GAP CLOSING

2D Measurement

Intermediate / Senior
Student Book

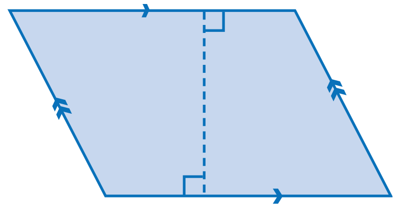
2-D Measurement

This entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/8-2Dmeasurement_SB_IS.pdf. The Facilitator's Guide for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/8-2DMeasurement_FG_IS.pdf.

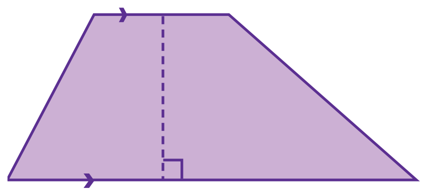
Diagnostic

1. Use a marker or a coloured pencil to indicate the lengths needed to calculate the area of each shape. Mark only the necessary measurements on each shape.

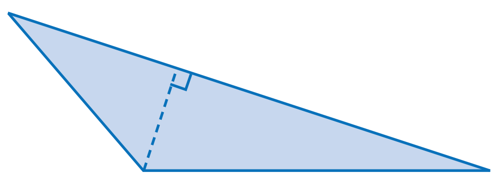
   a) Parallelogram



   b) Trapezoid

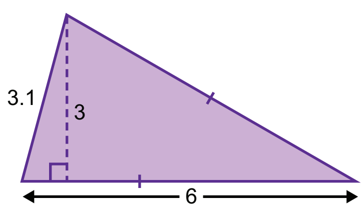


   c) Triangle



2. Calculate each area:

   a)



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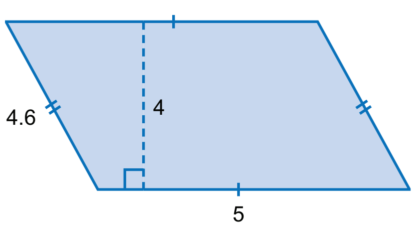
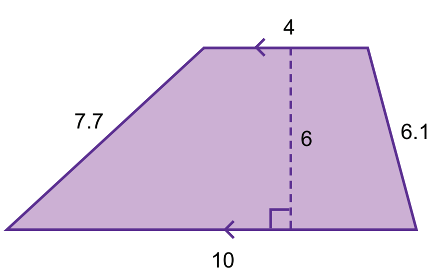


   c)

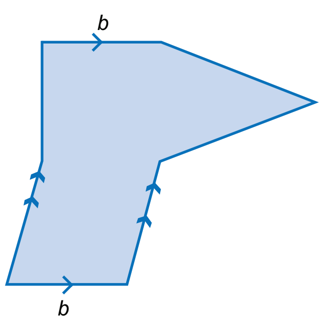


3. A triangle has a base of 5 cm and a height of 10 cm. A parallelogram with a base of 5 cm has the same area. What is the height of the parallelogram?

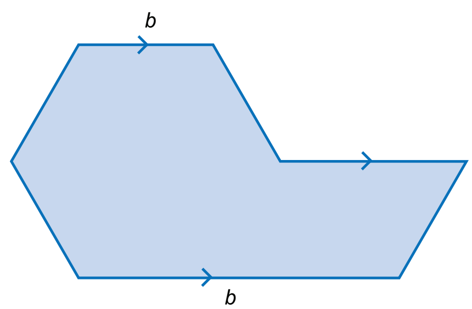
4. A trapezoid has an area of 20 cm^2. What could the height and base lengths be?

5. Show a way to divide each shape into any combination of triangles, rectangles, parallelograms, or trapezoids.

a)

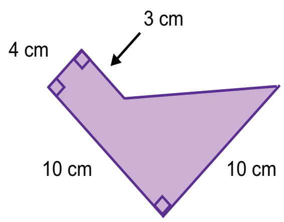


b)



6. Calculate the area of each shape. Show your work.

a)



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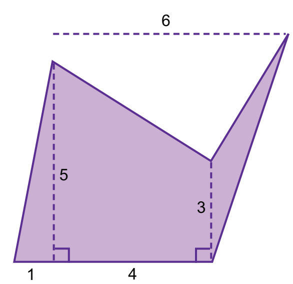
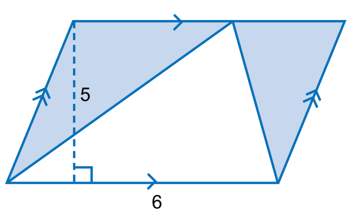


7. What is the area of the shaded space?

a)



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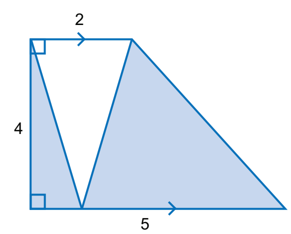
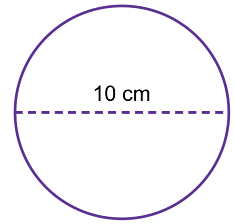


8. The diameter of a circle is 10 cm.

a) What is the area of the circle?



b) What is the circumference of the circle?

9. The circumference of a circle is about the same as the perimeter of a square with side length 3 cm. What is the diameter of the circle? Show your thinking.

10. A string is three times the length of the diameter of a circle. Circle the statement that is true:

A That string would go about halfway around the circle.

B That string would go almost all the way around the circle.

C That string would go around the circle and a bit more.

D That string would almost go around the circle twice.

11. The area of a circle is about 20 cm^2. Circle all of the statements that are true.

a) The radius is about 2.5 cm.

b) The radius is about 5 cm.

c) The diameter is about 5 cm.

d) The diameter is about 10 cm.

e) The circumference is about 15 cm.

f) The circumference is about 30 cm.

