

GAP CLOSING

Solving Equations

**Intermediate/ Senior
Student Book**

Solving Equations

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Diagnostic

1. Hassan is solving some equation by guess and test. Is his first guess a good one? Explain.

a) $3x + 24 = 95$ First guess: 25

b) $2x - 18 = 146$ First guess: 73

c) $500 - 6s = 116$ First guess: 50

2. Raina was solving some equations by guessing and testing. Her first guess is listed. What would a good second guess be? Why?

a) $5x - 30 = 65$ First guess: 20

b) $124 = 200 - 4s$ First guess: 30

c) $7m + 18 = 165$ First guess: 10

3. Solve each equation using guess and test. List all of your guesses and show how you tested each guess.

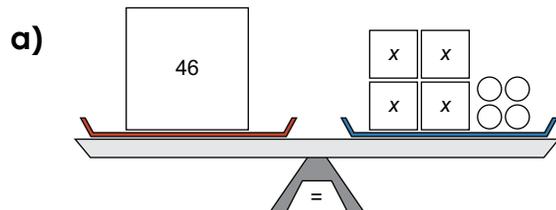
a) $6g + 23 = 101$

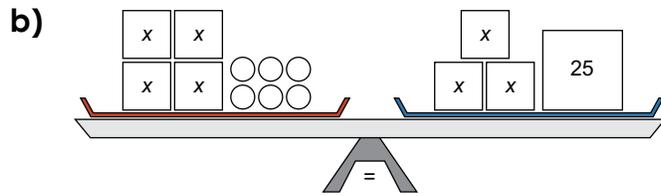
b) $12m - 80 = 64$

c) $348 = 500 - 8m$

4. What equation does each pan balance model?

NOTE: The box marked 46 would contain 46 small balls like the 4 balls on the right of the balance. It is just quicker to show than drawing 46 separate balls.





5. Imagine you are going to solve the equations shown on the pan balances in Question 4.

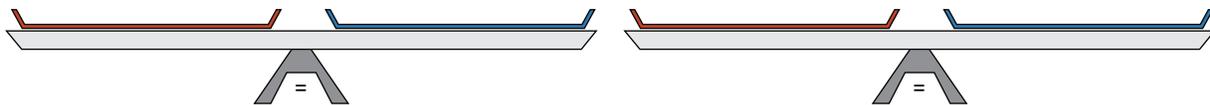
- a) What is your first step for the first equation?
- b) What is your first step for the second equation?

6. Show how to model each of these equations using a pan balance.

Then show how to adjust the balance to solve it.

a) $5g + 17 = 63$

b) $3m + 25 = 2m + 31$



7. What is the first operation you would perform to solve the equation?

a) $6m + 14 = 86$

b) $9m - 600 = 228$

c) $154 - 3m = 58$

d) $5m + 8 = 2m + 149$

8. Solve each equation in Question 7 using opposite operations.

9. List two equations you might solve by doing these operations:

- a) dividing by 4

- b) subtracting 12

- c) adding 12 and then dividing by 3

10. Suppose you wanted to change the equation to the form $x = \dots$

What is the first step you would take in each situation?

- a) if $y = 2x$

- b) if $y = x + 50$

- c) if $y = 2x - 80$

11. Write an equation that tells how to calculate t if you know m .

- | | |
|------------------|-------------------|
| a) $m = t + 10$ | b) $m = 4t$ |
| c) $m = 3t - 9$ | d) $3m = 4t$ |
| e) $3m = 6t + 8$ | f) $2m + 4t = 60$ |

Using Guess and Test

Learning Goal

- selecting appropriate guesses when using a guess and test strategy to solve equations.

Open Question

Sometimes you can solve an equation by guessing a solution.

- Create five different equations involving the variable k where a guess of $k = 15$ would make sense as a first guess. Make sure 15 is not the actual solution.

