required to determine areas of bases of prisms and/or cylinders
- mixing up the variable h, representing the height of the entire prism, with the variable h used to determine the area of a triangular or parallelogram base
- mixing up the variable B, representing the area of the base of the entire prism, with the variable b used to determine the area of a triangular base
- not including all faces when determining surface area
- confusing volume and surface area or not recognizing which is needed in a particular situation
- difficulty deducing information to indirectly determine necessary measurements of a shape when they are not provided, e.g., determining a fourth side of a right trapezoid when only three side lengths are given, recognizing that the heights of all of the rectangles of the prisms must be equal, or deducing that the rectangle that represents the lateral surface of a cylinder has a length that is the circumference of the base
- inability to apply either 2-D or 3-D measurement formulas in more complex situations

DIAGNOSTIC

Administer the diagnostic

Allow students access to the Formula Sheet template and calculators.

Materials
- calculators
- Formula Sheet

Using diagnostic results to personalize interventions

Intervention materials are included on each of these topics:
- volumes of prisms
- volumes of cylinders
- surface areas of prisms and cylinders

You may use all or only part of these sets of materials, based on student performance with the diagnostic. If students need help in understanding the intent of a question in the diagnostic, you are encouraged to clarify that intent.

Evaluating Diagnostic Results	Suggested Intervention Materials
If students struggle with Questions 1–3	use *Volumes of Prisms*
If students struggle with Questions 4–6	use *Volumes of Cylinders*
If students struggle with Questions 7–9	use *Surface Areas of Prisms and Cylinders*

Solutions

1. **a)** First volume is $3 \times 5 \times 8$ cm^3. That is 120 cm^3. Second volume is $3.5 \times 4 \times 7.5$ cm^3 = 105 cm^3. The first one is 15 cm^3 more.
b) The hexagonal prism has a volume of 270 cm^3. The trapezoidal-based prism has a base area of 24 cm^2 and the volume is 264 cm^3. The first one is 6 cm^3 more.
c) The rhombus base has an area of $6 \times 3 = 18$ cm^2. So the prism has a volume of $18 \times 8 = 144$ cm^3. The triangular base has an area of 24 cm^2 since the missing side length is 6 cm. The volume of the prism is $24 \times 6 = 144$ cm^3. The volumes are equal.

2. e.g., Both have a prism height of 10 cm. One has a rectangular base with length 5 cm and width 4 cm and the other has a parallelogram base with base 5 cm and base height of 4 cm.

3. I know that the area of the base is 25 cm^2.

4. **a)** The first volume is 25π cm$^2 \times 10$ cm = 250π cm^3. The second volume is 100π cm$^2 \times 5$ cm = 500π cm^3. The second volume is double the first.
b) The first volume is 25π cm$^2 \times 8$ cm = 200π cm^3. The second volume is 36π cm$^2 \times 6$ cm = 216π cm^3. The second one is bigger by 16π cm^3; that is about 50 cm^3.
c) The first radius is $\frac{30\pi \text{ cm}}{2\pi}$ = 15 cm, so the first volume is 225π cm$^2 \times 7$ cm = 1575π cm^3. The second radius is 10 cm, so the second volume is 100π cm$^2 \times 16$ cm = 1600π cm^3. The second one is bigger by 25π cm^3, which is about 80 cm^3.

5. The top cylinder has a volume of 250π cm^3 and the bottom one has a volume of 1000π cm^3, so the total volume is 1250π cm^3, which is about 3925 cm^3.

6. No, e.g., The volume of a cylinder is the product of the area of the base and the height. That means the area of the base is the volume divided by the height. If you use the same volume and same height, you get the same area.

7. **a)** 1 cm **b)** 6 cm **c)** 72 cm^2

8. About 7.1 cm

9. **a)** The first prism has a surface area of 2 × 28 cm^2 + 2 × 40 cm^2 + 2 × 70 cm^2 = 276 cm^2. The second prism has a surface area of 4 × 40 cm^2 + 2 × 25 cm^2 = 210 cm^2. The first prism has 66 cm^2 more surface area.

b) The triangular prism has two bases with an area of 80 cm^2, one rectangle with an area of 400 cm^2, and one with an area of 160 cm^2. The hypotenuse of the triangle had a length of 21.54 cm, so the rectangle has an area of 430.8 cm^2. The total surface area is 1150.8 cm^2. The cylinder has a surface area of 2 × 100π cm^2 + 20π × 6 cm^2 = 320π cm^2, which is about 1004.8 cm^2. The prism has more surface area – about 145 cm^2 more.

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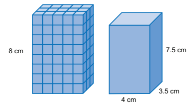
Diagnostic

Note: If an answer involves the number π, you may leave the answer in terms of π or you may estimate the answer using the value 3.14.

1. Which prism has a greater volume? How much greater? Show your work.

a)



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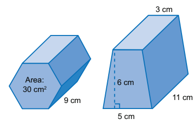

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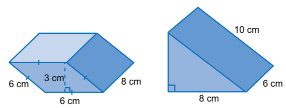


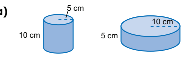
2. Give an example of two prisms that have different shaped bases but have the same volume.

3. The volume of a prism is 100 cm^3. Its height is 4 cm. What else do you know about the prism?

Diagnostic (Continued)

4. Which cylinder has a greater volume? How much greater? Show your work.

a)



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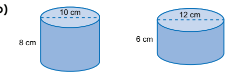
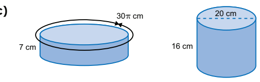
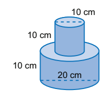


c)



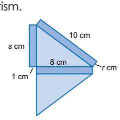
5. What is the volume of this container?



6. Two cylinders have the same height and same volume. Could they have different base areas? Explain.

Diagnostic (Continued)

7. This is a net of a triangular prism.



a) What is the value of *r*?

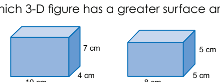
b) What is the value of *s*?

c) What is the surface area (the total area of all of the faces) of the prism?

8. The surface area of a cube is 300 cm^2. Estimate the side length of the cube to the nearest tenth.

9. Which 3-D figure has a greater surface area? How much greater? Show your work.

a)



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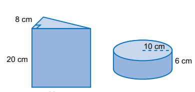
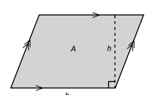


Formula Sheet

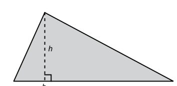
Area of a parallelogram

$A = bh$



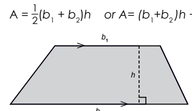
Area of a triangle

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$



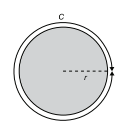
Area of a trapezoid

$A = \frac{1}{2}(b_1 + b_2)h$ or $A = (b_1 + b_2)h \div 2$



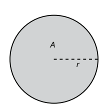
Circumference of a circle

$C = 2\pi r$ or $C = \pi d$



Area of a circle

$A = \pi r^2$



USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for successfully working with volumes and surface areas of more complex shapes including cones, pyramids, spheres, and composite shapes.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

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Volumes of Prisms

Learning Goal

- connecting the formula for the volume of any prism to the formula for the volume of a rectangular prism.