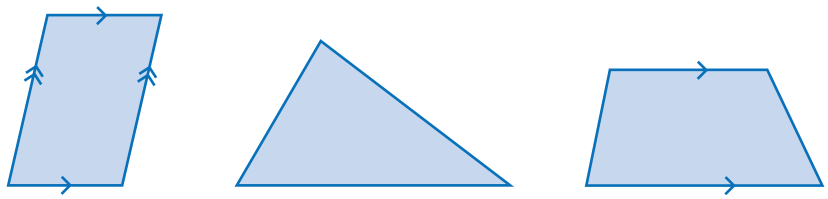
Areas of Parallelograms, Triangles, and Trapezoids

Learning Goal

- recognizing the efficiency of using formulas to simplify area calculations.

Open Question

A parallelogram, a triangle, and a trapezoid (none with a right angle), all have the same area.



- **Step 1:** Choose the area; ________ cm^2.

  Draw sketches and mark the dimensions (bases, heights) so that your three shapes will have this area. Show why the shapes have the required area.

- **Step 2:** Repeat using the same area, but different dimensions, for your three shapes.

- **Step 3:** Repeat Steps 1 and 2, using two different areas.

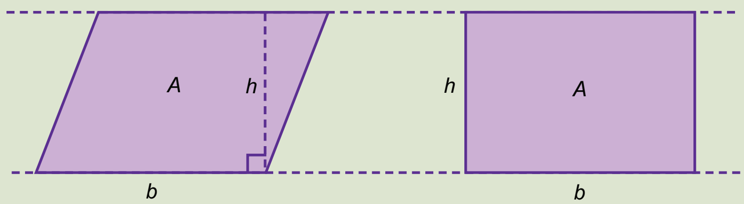
Think Sheet

One way to measure the area of a shape is to place that shape on a grid and count squares.

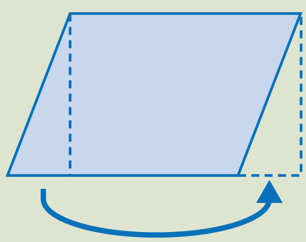
We can also use a formula that allows us to calculate the area using only length measures.

Area of a parallelogram

Consider a rectangle with the same base and the same height as the parallelogram. We know that the area of the rectangle is its length multiplied by its height.



The parallelogram can be rearranged into the rectangle. So knowing a parallelogram's base length and height allows us to calculate its area using that same formula we use to calculate the area of a rectangle.



$A = b \times h$

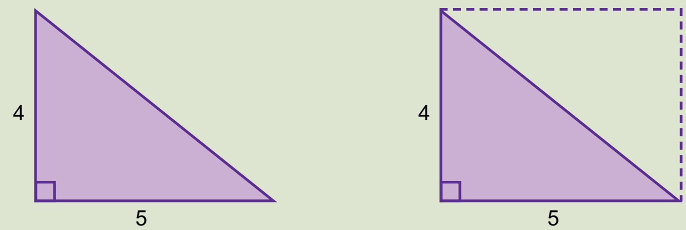
Notice that the height of the parallelogram is shorter than the slanted side length. It is important to use a base and the height, and not two side lengths, in the formula.

If we know the area of a parallelogram and the base or height, we can figure out the other dimension by dividing. For example, a parallelogram with an area of 20 cm^2 and a base of 4 cm has to have a height of $20 \div 4 = 5$ cm.

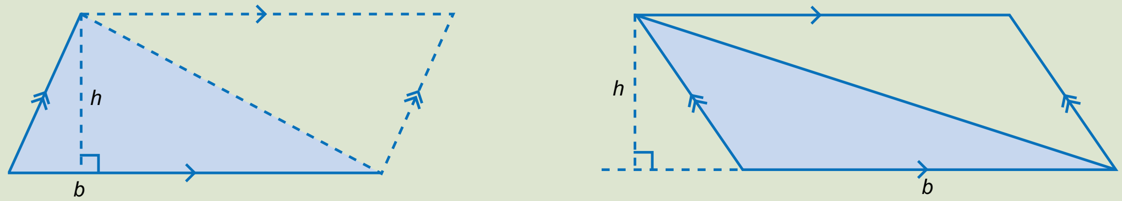
Area of a triangle

To figure out the areas of right triangles we might notice that they are halves of rectangles.

For example, the area of the triangle has to be $\frac{1}{2}$ of 4 × 5:

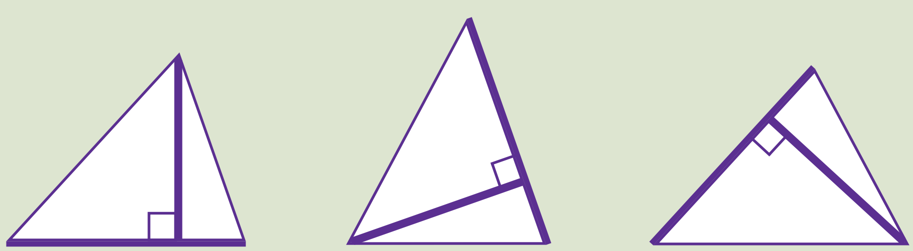


Other triangles are not as easy to visualize in terms of rectangles. These triangles might be viewed as half parallelograms.



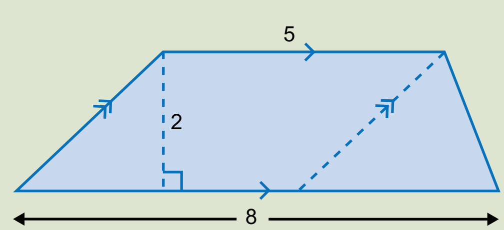
Each triangle is half of a parallelogram with a base that matches the triangle's base and a height that matches the triangle's height. [Sometimes, as with the triangle on the right, the height has to be drawn outside of the shape by extending the base.]

