GAP CLOSING

Solving Equations

Intermediate / Senior
Facilitator Guide

Solving Equations

The Facilitator's Guide for this entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/7-Solving Equations FG IS.pdf. The Student Book for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/7-SolvingEquations_SB_IS.pdf.

SOLVING EQUATIONS

Relevant Learning Expectations for Grade 9

MPM1D

Number Sense and Algebra

- solve first-degree equations, including equations with fractional coefficients, using a variety of tools and strategies
- rearrange formulas involving variables in the first degree, with and without substitution
- solve problems that can be modelled with first-degree equations and compare algebraic methods to other solution methods

Linear Relations

- construct tables of values, graphs, and equations, using a variety of tools to represent linear relations derived from descriptions of realistic situations

MPM1P

Number Sense and Algebra

- solve first-degree equations with non-fractional coefficients, using a variety of tools and strategies
- substitute into algebraic equations and solve for one variable in the first degree

Linear Relations

- solve problems that can be modelled with first-degree equations and compare the algebraic method to other solution methods

Possible reasons a student might struggle when solving first-degree equations

Students may struggle in solving first-degree equations.

Some of the problems include:

- lack of fluency with the integer or fraction operations required to solve an equation;
- dividing through only some of the terms of an equation, e.g., changing $4x - 2 = 28$ to $x - 2 = 7$ instead of $x - 0.5 = 7$;
- discomfort with equations where the coefficient is negative, e.g., equations such as $14 - 2m = 26$ (as opposed to $14 + 2m = 26$);
- not knowing what to do when solutions are neither whole numbers nor integers;
- lack of familiarity with equations where the unknown is not on the left (e.g., not sure what to do with $3 = 2 + 4m$ as compared to $4m + 2 = 3$);
- discomfort with equations where the variable appears on both sides;
- lack of understanding that there can be more than one solution in certain circumstances, e.g., $2n - 1 = n + 3 + n - 4$ has many solutions.

DIAGNOSTIC

Administer the diagnostic

Using the diagnostic results to personalize interventions

Materials
- a pan balance with counters
- small paper bags

Intervention materials are included on each of these topics:
- using the guess and test method
- using a balance model
- using opposite operations
- rearranging equations and formulas

You may use all or only part of these sets of materials, based on student performance with the diagnostic. If students need help in understanding the intent of a question in the diagnostic, you are encouraged to clarify that intent.

Evaluating Diagnostic Results	Suggested Intervention Materials
If students struggle with Questions 1–3	use *Using Guess and Test*
If students struggle with Questions 4–6	use *Using a Balance Model*
If students struggle with Questions 7–9	use *Using Opposite Operations*
If students struggle with Questions 10 and 11	use *Rearranging Equations and Formulas*

If students are successful with Using Opposite Operations, it is not essential to use the Using Guess and Test or Using a Balance Model approaches. However, if they are not successful with Using Opposite Operations, use the diagnostic to decide which earlier intervention would be valuable.

Solutions

1. a) e.g., Yes, since $3x = 75$ and 75 + 25 is close to 95.
   b) e.g., No, since that would be the answer without the –18, so a higher guess would be better.
   c) e.g., No, since 500 – 300 is 200 and that is high.

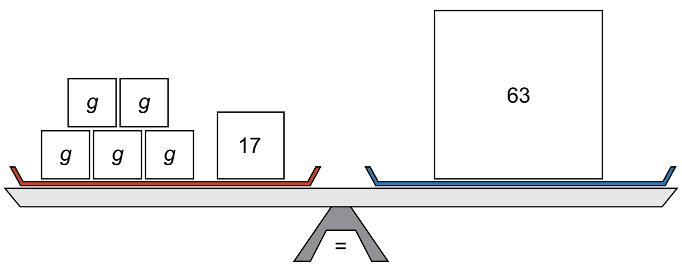
2. a) e.g., 19 since 5 × 20 – 30 was a bit high, so going lower is a good idea.
   b) e.g., 20; you need a higher number than 80, so you want to take less away.
   c) e.g., 20; you need a higher number than 88, so if m were 10 more, then 70 more is close.