As you create your equations, make sure:

- some of the equations involve addition and some involve subtraction.
- the coefficient of k is never 1, e.g. the equation involves $2k$ or $3k$, but not just k .
- some of the coefficients of k are negative, e.g. the equation involves $-2k$ or $-k$.
- some equations have k on both sides of the equal sign.
- some of the solutions are decimals.

- Solve each equation. Show your thinking.

Think Sheet

You can solve an equation using Guess and Test.

Guess and Test is a 3-step process:

- Step 1: Make a reasonable guess for a solution.
- Step 2: Substitute to test.
- Step 3: Adjust your guess, if needed.

For example, to solve $3x - 8 = 19$:

Start by guessing 10. 10 makes sense since it is an easy number to work with.

$3 \times 10 - 8 = 22$. 22 is too high. Try a lower number.

Try 9.

$3 \times 9 - 8 = 19$. 19 works, so the solution is $x = 9$.

Guess and test also works if the solution is a simple decimal.

For example, to solve $5x + 3 = 29$.

Start by guessing 5. This makes sense since $5 \times 5 = 25$ and is close to 29:

$5 \times 5 + 3 = 28$. 28 is too low, but not very low.

Try 5.3.

$5 \times 5.3 + 3 = 29.5$ 29.5 is too high.

Try 5.2.

$5 \times 5.2 + 3 = 29$. This is correct, so the solution is $x = 5.2$.

But if the solution to an equation were an infinite decimal or a decimal like 4.1286, it might take a lot of guesses or we might never get an exact solution.

1. Would a first guess of 10 make sense to solve each equation? Tell why or why not for each one.
 - a) $5x - 8 = 40$
 - b) $37 + 17x = 160$
 - c) $300 - 12s = 200$

2. What would be a good first guess to solve each equation?

Explain why.

a) $6x + 3 = 117$

b) $8x - 37 = 107$

c) $56 - 3x = 14$

d) $46 + 5x = 93$

e) $53 = 212 - 5x$

3. You are solving an equation involving $15x$. A first guess of $x = 20$ makes sense. List three possible equations.

4. You are solving equations using guess and test. The result of your first guess is shown. What would be a good second guess? Explain why.

a) $8x - 36 = 100$ First guess: 10 $8 \times 10 - 36 = 44$

b) $12m - 35 = 277$ First guess: 20 $12 \times 20 - 35 = 205$

c) $500 - 4c = 144$ First guess: 100 $500 - 4 \times 100 = 100$

5. You guess $x = 20$ as a first guess to solve both of these equations. Why would your next guess be higher for one of them, but lower for the other?

$$6x + 18 = 150$$

$$180 - 5x = 85$$

6. Use guess and test to solve each equation.

a) $9t + 17 = 224$

b) $15m - 123 = 387$

c) $400 - 6j = 262$

d) $134 = 8c + 58$

e) $516 = 4k - 38$

f) $48 + 6k = 10k - 164$

7. Give an example of an equation where guess and test might NOT be a good method and explain why it might not.

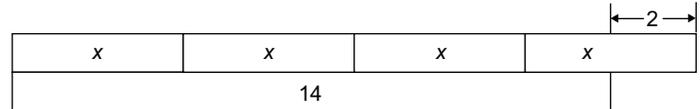
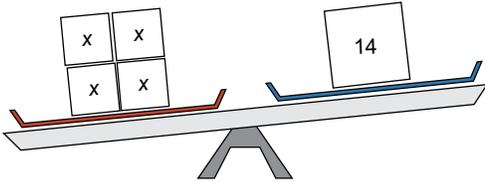
Using a Balance Model

Learning Goal

- representing an equation with a model as a strategy to solve it.

Open Question

The two models below can be used to represent the equation $4x - 2 = 14$.