Open Question

Materials
- Formula Sheet
- calculator
- about 40 linking cubes

Questions to Ask Before Using the Open Question

◊ *What does the volume of a figure mean?* (e.g., It tells how big it is.) *Big in what way?* (e.g., It means big overall; it includes how wide and deep and high the figure is.)

Build a single layer rectangular prism with linking cubes. Make the dimensions 4 × 3.

◊ *What is the volume of this prism?* (12) *What units would we use if these cubes were 1 cm on a side?* (cubic centimetres)

Build up the prism with two more identical layers.

◊ *What is the volume of the prism now?* (36) *How does the volume compare to the earlier volume?* (e.g., It is 3 times as much.)

◊ *Does that make sense?* (e.g., Yes, since there are three layers so you use three times as many cubes.)

◊ *If the prism was not made of cubes and the base of the prism was 1.5 cm by 6 cm and it was 4 cm high, how could you calculate the volume?* (e.g., I would get the area of the base by multiplying 1.5 × 6 and then I would have four layers, so I would multiply by 4.)

◊ *For what other kinds of prisms could you determine the volume?* (e.g., triangular prisms, hexagonal prisms, octagonal prisms)

◊ *How do you decide which is the base of the prism?* (e.g., It is the face that gives the prism its name, such as a triangle or a hexagon.)

◊ *Why might you write $V = Bh$ to describe the formula for the volume of a prism?* (e.g., You figure out how big the base is by getting its area and multiply by the height since more layers means more volume.)

Using the Open Question

Make sure students realize that they should keep changing the height for their three figures, even though the volume has not changed.

By viewing or listening to student responses, note if they:
- can determine the areas of the bases of various types of prisms;
- can apply the formula $V = Bh$ for various kinds of prisms;
- can create rectangular prisms with a given volume;
- can solve multi-step problems involving volume;
- can relate the volumes of certain prisms;
- recognize that dimensions can be whole numbers or not.

Consolidating and Reflecting on the Open Question

◊ *What volume did you select first?* (e.g., 1000 cm^3)
◊ *What dimension of the prism did you choose first? Why?* (e.g., I chose the height first and then divided the volume by the height. I knew that the answer had to be the area of the base. I always chose a height that divided into my volume easily.)
◊ *Once you knew your base area, what did you do for a rectangular prism?* (e.g., I multiplied two numbers that gave that number. For example, for 100, I used 10 × 10.)
◊ *What if the shape of the base was a triangle?* (e.g., I knew that the base of the triangle times the height of the triangle had to be double the number I had for the area. So if the base had an area of 50, I chose two numbers to multiply to 100 for the base and height.)
◊ *What if the shape of the base was a regular hexagon?* (e.g., I know that a hexagon is made up of six little triangles. So I would divide the base area by 6 to figure out the area of one of those little triangles. Then I would choose two numbers that multiplied to be twice that amount.)
◊ *How was your thinking the same for all of your solutions?* (e.g., Each time, I multiplied the area of the base by the height of the figure to get the volume.)

Solutions

a) e.g., This makes sense since the prism is made up of 16 cubic centimetres.
b) e.g., This makes sense since the bottom layer could be divided into 15 cm^3. And since there are 2 layers, that would be 30 cm^3.
c) e.g., This makes sense since it is half of the prism in part b).

e.g., 1000 cm^3

- First I picked a cube. I know that 10^3 = 1000, so the side length of the cube is 10 cm.
- Then I decided to use a prism height of 20 cm. That meant the area of the base had to be 50 cm^2. I decided to use a triangle with a base of 10 cm and a triangle height of 10 cm.
- Finally I decided to use a prism height of 25 cm. That meant the area of the base had to be 40 cm^2. I decided to use a trapezoid with a trapezoid height of 10 cm and bases of 2 cm and 6 cm.

240 cm^3

- First I picked a prism height of 10 cm, so the area of the base had to be 24 cm^2.
- I chose a parallelogram with a parallelogram height of 4 cm and base of 6 cm.
- Next I chose a prism height of 5 cm, so the area of the base had to be 48 cm^2.
- I chose a hexagon base made up of two equal trapezoids. The area of each trapezoid had to be 24 cm^2. I made the trapezoid height of each of the trapezoids 3 cm and the bases of each 6 cm and 10 cm.
- For the last shape, I chose a house shaped pentagon base. If the prism height this time was 3 cm, the area of the pentagon had to be 80 cm^2. The pentagon is made up of a square that has a side length of 8 cm and a triangle on top with a base of 8 cm and a triangle height of 4 cm.

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Think Sheet

Materials
- Formula Sheet
- calculator
- about 40 linking cubes

Questions to Ask Before Assigning the Think Sheet

◊ *What does the volume of a figure mean?* (e.g., It tells how big it is.) *Big in what way?* (e.g., It means big overall; it includes how wide and deep and high the figure is.)

Build a single layer rectangular prism with linking cubes. Make the dimensions 4 × 3.
◊ *What is the volume of this prism?* (12) *What units would we use if these cubes were 1 cm on a side?* (cubic centimetres)

Build up the prism with two more identical layers.
◊ *What is the volume of the prism now?* (36) *How does the volume compare to the earlier volume?* (e.g., It is 3 times as much.)
◊ *Does that make sense?* (e.g., Yes, since there are three layers so you use three times as many cubes.)
◊ *If the prism was not made of cubes and the base of the prism was 1.5 cm by 6 cm and it was 4 cm high, how could you calculate the volume?* (e.g., I would get the area of the base by multiplying 1.5 × 6 and then I would have four layers, so I would multiply by 4.)
◊ *For what other kinds of prisms could you determine the volume?* (e.g., triangular prisms, hexagonal prisms, octagonal prisms)
◊ *How do you determine the base of each prism?* (e.g., It is the face that gives the prism its name, such as a triangle or a hexagon.)
◊ *Why might it make sense to multiply the area of the base by the height to determine the volume?* (e.g., The area of the base affects how much space it takes up; if there were more height, there would be more layers, so there would be more volume.)

Using the Think Sheet

Read through the introductory box with the students and make sure they understand the material explained in the instructional box.

Provide the Formula Sheet and suggest that students may use their calculators for many of the calculations.

Assign the tasks.

By viewing or listening to student responses, note if they:
- can determine the area of bases of various types of prisms;
- can apply the formula $V = Bh$ for various kinds of prisms;
- can create rectangular prisms with a given volume;
- can solve multi-step problems involving volume;
- can relate the volumes of certain prisms;
- recognize that dimensions can be whole numbers or not.