3. a) Solution is 13; e.g., guess: 10 test: 83; guess: 13 test: 101
   b) Solution is 12; e.g., guess: 10 test: 40; guess: 13 test: 76; guess: 12 test: 64
   c) Solution is 19; e.g., guess: 20 test: 340; guess: 19 test: 348

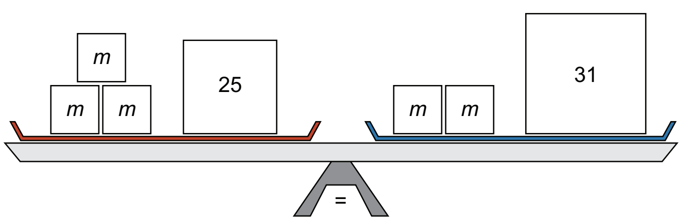
4. a) $46 = 4x + 4$
   b) $4x + 6 = 3x + 25$

5. a) e.g., Take 4 from both sides.
   b) e.g., Take $3x$ from both sides.

6. a)



   b)



7. a) e.g., Subtract 14 from both sides.
   b) e.g., Add 600 to both sides.
   c) e.g., Add $3m$ to both sides.
   d) e.g., Subtract $2m$ from both sides.

8. a) $m = 12$
   b) $m = 92$
   c) $m = 32$
   d) $m = 47$

9. a) e.g., $4s = 12$ or $4s = 20$
   b) e.g., $s + 12 = 50$ or $s + 12 = 39$
   c) e.g., $3s - 12 = 288$ or $3s - 12 = 12$

10. a) Divide both sides by 2.
    b) Subtract 50 from both sides.
    c) e.g., Add 80 to both sides.

11. a) $t = m - 10$
    b) $t = \frac{m}{4}$
    c) $t = \frac{(m + 9)}{3}$
    d) e.g., $t = 0.75m$
    e) $t = \frac{(3m - 8)}{6}$
    f) e.g., $t = 15 - \frac{m}{2}$

Diagnostic

1. Hassan is solving some equation by guess and test. Is his first guess a good one? Explain.
   - **a)** $3x + 24 = 95$ First guess: 25
   - **b)** $2x - 18 = 146$ First guess: 73
   - **c)** $500 - 6s = 116$ First guess: 50

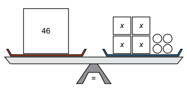
2. Raina was solving some equations by guessing and testing. Her first guess is listed. What would a good second guess be? Why?
   - **a)** $5x - 30 = 65$ First guess: 20
   - **b)** $124 = 200 - 4s$ First guess: 30
   - **c)** $7m + 18 = 165$ First guess: 10

3. Solve each equation using guess and test. List all of your guesses and show how you tested each guess.
   - **a)** $6g + 23 = 101$
   - **b)** $12m - 80 = 64$
   - **c)** $348 = 500 - 8m$

4. What equation does each pan balance model?

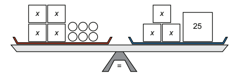
NOTE: The box marked 46 would contain 46 small balls like the 4 balls on the right of the balance. It is just quicker to show than drawing 46 separate balls.

a)



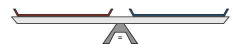
Diagnostic (Continued)

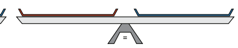
b)



5. Imagine you are going to solve the equations shown on the pan balances in Question 4.
   a) What is your first step for the first equation?
   b) What is your first step for the second equation?

6. Show how to model each of these equations using a pan balance. Then show how to adjust the balance to solve it.
   - **a)** $5g + 17 = 63$
   - **b)** $3m + 25 = 2m + 31$





7. What is the first operation you would perform to solve the equation?
   - **a)** $6m + 14 = 86$
   - b) $9m - 600 = 228$
   - **c)** $154 - 3m = 58$
   - d) $5m + 8 = 2m + 149$

8. Solve each equation in Question 7 using opposite operations.

Diagnostic (Continued)

9. List two equations you might solve by doing these operations:
   - **a)** dividing by 4
   - **b)** subtracting 12
   - **c)** adding 12 and then dividing by 3

10. Suppose you wanted to change the equation to the form $x = \ldots$
    What is the first step you would take in each situation?
    - **a)** if $y = 2x$
    - **b)** if $y = x + 50$
    - **c)** if $y = 2x - 80$

11. Write an equation that tells how to calculate t if you know m.
    - **a)** $m = t + 10$
    - **b)** $m = 4t$
    - **c)** $m = 3t - 9$
    - **d)** $3m = 4t$
    - **e)** $3m = 6t + 8$
    - **f)** $2m + 4t = 60$

USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for working with more complicated linear equations (e.g., ones involving decimals or fractions) and with more complicated formulas (e.g., surface area formulas).