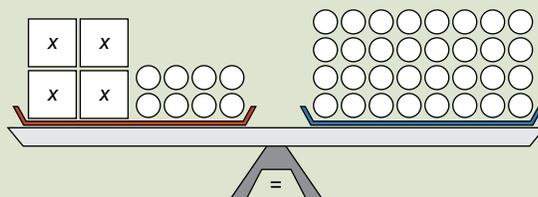


- Explain why both of them represent the equation.
- Describe how to use the models to solve the equation to determine the value of x .
- Create and model five equations. Use either or both models.
 - Make sure that some of the equations involve subtraction.
 - Make sure that the coefficient of the variable is never 1.
 - Make sure that some equations have the variable on both sides of the equal sign.
- Describe how to use your models to solve each of your equations.

Think Sheet

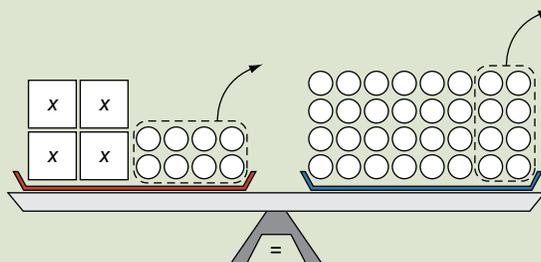
The two sides of an equation are equal; they balance. We can think of modelling the equation on a pan balance to figure out the unknown value that would make the two sides equal.

For example, if the equation is $4x + 8 = 32$, we could imagine

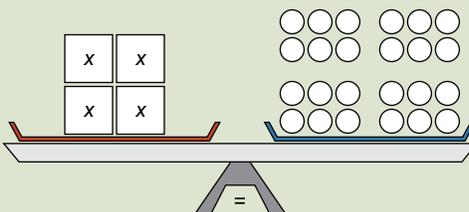


We could remove 8 ones from each side of the balance.

This is like subtracting 8 from each side of the equation: $4x = 32 - 8 = 24$.



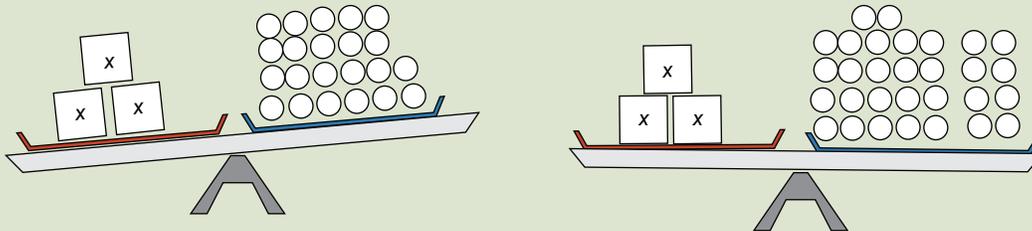
We can divide the 24 counters on the right into four equal groups — one group to match each x box. That means that we can divide both sides by 4 to see that $x = 24 \div 4$ or 6.



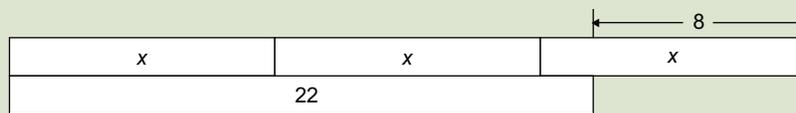
Note: We can draw a box that is marked 24 instead of drawing 24 counters.

If the equation involved subtracting from both sides, the model would have to be different. In this case, we might start with an unbalanced model and figure out what to do to make it balance.

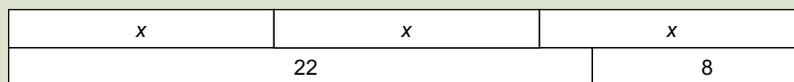
For example, if the equation were $3x - 8 = 22$, we would imagine that one side shows $3x$ and the other shows 22, but you must remove 8 from the $3x$ side to make the equation balance.