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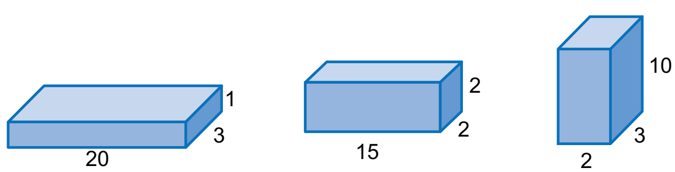
Consolidating and Reflecting: Questions to Ask After Using the Think Sheet
◊ *Why is it important to know how to calculate areas of polygons to calculate volumes of prisms?* (e.g., Part of the formula for the volume is the area of the base and the base is a polygon.) ◊ *How did you choose different rectangular prisms with the same volume in Question 3?* (e.g., I multiplied two numbers to make 60 different ways. For example, since 4 × 15 is 60, I used one prism with a prism height of 4 cm and a base area of 15 cm^2. But 60 is also 6 × 10, so I could use a prism height of 6 cm and a base area of 10 cm^2.) ◊ *Were you surprised with the size of your answer in Question 5?* (e.g., I was surprised, since the cube inside turned out not to use up very much of the volume of the big cube.) ◊ *How did you calculate the area of the base in Question 6a?* (e.g., I thought of the base as being made up of six identical triangles. Each one had a base of 12.6 cm and a triangle height of 10.8 cm.) ◊ *If I give you the volume of a prism, what would be a quick way to get the dimensions of a rectangular prism with that volume?* (e.g., Choose any number for the height of the prism and divide the volume by that number to get the area of the base; then choose a number for the length of the base and divide to get the width.) ◊ *What would be different if it were a different kind of prism?* (e.g., I could still divide to get the height, but I have to think about what shape the base is to figure out how to make sure the area is what it is supposed to be.)

Solutions

1. **a)** e.g., 9 cm^2 **b)** e.g., 50 cm^2 **c)** e.g., 2.0 cm^2

2. **a)** e.g., 72 cm^3 **b)** e.g., 200 cm^3 **c)** e.g., 1.0 cm^3

3. e.g.,



4. e.g., height of 1 cm; length of 50 cm; width of 1 cm

5. 448 cm^3

6. **a)** The area of each triangle is 68.0 cm^2 ($\frac{1}{2}$ of 10.8 cm × 12.6 cm). There are 6 triangles, so the area of the base is 408.2 cm^2.
b) The area of the bottom of the pentagon is 36 cm^2. To get the height of the triangle at the top, I used the Pythagorean theorem by drawing the height and marking two right triangles. The hypotenuse is 6 cm and one leg is 3 cm, so the other leg, which is the height, is 5.2 cm. So the top triangle area is $\frac{1}{2}$ of 6 × 5.2 cm = 15.6 cm. The total area is 51.6 cm^2.
c) The area of the base is 5 cm × $\frac{1}{2}$ of (6 cm + 8 cm) = 35 cm^2.
d) The missing base is $\sqrt{100 - 36}$ cm = 8 cm. So the area is 48 cm^2 ÷ 2 = 24 cm^2.
e) The area of each of the six triangles in the hexagon is $\frac{1}{2}$ of 10 cm × 8.66 cm, which is 43.3 cm^2. So the total area of the base is 6 × 43.3 cm^2 = 259.8 cm^2.

7. **a)** The volume is 408.2 cm^2 × 8 cm = 3265.6 cm^3. **b)** The volume is 8 cm × 51.6 cm^2 = 412.8 cm^3.
c) The volume is 10 cm × ($\frac{1}{2}$ × 5 × 14) cm = 350 cm^3. **d)** The volume is 5 cm × 24 cm = 120 cm^3.
e) The volume is 5 cm × 259.8 cm^2 = 1299 cm^3.

8. **a)** The volume is double **b)** The volume is 4 times as much.
c) The volume is double.

9. e.g., The height is 5 cm, one leg of the triangle is 8 cm and the other leg is 6 cm.

10. Disagree, e.g., if the height were 4.5 cm, the volume would be 90 cm^3.

Open Question

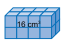
Volumes of Prisms

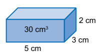
Learning Goal

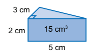
- connecting the formula for the volume of any prism to the formula for the volume of a rectangular prism.

Open Question

The volume of each prism is indicated.

a) 

b) 

c) 

- Why does each make sense?

- Choose a volume.

  Create a set of three different prisms, with different types of bases and different heights, each of which has that volume.

- Repeat once more with a new volume and new types of 3-D figures.

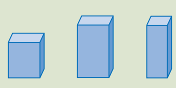
  Make sure to communicate your thinking.

Think Sheet

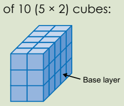
Volumes of Prisms (Continued)

Think Sheet

The **volume** of a 3-D figure tells how much space it takes up. We might also want to know the volume of a 3-D solid to figure out how much it would cost for the material to build it.



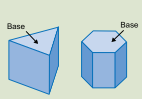
- The centre prism has more volume than the one on the left because the centre prism is taller, with the same "footprint"; the centre prism has more volume than the one on the right because the centre prism has the same height but a bigger "footprint."
- We measure the volume of a rectangular prism by figuring out how many centimetre cubes it would take to build it. This prism has a volume of 30 cubes because there are 3 layers of 10 (5 × 2) cubes:



If the prism were taller, it would have a greater volume. For example, if the base has 10 cubes and there are 6 layers, there would be 6 × 10 = 60 cubes in the volume.

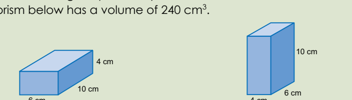
In general, if the area of the base is B, and h is the height of the prism, use this formula for the volume (V): ***V = Bh*** or ***V = (Area of Base) × height***

- Even if the prism is not rectangular, its volume is the area of its base multiplied by its height. It is important to remember that the base of the prism is the face used to name the prism — the base might be a triangle, a hexagon, an octagon, etc. The base will be a rectangle if the shape looks like a standard box.
- For example:



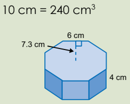
Volumes of Prisms (Continued)

- We can see that the space each prism takes up depends on how much area the base takes up as well as how high the prism is. So ***V = Bh*** or ***V = (area of base) × height*** for these prisms too.
- If the prism is a rectangular prism, any face can be used as the base. Either way, the prism below has a volume of 240 cm^3.

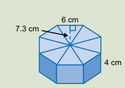


$V = 60\ cm^2 \times 4\ cm = 240\ cm^3$ $\quad$ $V = 24\ cm^2 \times 10\ cm = 240\ cm^3$

- There are times when, to determine the volume, we have to figure out the base's area. For example, suppose we have a regular (all the sides are equal on the base) octagonal prism and we know one side of the base is 6 cm, the distance from the middle of that side to the centre of the base is 7.3 cm, and the height of the prism is 4 cm.