Any side of a triangle can be considered the base of that triangle, but the height that is used in the formula must be the height that is perpendicular to that base. There are three possible base-height combinations for a triangle:



Area of a trapezoid

Every trapezoid can be divided into a parallelogram and a triangle. The area of the trapezoid can be calculated by adding the area of that parallelogram and the area of that triangle. For example:



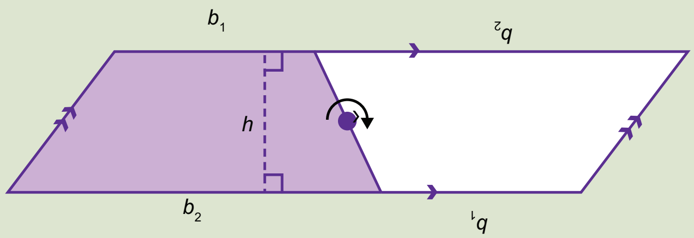
The 8 units in the base can be divided into 5 units for the base of the parallelogram and 3 for the base of the triangle. The heights of the parallelogram and triangle are 2 units.

The area of the parallelogram is 2 × 5 square units.

The area of the triangle is $\frac{1}{2}$ of 3 × 2 square units.

So the area of the trapezoid is 2 × 5 + $\frac{1}{2}$ of 3 × 2 = 13 square units.

We can also think of any trapezoid as half of a parallelogram; turn a copy of the trapezoid around 180° and let the sides of the original and the copy touch, as shown:



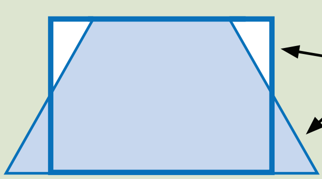
The base of the parallelogram is the sum of the two bases of the trapezoid ($b_1 + b_2$).

The height of the trapezoid is the height of the parallelogram.

But the area of the trapezoid is only half of the area of the parallelogram, so the area of the trapezoid is $\frac{1}{2}(b_1 + b_2)h$.

The area for the trapezoid above would be $\frac{1}{2}(5 + 8) \times 2 = 13$ square units.

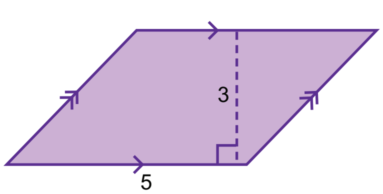
We can also think of the formula as: Take the average of the two bases and multiply by the height. This makes sense if you think of the rectangle with a base that is the average of the two bases and the same height as the parallelogram.



The extra white areas balance the areas of the small triangles on each side that are not inside the rectangle.

1. Determine each area.

a)



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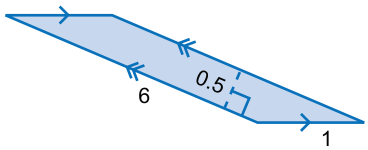
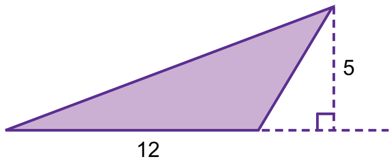


c)



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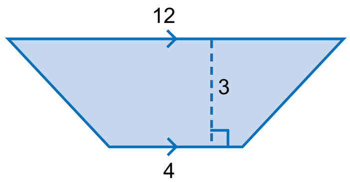
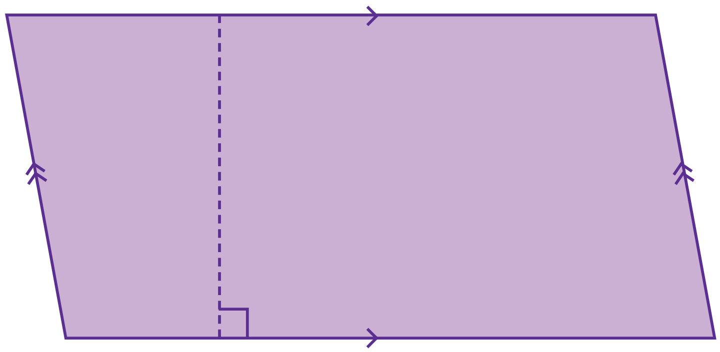
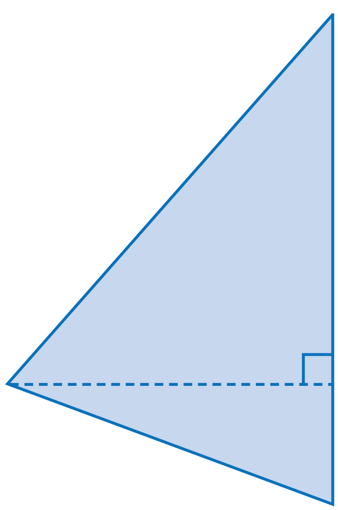


2. Use your ruler to measure the lengths you need to determine each area. Then estimate the area. Show your steps.

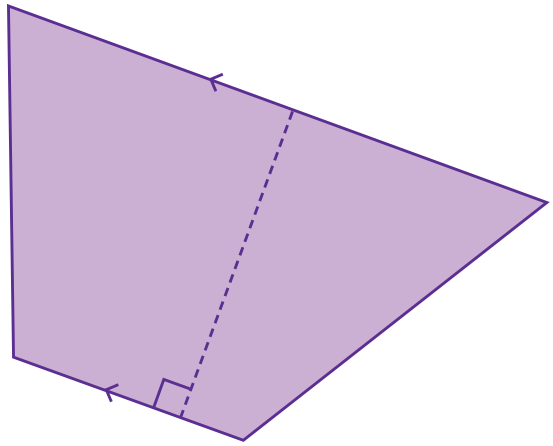
a)



b)



c)



3. What is the height of each shape?

a) a parallelogram with an area of 30 cm^2 and a base of 5 cm

b) a triangle with an area of 30 cm^2 and a base of 5 cm

c) a trapezoid with an area of 30 cm^2 and bases of 4 cm and 8 cm

4. a) Draw a non-right triangle and calculate its area.

b) Use a different side of the triangle as the base. Measure it.