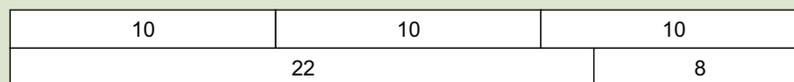
It is hard to visualize how to take the 8 away, so another model might be more helpful. If an amount is shown with arrows above an x box, it is taken away.



Notice that a section 8 units long is cut from $3x$ and the remaining section balances 22. If the 8 unit section were not removed from the top row and added to the bottom one, it would result in the full $3x$ on top and $22 + 8 = 30$ on bottom.



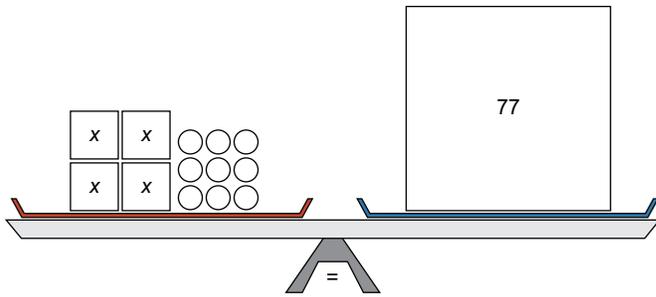
Since the 30 is made up of three sections labeled x , each section must be worth 10.



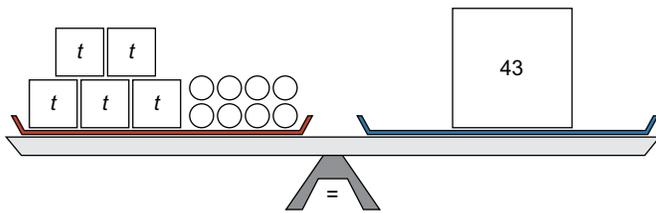
1. What equation does each model represent?

a)

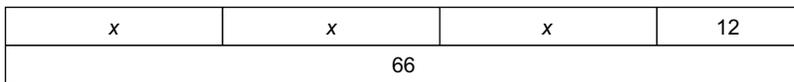
b)



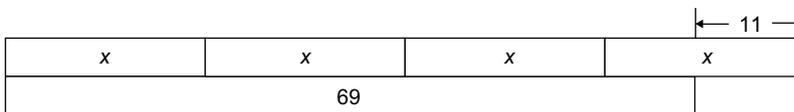
c)



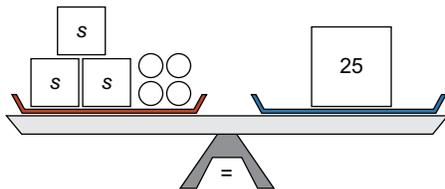
d)



e)



2.

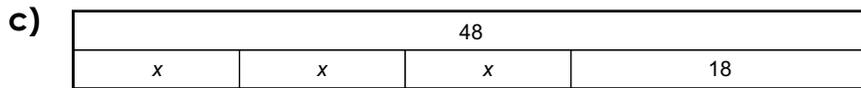
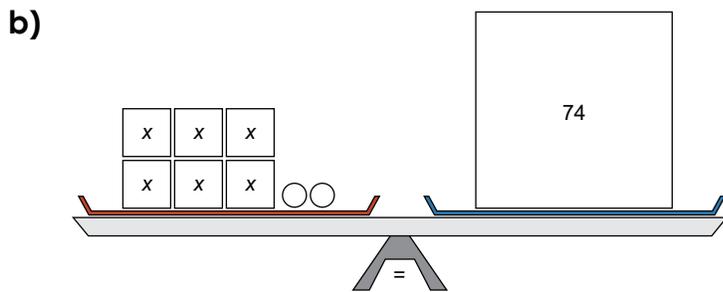
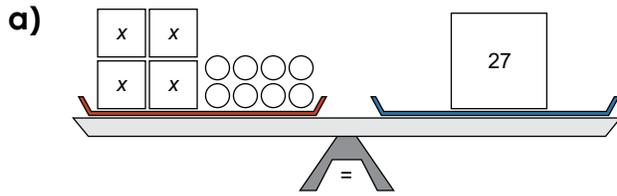


a) Why might it make sense to take 4 counters off of each side of the balance?

b) What is the resulting equation?

c) Solve the equation to determine the value of s .