We can think of the octagon as made up of 8 equal triangles, and we know that the base of each is 6 cm and the height of each triangle is 7.3 cm.



Using what we know about area, each triangle area is:
$A = \frac{1}{2} \times 6\ cm \times 7.3\ cm = 21.9\ cm^2$.

So B (area of base of prism) = $8 \times 21.9\ cm^2 = 175.2\ cm^2$.

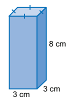
V (volume) = $Bh = 175.2\ cm^2 \times 4\ cm = 700.8\ cm^3$.

Note: The h in the formula for the volume is the height of the whole pyramid. Sometimes we need to use a different height to figure out the area of the base, as we did above with the octagon. If there is a triangular base, the b we use in the formula for that triangle is different from the B we use in the formula for the volume.

Volumes of Prisms (Continued)

1. What is the area of the base of each prism?

a)



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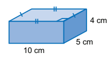
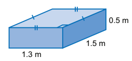


c)



2. Calculate the volume of each prism in Question 1.

a)

b)

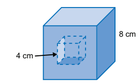
c)

3. Sketch three rectangular prisms, all with a volume of 60 cm^3. Label the length, width, and height of each.

Volumes of Prisms (Continued)

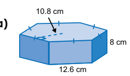
4. A long, narrow, but short, rectangular prism has a volume of 50 cm^3. What might its dimensions be?

5. A cube has a side length of 8 cm. Inside is a hole, also in the shape of a cube, with a side length of 4 cm. How much volume does the part that is still there have?

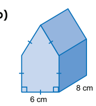


6. What is the area of the base of each of these prisms? Show your work.

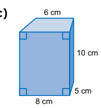
a)



b)

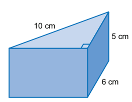


c)

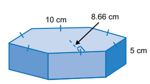


Volumes of Prisms (Continued)

d)



e)



7. Calculate the volume of each prism in Question 6.

a)

b)

c)

d)

e)

8. Another prism is just like the trapezoid-based prism in Question 6c) with one change. Tell how the new volume compares to the old one:

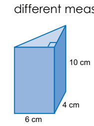
a) The base is the same, but the height of the prism is 20 cm (double the original)

b) The height is the same, but the sides of the base are all double the original side lengths.

c) The height is half the original prism's height, but the side lengths of the base are double the original side lengths.

Volumes of Prisms (Continued)

9. Describe the dimensions of (or sketch with labels) another triangular prism with different measurements but the same volume as this one:



10. Cain said that if the base of a prism has area 20 cm^2, then the volume could be 40 cm^3 or 80 cm^3, but could not be 90 cm^3. Do you agree or disagree? Explain.

Volumes of Cylinders

Learning Goal

- connecting the formula for the volume of a cylinder to the formula for the volume of a prism.

Open Question

Materials
- Formula Sheet
- calculator
- three cylinders – two with the same height, but different radii, and two with the same radius, but different heights

Questions to Ask Before Using the Open Question

Show the two cylinders with the same height.
◊ *Which cylinder has a greater volume? Why do you think that?* (e.g., the fatter one has the greater volume since it is fatter.) *How does being fatter affect the volume?* (e.g., It would take more material to make it.)

Show the two cylinders with the same radius.
◊ *Which of these cylinders do you think has a greater volume? Why?* (e.g., The taller one is has more volume, since it includes the short one and even more.)
◊ *Why does it make sense, then, that the formula for the volume of a cylinder involves the radius size and the height?* (e.g., How wide and how tall the figure is both matter.)
◊ *How do you calculate the area of the base?* (πr^2)

Provide the formula sheet.
◊ *Why does the formula for the volume of a cylinder that is shown on the sheet make sense?* (It includes the radius and the height.)

Using the Open Question

Suggest that students use their calculators.

By viewing or listening to student responses, note if they:
- can determine the area of bases of cylinders;
- can apply the formula $V = Bh$ or $V = \pi r^2 h$ for cylinders;
- can determine the volumes of rectangular prisms;
- can create a cylinder with a given volume;
- predict the effect of changes in r and h on the volume of a cylinder.

Consolidating and Reflecting on the Open Question

◊ *What volume did you select first?* (e.g., 20π cm^3)
◊ *What dimension of the cylinder did you choose first? Why?* (e.g., I chose the height first and then just divided the volume by the height. I knew that the answer had to be the area of the base.)
◊ *Once you knew your radius, how did you figure out your height?* (e.g., I divided the volume by πr^2.)
◊ *Why did you need to take a square root to figure out the radius if you chose the height first?* (e.g., I divided the volume by the height to get the area of the base and the area of the base involves r^2 and not just r.)
◊ *How did you make sure a rectangular prism had the same volume if you used π in your volume?* (e.g., I estimated π as 3.14, then I chose a height and divided. I chose one more number to be the width of the base and divided; the last number was the length of the base.)
◊ *Was the task as easy if you had started with a whole number volume?* (e.g., Yes, I just used an estimate for π.)

Solutions

- First volume is 20π cm^3

  If the cylinder is tall, I will make the height 20 cm.

  Then the area of the base is π cm^2, so the radius is 1 cm.

  If a rectangular prism has that volume, I could make the height of the prism 10 cm. Then the area of the base is 2π cm^2. That means I could make one side of the base 2 cm and the other π cm, which is about 3.14 cm.

- Second volume is 400 cm^3.

  The rectangular prism could have a height of 20 cm and a base with sides of 4 cm and 5 cm.

  The cylinder could have a height of 40 cm. That leaves 10 cm^2 for the area of the base. But since the area is πr^2, that means that I have to take the square root of $\frac{10}{\pi}$ to figure out the radius. It is 1.8 cm.

- Third volume is 500 cm^3.

  The rectangular prism could have a height of 10 cm. That means the area of the base is 50 cm^2. The length could be 10 cm and width could be 5 cm.

  The cylinder could have a height of 25 cm. That means the area of the base has to be 20 cm^2. If 20 cm^2 = πr^2 cm^2, then the radius is the square root of $\frac{20}{\pi}$. That means it is 2.5 cm.

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Think Sheet

Materials
- Formula Sheet
- calculator
- three cylinders – two with the same height, but different radii, and two with the same radius, but different heights

Questions to Ask Before Assigning the Think Sheet

Show the two cylinders with the same height.

◊ *Which cylinder has a greater volume? Why do you think that?* (e.g., the fatter one has the greater volume, since it is fatter.) *How does being fatter affect the volume?* (e.g., It would take more material to make it.)

Show the two cylinders with the same radius.

◊ *Which of these cylinders do you think has a greater volume? Why?* (e.g., The taller one has the greater volume, since it includes the short one and even more.)

◊ *Why does it make sense, then, that formula for the volume of a cylinder involves the radius size and the base?* (e.g., how wide and how tall the shape are both matter.)