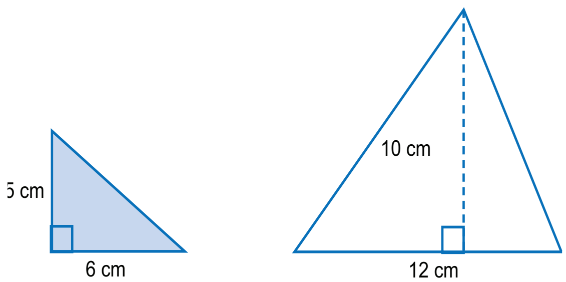
c) Tell what the height that matches that base must be and why.

d) Measure to check.

Areas of Parallelograms, Triangles, and Trapezoids (Continued)

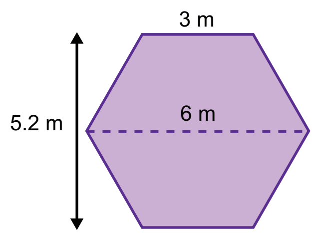
5. A parallelogram and a triangle have the same base and the same area. What do you know about their heights? Explain why.

6. How can you tell that the shaded triangle has $\frac{1}{4}$ the area of the white one without calculating each area?

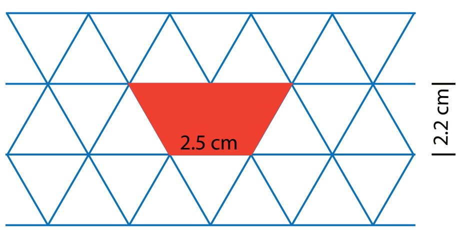


7. Sketch three different trapezoids, each with an area of 60 cm². Mark the base lengths and heights for each.

8. What is the area of this regular hexagon? How do you know?



9. What is the area of the red pattern block?



10. A trapezoid has an area of 100 cm^2. Its height is 10 cm. What else do you know about the measurements of the trapezoid?

11. Draw any trapezoid. Divide it into two triangles. Create a formula for the area of a trapezoid based on the areas of those two triangles. Show your reasoning.

Areas of Composite Shapes

Learning Goal

- composing and decomposing shapes to simplify area calculation.

Open Question

A composite shape is made up of a combination of some of these shapes: rectangles, triangles, parallelograms, and trapezoids.

Create composite shapes using these conditions:

- There is always at least one parallelogram or trapezoid as part of the shape.
- The shape fits inside a rectangle that is 30 cm × 20 cm.
- The total area of the shape is 500 cm^2.

- Draw at least three (or more) possible shapes. Use a marker or coloured pencil to mark on your drawings those measurements of the rectangles, triangles, parallelograms, and trapezoids you would need to know to be sure that the area is 500 cm^2 without using a grid.

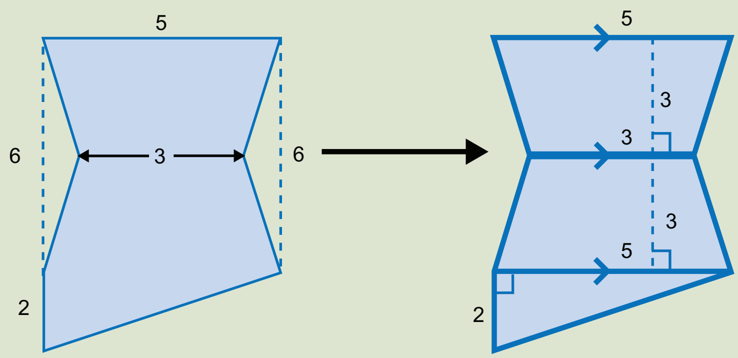
- Tell what the area of each component must be, using formulas for the areas of those particular shapes.

Think Sheet

Composite shapes are shapes that are made by putting together other shapes.

If a composite shape is made up of rectangles, triangles, trapezoids, and/or parallelograms, we can use what we know about the areas of the simpler shapes to figure out the area of the composite shape.

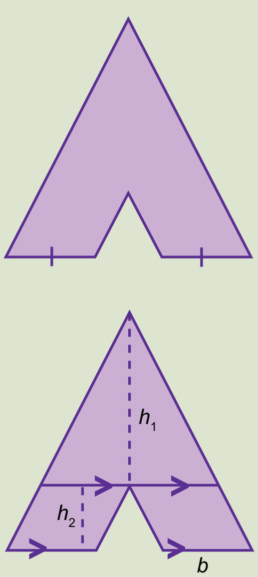
For example, it is challenging to figure out the area of the shape on the left unless we notice it can be divided into two trapezoids and a triangle.



The area of each trapezoid is $\frac{1}{2}(3 + 5) \times 3$ square units. The height of 6 is divided — half for each trapezoid. The area of the triangle is $\frac{1}{2}$ of $2 \times 5 = 5$ square units. So the total area is 29 square units.

It is important that the dimensions we need for the simpler shapes are given or that we have enough information to figure them out. Notice that we had to figure out that the height of each trapezoid was 3 from what was given.