3. What would be your first step to solve each of these?



4. You are solving an equation modelled on a pan balance. Your first step is to take 8 counters off of each side and your next step is to separate the counters on the right side into 6 equal groups. List three possible equations.

5. Sketch a model (pan balance or rectangles of equal lengths) you could use to help you to solve each equation. Then solve it.

a) $7k + 9 = 72$

b) $4t - 18 = 46$

c) $300 - 9j = 192$

d) $57 = 6m + 3$

e) $112 = 5k - 88$

f) $36 + 2k = 12 + 8k$

6. a) Create an equation with a solution of $k = -2$.

b) Explain whether you would be able to use a real pan balance to solve it.

c) Explain how to use a model to solve it.

Using Opposite Operations

Learning Goal

- working backwards to solve an equation.

Open Question

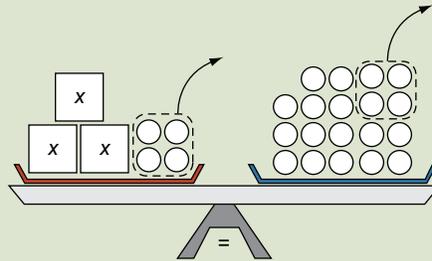
$6s + 20 = 68$ and

$6s = 68 - 20$ are equivalent equations.

- Explain how they give the same information.
- Why might someone think that the way you solve $6s + 20 = 68$ involves subtracting and then dividing?
- Create and then solve equations that might be solved by performing each of these operations:
 - adding and then dividing
 - only subtracting
 - only dividing
 - subtracting twice
 - dividing and then subtracting
 - dividing and then adding
 - adding and then multiplying

Think Sheet

If an equation such as $3x + 4 = 19$, was modelled on a pan balance, we would subtract four counters from each side to get a simpler equivalent equation.



This means that

$$3x + 4 = 19 \text{ and}$$

$$3x = 19 - 4 \text{ are equivalent equations.}$$

The equation was simplified to $3x = 15$ by performing an opposite operation.

4 had been added to $3x$, so now we subtract it.

If $3x = 15$, you multiplied x by 3.

The opposite operation is division.

So if

$$3x = 15, \text{ then}$$

$$3x \div 3 = 15 \div 3$$

$x = 5$ is a very simple equivalent equation.

This is exactly what you would have done on the pan balance, without using the balance.

When using opposite operations, it is useful to think of the order of operations (BEDMAS) in reverse.

If you add and subtract last, it might be easier to do the opposite first.

To solve $3x + 4 = 19$, first subtract and then divide.

You can divide first, but you have to divide every term.

$$3x + 4 = 19 \text{ is equivalent to } x + 4/3 = 19/3.$$

1. For which of these equations would you add something first to solve using opposite operations? Tell why.
 - a) $3x - 9 = 21$
 - b) $62 = 5x + 22$
 - c) $120 - 9t = 18$

2. What is the first operation you would perform in solving each equation? Why?
 - a) $7x + 8 = 71$

 - b) $9s - 18 = 63$

 - c) $120 - 8s = 48$

 - d) $52 + 9t = 151$

 - e) $73 = 115 - 3m$

3. You are solving an equation involving the term $8x$. Each time, your first step is to subtract.
 - a) List three possible equations.

 - b) Tell what your next step would be and why.

7. Kyla said that to solve $4x + 20 = 56$, you have to subtract 20 first.
Eric said you could divide by 4 first as long as you divide everything by 4.
With whom do you agree? Why?

Rearranging Equations and Formulas

Learning Goal

- representing an equation or formula in a different way to make it easier to solve.

Open Question

Sometimes equations describe the relationship between different variables.