◊ *How do you calculate the area of the base?* (πr^2)

Using the Think Sheet

Read through the introductory box with the students and make sure they understand the material explained in the instructional box.

Provide the Formula Sheet and suggest that students may use their calculators for many of the calculations.

Assign the tasks.

By viewing or listening to student responses, note if they:
- can determine the area of bases of cylinders;
- can apply the formula $V = Bh$ or $V = \pi r^2 h$;
- recognize the effect of changes in r and h on the volume of a cylinder;
- can create a cylinder with a given volume;
- can solve multi-step problems involving volumes of cylinders;
- can relate the volume of a cylinder to a related prism or other cylinder.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How did you choose different cylinders with the same volume in Question 3?* (e.g., I multiplied two numbers to make 300 different ways. For example, since 3 × 100 is 300, I used one cylinder with a height of 3 cm and a radius of 10 cm. But 12 × 25 is also 300, so I could use a height of 12 cm and a radius of 5 cm.)

◊ *Why would you need to take a square root to figure out the radius in Question 3 if you chose the height first?* (e.g., You divide the volume by the height to get the area of the base and the area of the base involves r^2 and not just r.)

◊ *How did you know that the prism in Question 6 had a side length equal to the diameter of the circle?* (e.g., The cylinder would not fit in if the prism was smaller, but if you want the smallest possible prism, you do not want any extra width.)

◊ *How can a cylinder have a volume of a whole number of cubic centimetres, e.g., 64 cm³, when the area of the base involves π?* (e.g., You choose a height, and divide it into 64. Then you divide the quotient by 3.14 to figure out the square of the radius.)

◊ *Why did it make sense that the prism was shorter in Question 9?* (e.g., If they were the same height the cylinder would fit in and there would be empty space around the round parts. So the prism needs to be shorter so that the volumes would be the same.)

Note: Volumes are given based on π as well as using an estimate of 3.14.

Solutions

1. **a)** 250π cm^3 (785 cm^3) **b)** 250π cm^3 (785 cm^3)
   c) 176.4π cm^3 (553.9 cm^3) **d)** $\frac{800}{\pi}$ cm^3 (254.78 cm^3)

2. e.g., The heights were the same, but one had a smaller radius, so it had to have a smaller volume.

3. e.g., height of 10 cm, radius of about 5.5 cm; height of 15 cm, radius of 4.5 cm; height of 5 cm, radius of about 7.8 cm

4. The bottom part is a rectangular prism with a volume of 6 cm × 20 cm × 11 cm = 1320 cm^3. The top part is half a cylinder. The whole cylinder would have a height of 20 cm and a radius of 5.5 cm; the volume of the whole cylinder would be 30.25π cm^2 × 20 cm, so a half-cylinder would have a volume of 302.5π cm^3, which is 949.85 cm^3. The full volume is 2269.85 cm^3.

5. If the pipe had not been hollow, the volume would have been 300 cm × 16π cm^2. But the hole has a volume of 300 cm × 6.25π cm^2, so what is left is 2925π cm^3, which is about 9184.5 cm^3.

6. 80 000 cm^3

7. One cylinder has a volume of 121π cm^2 × 29 cm; the other has a volume of 210.25π cm^2 × 22 cm. The first volume is 3509π cm^3 and the other has a volume of 4625.5π cm^3. The volumes are different.

8. The cube has a side length of 4 cm. The cylinder might have a height of 8 cm and a base with area 8 cm^2. That means $\pi r^2 = 8$, so $r = 1.6$ cm.

9. e.g., The prism's volume is $d^2 \times h_p$. The cylinder's volume is $\pi(\frac{d}{2})^2 h_c$.
   That means that $h_c = 4 \div \pi \times h_p$. Since $4 \div \pi$ is about 1.3, then the cylinder height is 30% more.

10. e.g., No. It could be shorter, but much wider.

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Open Question

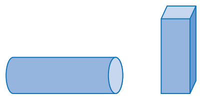
Volumes of Cylinders

Learning Goal

- connecting the formula for the volume of a cylinder to the formula for the volume of a prism.

Open Question

A very tall cylinder has the same volume as a rectangular prism.



- Choose three possible volumes. For each choice, describe the dimensions of both a cylinder and a rectangular prism with that volume. Show your reasoning.

Note: If an answer involves the number π, you may leave the answer in terms of π or may estimate using the value 3.14.

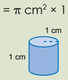
Think Sheet

Volumes of Cylinders (Continued)

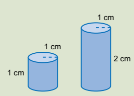
Think Sheet

A cylinder is a lot like a prism, only with a round base, so it makes sense that to calculate the volume of a cylinder, we think the same way as we do about a prism.

For example, a cylinder with a base of radius 1 cm and a height of 1 cm would have V (volume) = π since the area of the base $B = \pi \times (1 \text{ cm})^2$. If we multiply by the height, which is 1 cm, we get $V = \pi \text{ cm}^2 \times 1 \text{ cm} = \pi \text{ cm}^3$.



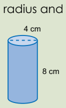
If the cylinder was twice as tall, it would be like stacking two of the original cylinders on top of each other, so the new cylinder's volume would be twice as much as the original volume.



In general, the volume of a cylinder with base radius of r and height h is: $V = \pi r^2 h$.

Notice that we need two values, r and h, to calculate the volume of a cylinder. But we can calculate the volume if we have other measures instead of the radius and height.

For example, if we know the diameter of the base, instead of the radius, we could divide that distance by 2 to get the radius and then use the formula.



$d = 4$ cm, so $r = 2$ cm, so $V = \pi(2 \text{ cm})^2 \times 8 \text{ cm} = 32\pi \text{ cm}^3$ (or 100.48 cm^3).

Even if we know the circumference of the base, we can figure out the radius using the formula $C = 2\pi r$ and proceed to calculate the volume.

If $C = 30$ cm, then $2\pi r$ cm = 30 cm, so $\pi r = 15$ cm and $r = \frac{15}{\pi}$ cm (or 4.78 cm).

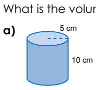
We can now use that value in the regular formula.

Volumes of Cylinders (Continued)

Note: If an answer involves the number π, you may leave the answer in terms of π or you may estimate the answer using the value 3.14.

1. What is the volume of each cylinder?

a)



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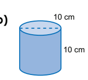

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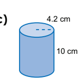

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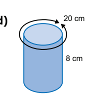
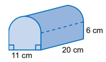


2. Why did it make sense that the volume in Question 1c was less than the volume in Question 1a?

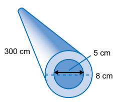
3. The volumes of three different cylinders are each about $300\pi \text{ cm}^3$ (or 942 cm^3). What could the dimensions be?

Volumes of Cylinders (Continued)

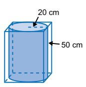
4. What is the volume of this loaf of bread, assuming that the top part of the end of the loaf is a semi-circle? Show your work.