The area of this shape can be calculated if only a few lengths are known:

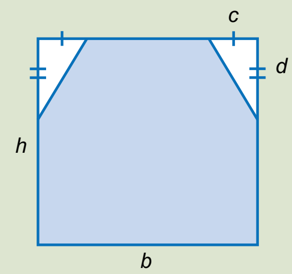


We can divide the shape into a triangle and two parallelograms. If we know the base of one of the parallelograms, *b*, and the two indicated heights, we can calculate the area. That is because the base of the triangle is twice the base of the parallelogram. So if we know the base of the triangle, we can calculate the base of the parallelogram.

$$A = bh_2 + bh_2 + \frac{1}{2}h_1 \times 2b$$

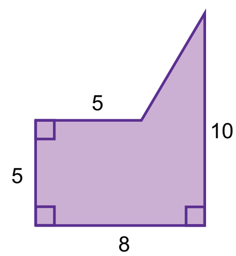
$$A = 2bh_2 + bh_1$$

Sometimes we take away the area of a known shape to calculate the area of a more complicated shape. For example, the area of the shaded shape below is $bh - 2 \times \frac{1}{2}cd$.



1. What simpler shapes could you use to describe the shaded shape?

a)



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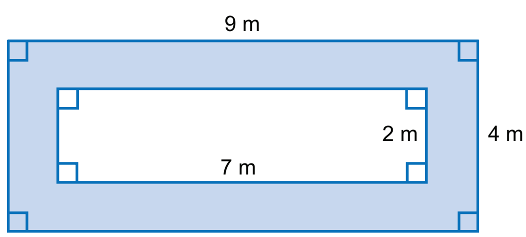

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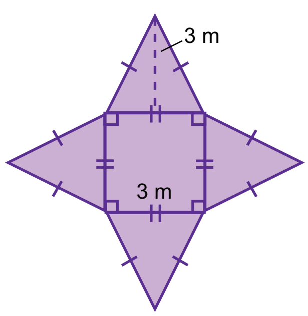

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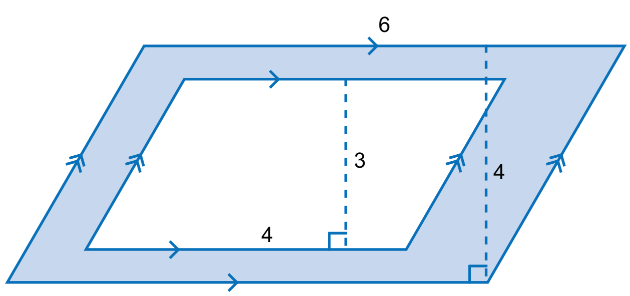
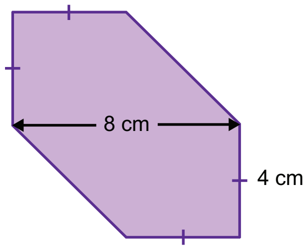


e) 

2. Calculate the areas of the shapes in Question 1.

a)

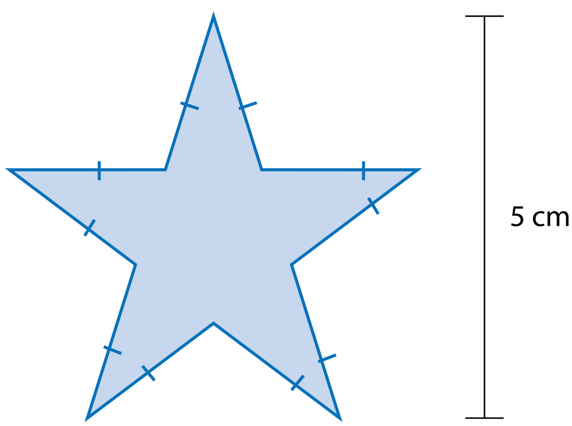
b)

c)

d)

e)

3. a) How would you divide up this star into simpler shapes to make it easier to calculate its area?



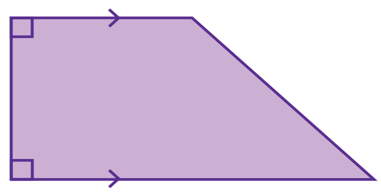
b) Use your ruler to measure. Mark the measurements that you would need to calculate the areas of the pieces of the star.

c) Estimate the area of the star?

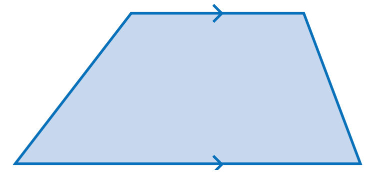
4. a) Prove that every trapezoid can be divided into three triangles.

b) What dimensions would you need to know to calculate the area of the trapezoid based on how you divided it up in part a)? Explain.

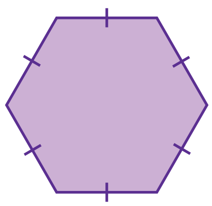
5. a) If you know all of the side lengths of a trapezoid with two right angles, but no other measurements, can you figure out the area without doing any more measurements? Explain.



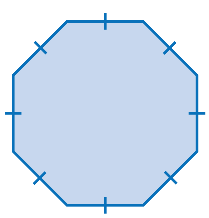
b) If you know all of the side lengths of a non-right trapezoid but no other measurements, can you figure out the area without doing any more measurements? Explain.