Some examples are:

$E = 15h$ (the total earnings for h hours if you earn \$15 an hour)

$A = lw$ (the formula for the area of a rectangle)

$2t + 5f = 100$ (the total value of t toonies and f \$5 bills)

Sometimes you know one of the variables and want to solve for the other.

For example, for the first equation, you might know the number of hours and want to figure out the earnings or you might know the amount earned and try to figure out how many hours.

For the second equation, you might know the area and length and want to figure out the width or you might know the length and width and want to figure out the area.

- Think of at least five formulas you know or create situations that you could describe with two or more variables.
 - Make sure that some of the equations involve coefficients other than 1.
 - Make sure that different operations are used in different equations.
 - Do not use the equations or situations from the background section above.

- Show how you would solve for each variable in terms of all of the others.

Think Sheet

When an equation involves two or more variables, you can solve for any one of them using the other(s).

For example, if $y = x + 2$, then y is 2 more than x .

That means that x is 2 less than y .

You can use opposite operations to see this:

$$y = x + 2, \text{ so}$$

$$y - 2 = x + 2 - 2$$

$$y - 2 = x$$

In a triangle, $A = bh \div 2$.

You can use opposite operations to solve for h if you know b and A .

$$A = bh \div 2, \text{ so}$$

$$2A = bh, \text{ so}$$

$$2A \div b = h$$

If you know actual values to substitute, you can substitute and then solve. For example, if the area of a triangle is 12 square units and the base is 4 units, you could write:

$$A = bh \div 2, \text{ so}$$

$$12 = 4h \div 2, \text{ so}$$

$$2 \times 12 = 4h, \text{ so}$$

$$24 \div 4 = h.$$

1. For which of these equations would you add something first to solve for x in terms of y ? Why?

a) $y = 4x - 8$

b) $y = 3x + 12$

c) $6x - 12 = y$

d) $500 - 2x = y$

2. What is the first step you would perform to solve for m in terms of p ?

a) $p = 6m - 8$

b) $p = 15 - 3m$

c) $p = 3m - 2$

d) $2p = 5 - 4m$

e) $10p = 18 + 3m$

3. Complete this table to determine the values of r if $C = 2\pi r$.

	C	r
a	10π	
b	20π	
c	30π	
d	50	
e	100	

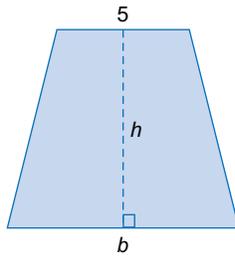
4. The formula for the area of a parallelogram is $A = bh$

a) How would you solve for h in terms of A and b ?

b) How would you solve for b in terms of A and h ?

c) What do you notice about your two strategies?

5. The formula for the area of this trapezoid is $A = \left(\frac{b+5}{2}\right)h$.



- a) How would you solve for h if you knew A and b ?
- b) How would you solve for b if you knew A and h ?
6. For each equation, solve for the bolded variable in terms of the non-bolded variable.
- a) $V = 12\mathbf{h}$
- b) $P = 12 + 2\mathbf{w}$
- c) $SA = 30 + 6\mathbf{w} + 10\mathbf{w}$
- d) $4y = 8\mathbf{x} + 12$
- e) $2y = 4(\mathbf{x} + 5)$
- f) $3y + 9\mathbf{x} = 54$

7. The formula for the perimeter of a rectangle is $P = 2l + 2w$.
- a) To solve for w if you know that $P = 20$ and $l = 4$, which of the following strategies could you choose?

Strategy A: Write $20 = 2 \times 4 + 2w$ and solve the equation $20 = 8 + 2w$.

Strategy B: Solve for w in terms of P and l . $w = (P - 2l) \div 2$. Then substitute for P and l , to get $w = (20 - 2 \times 4) \div 2$.

- b) If the perimeter were 20 but you had to solve for a lot of values of l for a lot of different values of w , what strategy would you choose?