5. How much material is required to make a pipe that is 300 cm long whose outer diameter is 8 cm with an inner diameter of 5 cm? Show your thinking.



6. What is the volume of the smallest square-based prism that would hold this cylinder?



Volumes of Cylinders (Continued)

7. You roll a piece of loose-leaf paper (22 cm × 29 cm) the long way to make an open-ended cylinder. You roll an identical piece of paper the other way to make a different open-ended cylinder that is shorter. Do the cylinders hold exactly the same amount? Explain.

8. A cube and a cylinder have volumes of 64 cm^3. What could be the dimensions of each figure?

9. A cylinder has a diameter that is the same as the side length of a square-based prism. Their volumes are almost the same. How do their heights compare? Show your work.

10. One cylinder has double the volume of another. Does the one with double volume have to be taller? Explain.

Surface Areas of Prisms and Cylinders

Learning Goal

- composing and decomposing shapes to simplify surface area calculation.

Open Question

Materials
- Formula Sheet
- calculator
- rectangular prism
- triangular prism
- cylinder
- solid shapes (optional)

Questions to Ask Before Using the Open Question

Show the rectangular prism.
◊ *Suppose I wanted to cut out wrapping paper to just cover this prism? What information would I need?* (e.g., You would need to know how big the box is.) *In what way?* (e.g., I would need to know the areas of all of the faces.)
◊ *Which dimensions of the prism would you need to know to do that?* (e.g., You need to know how high it is, how wide, it is and how deep it is.)

Show the triangular prism.
◊ *What if you were covering this figure?* (e.g., I would need to know the areas of each rectangle and the area of the top and bottom.)
◊ *How would you calculate those areas?* (e.g., for the rectangles, I would have to know each side length of the triangle and multiply by the height. For the triangles, I would have to take half the base times the triangle height.)
◊ *If it was a right triangle, how would you figure out the side length you do not know if you knew the base and hypotenuse?* (I would use the Pythagorean theorem.)

Show the cylinder.
◊ *Which surface area of the cylinder would be trickiest for you to calculate?* (e.g., It would be the tall part.)
◊ *If you removed the top and bottom and cut straight down the side and opened the cylinder up, what shape would you see?* (a rectangle) *How wide is the rectangle and how long?* (The height is the length and the width is actually the circumference of the base.)

Have students examine the surface area formulas on the formulas sheet.
◊ *Why does the first prism formula make sense to you?* (e.g., There are two bases with the same area and then there are different numbers of rectangles for different prisms.)
◊ *Why does the cylinder surface area formula make sense?* (e.g., it includes both the top and the bottom and the round tall part.)

Using the Open Question

You may choose to make a variety of prisms and cylinders available for students to view.

By viewing or listening to student responses, note if they:
- can calculate areas of various 2-D shapes and circles;
- are careful to include all faces when calculating a surface area;
- recognize when faces have the same area and shortcut the calculations for surface area using that knowledge;
- realize they need to know each side length of the base to calculate surface area;
- understand why the lateral surface of a cylinder has a length and width of the height of the prism and circumference of its base;
- can adjust one figure to create another with the same surface area.

Consolidating and Reflecting on the Open Question

◊ *Why did you choose the surface area you did?* (e.g., I knew that a cube with six faces with each side length of 10 cm would have that volume, so one shape was already done.)
◊ *Once you had selected that one, how did you choose a trapezoid-based prism that would work?* (e.g., I chose an isosceles trapezoid and decided on the two bases and the height. That way, I knew the area of the top and bottom and subtracted that total from 600. I used the Pythagorean theorem to figure out the slanted sides. I knew that if I multiplied by the height, I would get the area of each rectangle. So I used the variable h for the height. I used an equation involving h where the total area was the number I wanted and then solved it for h.)
◊ *Which shape, other than the cube, was easiest for you?* (e.g., the cylinder, since I wrote an equation and solved it for r.)
◊ *Why did you need to know all of the side lengths for every prism?* (e.g., The surface area includes the rectangles and the widths of the rectangles change depending on each side length.)

Solutions

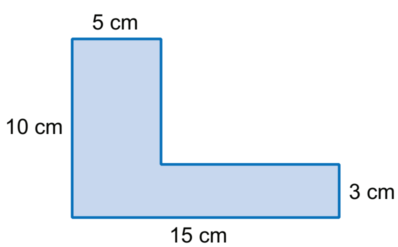
e.g., I chose 600 cm^2.

My square-based prism is a cube. It has six faces that are 10-cm squares.

My rectangle-based prism had a base 4 cm × 25 cm. That means Ph = 400 cm^2, since 200 of the 600 units are used in the two bases. So since P = 58, then h = 6.9 cm

I chose a right trapezoid with parallel bases of 5 cm and 7 cm and a height of 6 cm. That meant its area would be 36 cm^2, so the two bases would have a total area of 72 cm^2. The fourth side of the trapezoid is $\sqrt{40}$ cm, which is about 6.3 cm. That means the perimeter is 24.3 cm. Since SA = 72 + 24.3h and that is 600, that means h = 21.7 cm.

I chose a hexagon that is shaped like this:



Its area is 80 cm^2, so two bases have a total area of 160 cm^2.
The perimeter of this shape is 50 cm.
That means that 160 + 50h = 600.
So 50h = 440; h = 8.8 cm.

For the cylinder, I chose a radius of 5 cm.
That means the area of one base is 25π cm^2 and the area of both bases is 50π cm^2.
The rest of the surface area is 10πh.
So 50π + 10πh = 600.
That means that 31.4h = 443, so h = 14.1 cm.

Think Sheet

Materials
- rulers
- calculators
- rectangular prism
- triangular prism
- cyinder
- string, scissors
- Formula Sheet

Questions to Ask Before Assigning the Think Sheet

Show the rectangular prism.

◊ *Suppose I wanted to cut out wrapping paper to just cover this prism? What information would I need?* (e.g., You need to know how big the box is.) *In what way?* (e.g., I would need to know the areas of all of the faces.)

◊ *Which dimensions of the shape would you need to know to do that?* (e.g., You need to know how high it is, how wide it is, and how deep it is.)

Show the triangular prism.

◊ *What if you were covering this figure?* (e.g., I would need to know the areas of each rectangle and the area of the top and bottom.)

◊ *How would you calculate those areas?* (e.g., For the rectangles, I would have to know each side length of the triangle and multiply by the height. For the triangles I would have to take half the base times the triangle height.)

◊ *If it was a right triangle, how would you figure out the side length you do not know if you knew the base and hypotenuse?* (I would use the Pythagorean theorem.)

Show the cylinder.

◊ *Which surface of the cylinder would be trickiest for you to calculate the area of?* (e.g., It would be the round part.)

◊ *If you removed the top and bottom and cut straight down the side and opened it up, what shape would you see?* (a rectangle) *How wide is the rectangle and how long is it?* (The height is the length and the width is actually the circumference of the base.)