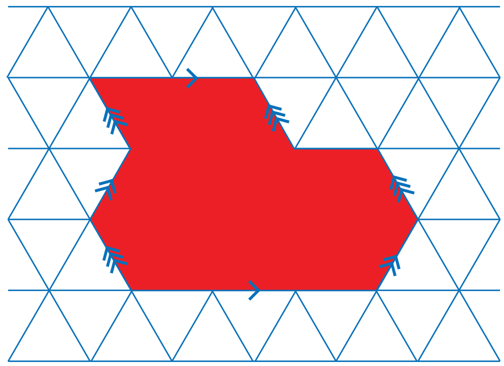
6. a) How could you divide a regular hexagon into simpler shapes to calculate its area? Mark the fewest dimensions you would need to know to figure out the area and explain.



b) Repeat part a) with a regular octagon.

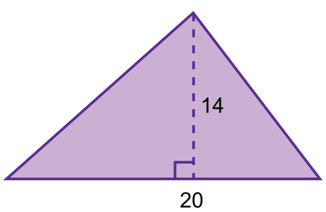


7. Show two different ways to measure lengths so you can calculate the total area. Explain your thinking.

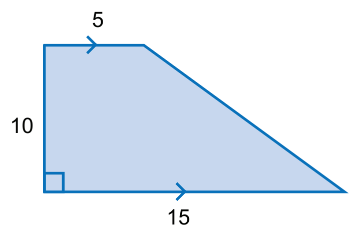


8. Divide each shape into two shapes of equal area. Prove that the shapes have equal area.

a)



b)



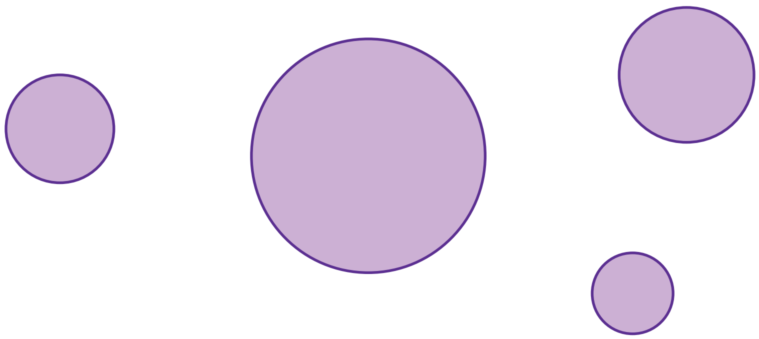
Circumferences and Areas of Circles

Learning Goal

- recognizing that knowing one measure of a circle provides information about all other measures.

Open Question

Draw at least four circles of different sizes.



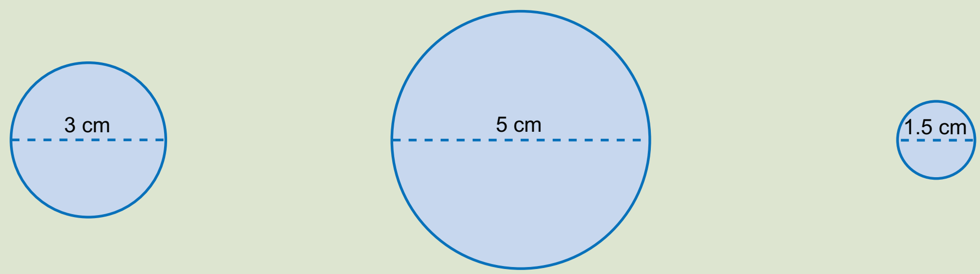
- For each circle, measure the distance around it. Compare that measurement to the diameter of that circle.
- For each circle, estimate the area of the circle using grid paper. Compare it to the area of a square whose side length is the radius of the circle.
- Describe what you observed. Draw pictures to show why what you observed makes sense.

Think Sheet

Measuring the perimeter of circles is more challenging than measuring the distance around polygons because we cannot use a ruler. Even a tape measure is awkward. It is for this reason that formulas for the circumference (perimeter) of a circle and its area are useful.

Circumference

The circumference of a circle is the distance around it. A string was used to measure the circumferences of these circles. We might notice that for each one, the ratio of the circumference to the diameter (the distance across the circle) is the same.



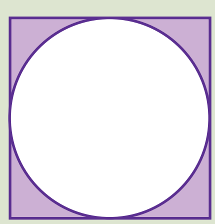
Estimated Circumference = 9.4 cm	Estimated Circumference = 15.7 cm	Estimated Circumference = 4.7 cm
Ratios: $\frac{9.4}{3} = 3.14$	$\frac{15.7}{5} = 3.14$	$\frac{4.7}{1.5} = 3.14$

The exact ratio is a number called π that is pronounced ***pi***.

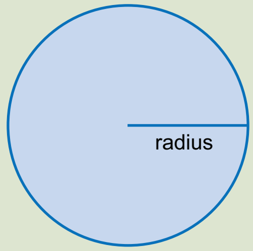
For any diameter, we multiply by π to figure out the circumference.

This diagram of a circle inside a square might help us see why this ratio makes sense.

Notice that the perimeter of the square with a side length the diameter of the circle is 4 times the diameter. It seems reasonable the circle's circumference is less than 4 times that diameter.



The radius of the circle is the distance from the centre to the edge. It is always half of the diameter.

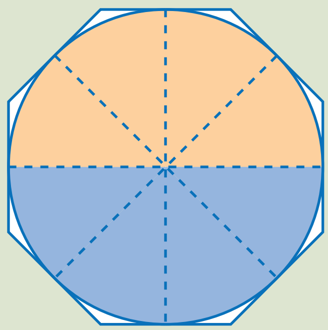


So $C = \pi d$ or $C = 2r \times \pi$ ($C = 2\pi r$)

Area

Knowing the radius or diameter of a circle allows us to figure out its area.