Using the Think Sheet

Read through the introductory box with the students and make sure they understand the material explained in the instructional box.

Assign the tasks on the page.

By viewing or listening to student responses, note if they:
- can calculate areas of various 2-D shapes and circles;
- are careful to include all faces when calculating a surface area;
- recognize when faces have the same area and shortcut the calculations for surface area using that knowledge;
- realize they need to know each side length of the base to calculate surface area;
- understand why the lateral surface of a cylinder has a length and width of the height of the prism and circumference of its base;
- realize that one dimension of a prism or cylinder can be greater than another's corresponding dimension even if its surface area is less;
- can solve multi-step problems involving surface area;
- recognize why a shape might have a big surface area.

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Consolidating and Reflecting: Questions to Ask After Using the Think Sheet
◊ *Why are there always at least two faces with the same area when you work with prisms and cylinders?* (The bases always have the same area.) ◊ *Why do you need to do more area calculations to determine the surface area of some prisms than others, even if they have the same number of faces?* (e.g., If some of the sides of the bases are equal, the areas attached to those sides are equal, so you only need to calculate one of them and multiply.) ◊ *Why did you need to know all of the side lengths for every prism?* (e.g., The surface area includes the areas of the rectangles and the widths of the rectangles change depending on each side length.) ◊ *What did you have to consider when calculating the surface area for Question 8c? (e.g. whether different parts of the shape were missing or sitll there)* ◊ *What makes a shape have a big surface area?* (e.g., It will have a big surface area if at least some of its faces are big.) ◊ *Which surface area formula do you find most convenient?* (e.g., I like the one that says 2*B* + P*h* since it works for all the shapes and is quite simple to remember.)

Solutions

Note: A number of solutions are given both in terms of π as well as using an estimate of 3.14 for π.

1. **a)** e.g., 25 cm^2 **b)** 44 cm^2
c) 16π cm^2 (or 50.24 cm^2)
d) $6 \times \frac{1}{2}$ of 6 cm × 5.2 cm = 93.6 cm^2
e) 25π cm^2 (or 78.5 cm^2)

2. **a)** 2: I need one of the height rectangles (which I can multiply by 4) and one of the bases (which I can multiply by 2).
b) 5: I can use the base twice, but each of the rectangles that include the height has the same height, but a different width.
c) 2: I need to figure out the area of the base and double it and figure out the lateral surface area.
d) 2: I need one of the height rectangles (which I can multiply by 6) and one of the bases (which I can multiply by 2).
e) 2: I need to figure out the area of the base and double it and figure out the lateral surface area.

3. **a)** 4 × 50 cm^2 + 2 × 25 cm^2 = 250 cm^2
b) 2 × 44 cm^2 + 5 cm × 20 cm + 6 cm × 20 cm + 16 cm × 20 cm + 8.1 cm × 20 cm = 790 cm^2. I figured out the area of the last rectangle by using the Pythagorean theorem to get the side length of the side on the right. The legs of that right triangle are 4 cm and 7 cm.
c) 2 × 16π + 96 π cm^2 = 128π cm^2 = 401.9 cm^2
d) I figured out that the area of the base was 93.6 cm^2 and there are two of them. Each side rectangle is 48 cm^2, so the total area is 6 × 48 cm^2 + 2 × 93.6 cm^2 = 475.2 cm^2
e) Each base has an area of 25π cm^2 and the lateral surface has an area of 100π cm^2, so the total surface area is 150π cm^2, which is 471 cm^2.

4. A could be true if it was a cube.
B would always be true since the two bases have to be identical.
C could be true if it was a square-based prism.

5. **a)** a cube – each face with an area of 25 cm^2, or a side length of 5 cm
b) a rectangular prism with a base of 3 cm × 4 cm and a height of 5 cm
c) a right triangular prism with a base with sides 3 cm, 4 cm, and 5 cm and a height of 10 cm
d) a cylinder with a radius of 3 cm and a height of 10 cm

6. Yes, e.g., A prism with a base of 1 cm × 1 cm that is 100 cm tall has a surface area of 402 cm^2. A short prism that is 1 cm high with a base of 100 cm × 100 cm has much greater surface area – it is 20 400 cm^2.

7. The front and back triangles have an area of 200 cm^2 each. The back rectangle has an area of 3400 cm^2. The hypotenuse of the triangle is $\sqrt{800}$ cm, which is about 28.28 cm, so the other rectangle has an area of 28. 28 × 170 cm^2, which is 4807.6 cm^2. The total area is 8607.6 cm^2.

8. **a)** $2 \times 2 \times 1.7\ m^2 + 2 \times 2 \times 3\ m^2 + 2 \times 3 \times 1.7\ m^2 = 29\ m^2$
   b) 0.9π m × 3 m = 2.7π m2 = 8.5 m^2
   c) 36.2 m^2; I added the two surface areas and subtracted the area of the two circles.

9. e.g., I think it will be the number of square centimetres in the surface area. Since it is so short and thin, the volume will not be much different than the long length of it. But the surface area will be more than four of those lengths.

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Open Question

Surface Areas of Prisms and Cylinders

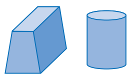
Learning Goal

- composing and decomposing shapes to simplify surface area calculation.

Open Question

The **surface area** of a 3-D figure is the total of the area of all of its surfaces.

We might use it to figure out how much material it would take to exactly cover a box or can.



- Choose a number of square centimetres between 100 and 1000.
- Use that number to create five different figures with that surface area. Show your thinking.
  1. A prism with a square base
  2. A prism with a non-square rectangular base
  3. A prism with a trapezoid base
  4. A prism with a hexagon or octagon base
  5. A cylinder

Note: If an answer involves the number π, you may leave the answer in terms of π or you may estimate using the value 3.14.

Think Sheet

Surface Areas of Prisms and Cylinders (Continued)

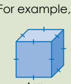
Think Sheet

The **surface area** of a 3-D figure is the sum of all of the areas of its faces. We might use the surface area to know how much wrapping paper is needed to wrap a box or perhaps how much aluminum is needed to make a can.

Surface area is measured in square units, like square centimetres, square metres, etc.

When we know something about the relationship between the faces of the prism there are fewer calculations to determine the surface area than in other situations.

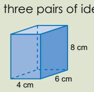
For example, suppose there is a cube with a side length of 4 cm.



Each face has an area of 4 cm × 4 cm = 16 cm^2, so instead of adding six numbers, we can multiply one number by 6.

The total area is 6 × 16 cm^2 = 96 cm^2.

But if the prism is a rectangular prism that was not a cube, the six faces would not be identical. There would be three pairs of identical faces:



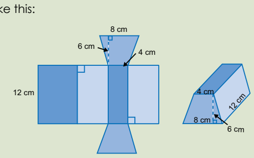
Two of the faces (front and back) have areas of 4 cm × 8 cm.

Two of the faces (sides) have areas of 8 cm × 6 cm.

Two of the faces (top and bottom) have area of 4 cm × 6 cm.

So the total surface area is 2 × 32 + 2 × 48 + 2 × 24 cm^2 = 208 cm^2.

One way to visualize the total surface area is to use a net of the 3-D figure — a flattened-down representation of it. For example, the net of a trapezoidal-based prism might look like this:



Surface Areas of Prisms and Cylinders (Continued)

We would determine the area of each shape in the net and add those values.

For the trapezoid-based prism, the height of each rectangle is 12 cm. The width varies, depending on the side to which each rectangle attaches. One width is 8 cm, one width is 4 cm, and two widths are the lengths of the slanted sides of the trapezoid.

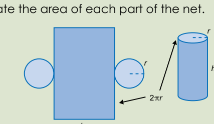
To determine those values, we could use the Pythagorean theorem, realizing that one leg of a right triangle is 6 cm and the other leg is half of 8 cm – 4 cm. The hypotenuse (the side we need) is $\sqrt{36 + 4}$ cm = 6.32 cm.

Each trapezoid has an area of $\frac{1}{2}$(8 cm + 4 cm) × 6 cm, or 36 cm^2.

The total area is
2 × 36 cm^2 + 12 cm × 8 cm + 12 cm × 4 cm + 12 cm × 6.32 cm + 12 cm × 6.32 cm.
That is 367.68 cm^2.

We could have written 2 × 36 cm^2 + 12 cm (4 cm + 6.32 cm + 8 cm + 6.32 cm). The expression in brackets is the perimeter of the trapezoid base of the prism. So another formula for the surface area of a prism is $SA = 2B + Ph$ where B is the area of the base, P is the perimeter of the base and h is the height of the prism.

- We can calculate the surface area of a cylinder in a similar way. We could create a net showing its two circular bases and its lateral surface (the tall round part) and calculate the area of each part of the net.



The rectangle is the result of cutting straight down the height of the cylinder.

Notice that one dimension of the rectangle is the height of the cylinder but the other dimension is the circumference of the base of the cylinder; this happens to ensure that the lateral surface wraps all the way around the top and bottom circular bases.

That means the area of the rectangle is $2\pi rh$, where r is the radius of the base and h is the height of the cylinder.

The full surface area (SA) is twice the area of the base (for a cylinder with a top and bottom) and the area of the rectangle cut from its lateral surface.

So $SA = 2B + Ch$, where B is the area of the base, C is the circumference of the base and h is the height of the cylinder. Since the circumference is really a perimeter, it is the same formula as for a prism.

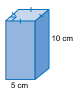
Two other versions of the formula for the surface area of a cylinder are based on being more specific about the area of the base and circumference:

$SA = 2\pi r^2 + 2\pi rh$ or $SA = 2\pi r(r + h)$.

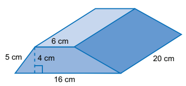
Note: If an answer involves the number π, you may leave the answer in terms of π or you may estimate the answer using the value 3.14.

1. What is the area of the base of each of these prisms or cylinders? Show your work.

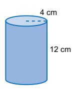
a)



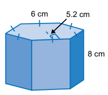
b)



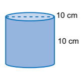
c)



d)



e)



2. Tell how many separate areas you must figure out in order to determine the surface area of each prism or cylinder in Question 1. Explain your thinking.

3. Calculate the surface areas of each prism or cylinder in Question 1.

4. Decide whether each statement is sometimes, always or never true when calculating the surface area of a prism with a 4-sided based. Explain.

A: All the faces could have the same area.

B: At least two faces have to have the same area.

C: There could be exactly four faces with the same area.

5. The calculations for the surface areas of different prisms or cylinders are described. Sketch or tell about the dimensions and faces of the prism.

a) $6 \times 25\ \text{cm}^2$

b) $2 \times 12\ \text{cm}^2 + 2 \times 15\ \text{cm}^2 + 2 \times 20\ \text{cm}^2$

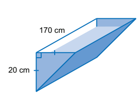
c) $2 \times \frac{3 \times 4}{2}\ \text{cm}^2 + 3 \times 10\ \text{cm}^2 + 4 \times 10\ \text{cm}^2 + 5 \times 10\ \text{cm}^2$

d) $18\pi\ \text{cm}^2 + 60\pi\ \text{cm}^2$

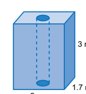
6. Could a tall prism have a smaller surface area than a shorter one? Explain.

7. How many square centimetres of metal would be needed to make this water trough? Remember there is no top.

Show your thinking.



8. A hole in the shape of a cylinder is drilled all the way through from top to bottom inside this wooden rectangular prism. The radius of the hole is 0.45 m.



a) What would be the surface area of the prism if there were no hole?

b) What is the surface area of the cylinder inside not including the top and bottom?

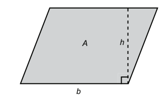
c) What is the surface area of the block of wood remaining? Explain.

9. A rectangular prism is very, very long, but not very high and not very wide. Which number do you think will be greater — the number of square centimetres in its surface area or the number of cubic centimetres in its volume? Explain your thinking.

Formula Sheet

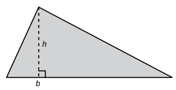
Area of a parallelogram

$A = bh$



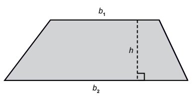
Area of a triangle

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$



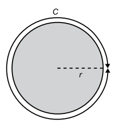
Area of a trapezoid

$A = \frac{1}{2}(b_1 + b_2)h$ or $A = (b_1 + b_2)h \div 2$



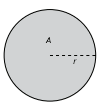
Circumference of a circle

$C = 2\pi r$ or $C = \pi d$



Area of a circle

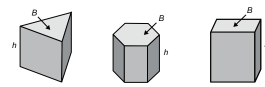
$A = \pi r^2$



Formula Sheet cont.

Volume of any prism

$V = Bh$ where B is the area of the base and h is the height of the prism

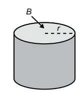


Volume of a cylinder

$V = Bh$ where B is the area of the base and h is the height of the cylinder

OR

$V = \pi r^2 h$ where r is the radius of the base and h is the height of the cylinder

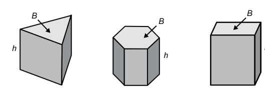


Surface area of any prism

$SA = 2B +$ *areas of all rectangles* where B is the area of the base

OR

$SA = 2B + Ph$ where B is the area of the base, P is the perimeter of the base and h is the height of the prism



Formula Sheet cont.

Surface area of a cylinder

$SA = 2B + Ch$ where B is the area of the base, C is the circumference of the base and h is the height of the cylinder

OR

$SA = 2\pi r^2 h + 2\pi rh$ where r is the radius of the base and h is the height of the cylinder
OR

$SA = 2\pi r(r + h)$ where r is the radius of the base and h is the height of thecylinder