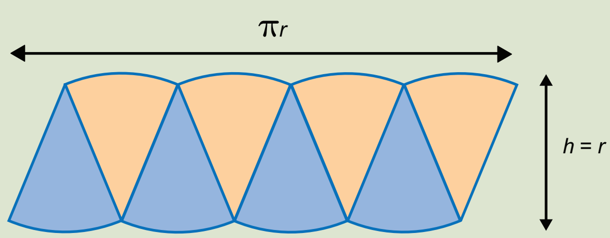
It helps to draw the circle inside a polygon. For example, we could draw the circle inside an octagon and cut the circle into eight pieces.



If we rearrange the eight pieces, we can create a shape that is "almost" a parallelogram. If a polygon with more sides were used, the curve on the bases would be less.

The height of the parallelogram is the radius of the circle.

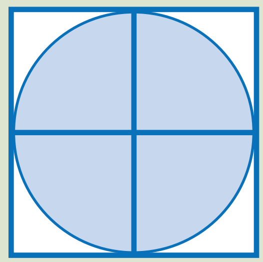
Each base length of the parallelogram is half the circumference since the two bases make up the whole circumference.



$C = 2\pi r$, so $\frac{1}{2}C = \pi r$.

The area is the product of the base and height, so $A = \pi r \times r = \pi r^2$.

This picture might help us see why it makes sense that the area of a circle is about three times the area of a square on its radius. The circle's area is less than the area of four squares with a side length equal to the radius, since there are white sections not covered by the circle. But it is not a lot less.

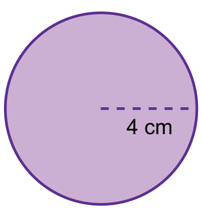


If we know the area of the circle, we can calculate the radius.

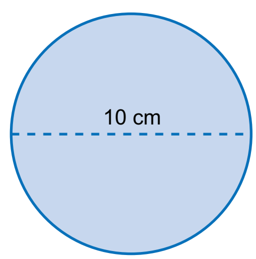
$r^2 = \frac{A}{\pi}$ so $r = \sqrt{\frac{A}{\pi}}$.

1. Calculate the circumference of each circle.

a)

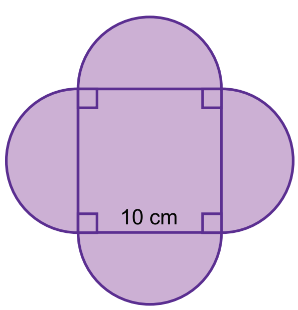


b)

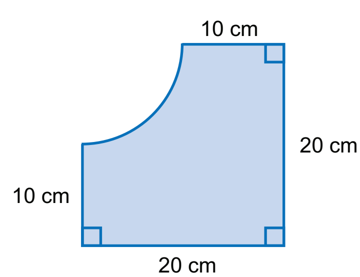


2. Determine the perimeter of each shape.

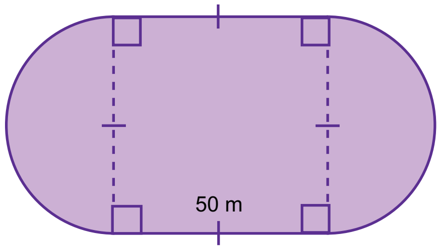
a)



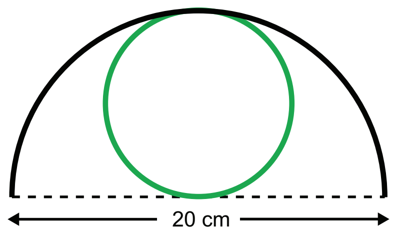
b)



c)



3. Black ribbon is used for the semi-circle of this design. Green ribbon is used for the circle. How do the two ribbon lengths compare? How do you know?



4. A circle has a circumference of 20 cm. What is its radius? How do you know?

5. If the diameter of one circle is double the diameter of another, how do their circumferences compare? Explain.

6. Determine the area of each shape in Question 2.

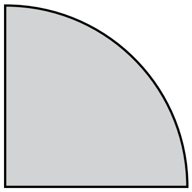
a)

b)

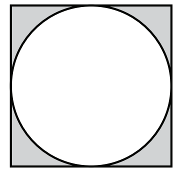
c)

7. Calculate the area of the shaded space. Show your thinking.

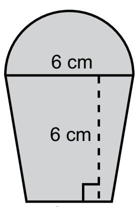
a)



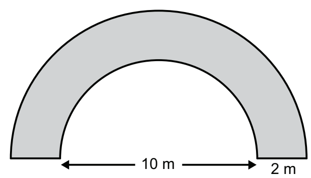
b)



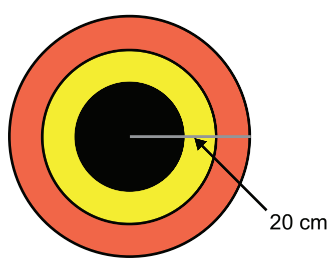
c)



d)

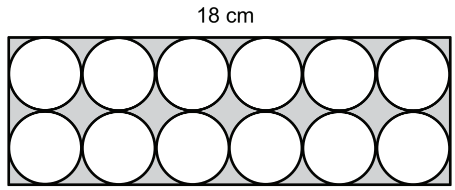


8. The black section of the dart board is $\frac{1}{5}$ of the total area. What is the radius of the black section?

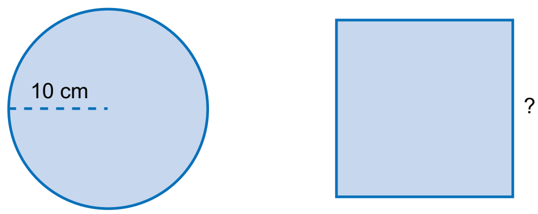


9. A circle has an area of 10 cm^2. What is its radius?

10. What is the area of the grey section?



11. A square has almost the same area as a circle with radius 10 cm. What is the side length of the square?



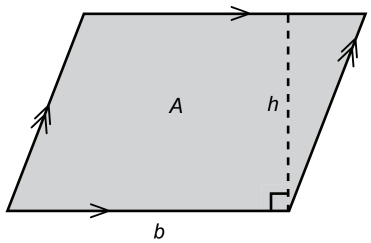
12. Circle A has an area 9 times the area of Circle B. What do you know about their radii (the plural of radius)? Explain why your answer makes sense.

13. Why do you only need one of the radius, diameter, circumference, or area of a circle to figure out all of the others?

Formula Sheet

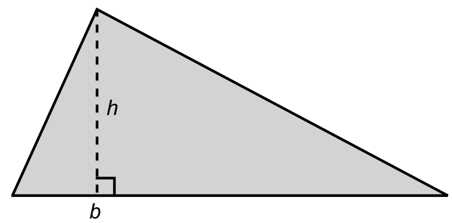
Area of a parallelogram

$A = bh$



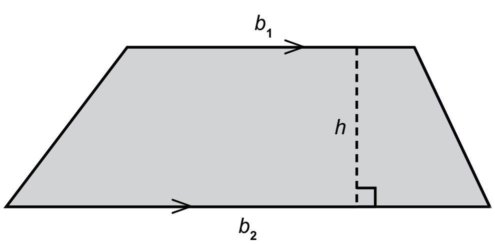
Area of a triangle

$A = \frac{1}{2}bh$ or $A = bh \div 2$



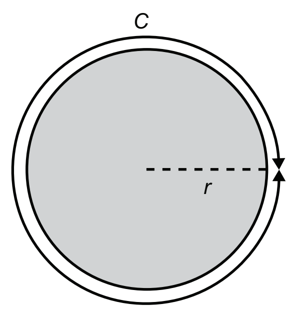
Area of a trapezoid

$A = \frac{1}{2}(b_1 + b_2)h$ or $A = (b_1 + b_2)h \div 2$



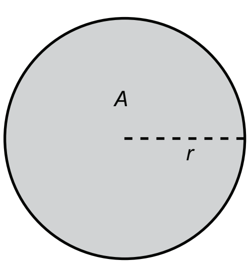
Circumference of a circle

$C = 2\pi r$ or $C = \pi d$



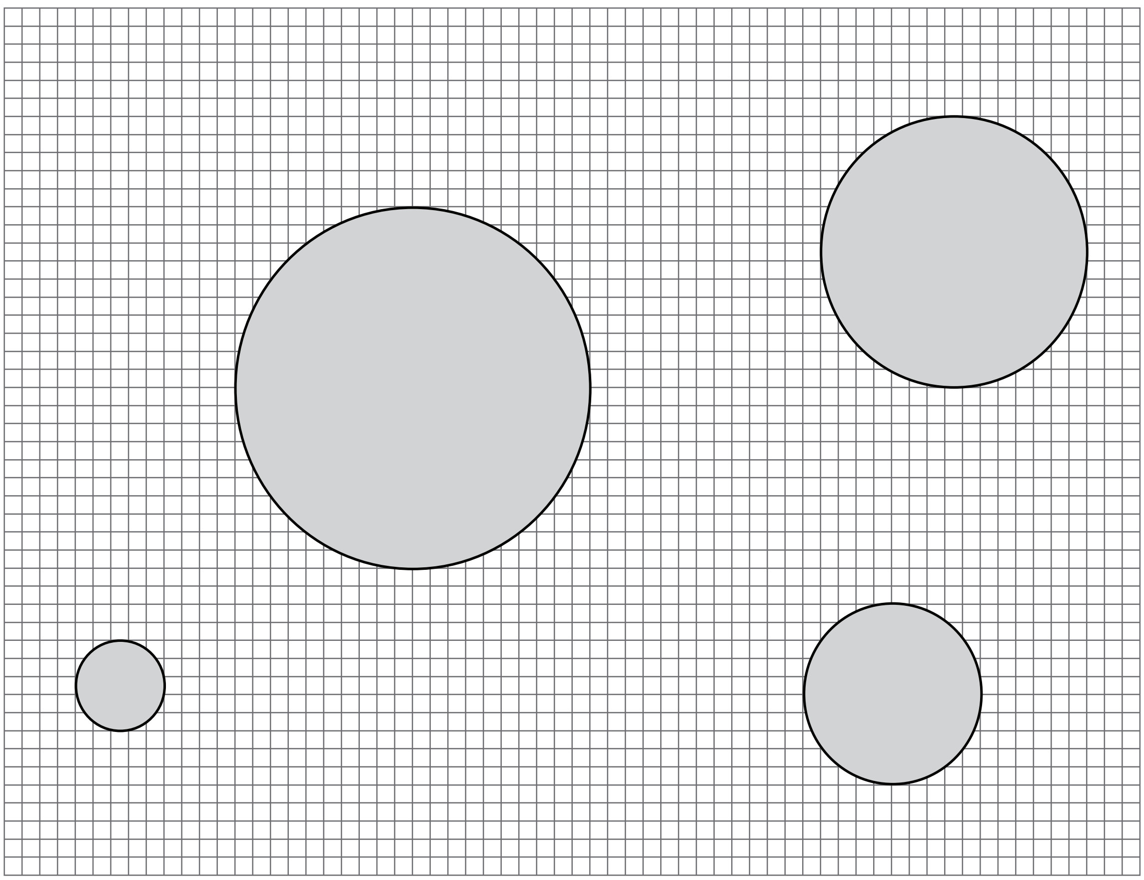
Area of a circle

$A = \pi r^2$



Rectangle and Parallelogram

Four Circles



Divided Circle