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using you choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

Using Guess and Test

Learning Goal

- selecting appropriate guesses when using a guess and test strategy to solve equations.

Open Question

Materials
- calculators

Questions to Ask Before Using the Open Question

◊ *Why might you guess that 20 would be close to the solution for 3m – 40 = 28?* (e.g., Since 3 × 20 is 60 and 60 – 40 is not that far from 28.)
◊ *Would your second guess be higher or lower than 20? Why?* (Higher since 20 is too low; you have to have 68 to subtract 40 from.)
◊ *What other equation involving multiplication and addition might have a solution that is not far from 20?* (e.g., 2*x* + 4 = 48)

Using the Open Question

Make sure students understand that the group of five equations must satisfy the conditions listed.

By viewing or listening to student responses, note:
- how they use the test after the first guess to get a better second guess;
- how they operate when coefficients are negative;
- whether they can handle equations where solutions are not whole numbers.

Consolidating and Reflecting on the Open Question

◊ *How did you create your first equation?* (e.g., I multiplied 15 by 4 and added 8; then I wrote the answer on the other side, but I added a little bit to it) *Why did you add the little bit?* (so the solution was not exactly 15)
◊ *How did you create an equation with k on both sides* (e.g., I wrote two expressions involving *k* that were the same for *k* = 18)
◊ *How did you solve the equation when the answer was a decimal?* (e.g., I worked it out so that the solution was an integer, I changed the constant, and then I tried decimal tenths that were close to the integer solution I started with.)

Solutions

e.g.

$6k + 12 = 108$	$k = 16$
$200 - 3k = 149$	$k = 17$
$2k + 38 = 3k + 22$	$k = 16$
$80 - 5k = 6$	$k = 14.8$
$140 + 10k = 20k - 20$	$k = 16$

Think Sheet

Questions to Ask Before Assigning the Think Sheet
◊ *Why might you guess that 20 would be close to the solution for 3m – 40 = 28?* (e.g., Since 3 × 20 is 60 and 60 – 40 is not that far from 28.) ◊ *Would your second guess be higher or lower than 20? Why?* (Higher, since 20 is too low; you have to have 68 to subtract 40 from.) ◊ *What other equation involving multiplication and addition might have a solution that is not far from 20?* (e.g., 2*x* + 4 = 48)
Using the Think Sheet
Read through the introductory box with the students or clarify any questions they might have. Make sure they recognize the importance of testing after a guess and using the results to decide whether to go up or down. Assign the tasks. By viewing or listening to student responses, note if they: • make a reasonable first guess; • can figure out whether to increase or decrease the number for a second guess; • can tell when an equation has been solved.
Consolidating and Reflecting: Questions to Ask After Using the Think Sheet
◊ *Why is it a good idea to start with guesses like 10 or 20 rather than 12 or 23?* (e.g., So that you can use mental math to test how close you are to the value you want.) ◊ *You are trying to solve 4x – 30 = 80. What would be a good first guess?* (e.g., 25, since 4 × 25 – 30 is 70, which is close to 80.) *Would you try a higher or lower number next? Why?* (Higher, since you want to start with more to take the 30 from.) ◊ *If you had guessed x = 30 in an equation involving 8x and you were 72 too low, what might your next guess be? Why?* (39, since 8 × 9 = 72 and you want the extra 72.)

Solutions

1. **a)** Yes, e.g., since 50 – 8 is close to 40.
b) No, e.g., since 17 × 10 is already more than 160, even before you add 37.
c) Yes, e.g., since 180 is close to 200.

2. **a)** e.g., $x = 20$ since 123 is close to 117.
b) e.g., $x = 20$ since 160 – 37 is close to 120 and that is not too far from 107.
c) e.g., $x = 10$ since 56 – 30 is not too much higher than 14.
d) e.g., $x = 10$ since 46 + 50 is close to 93.
e) e.g., $x = 30$ since 212 – 150 is 62 and that is close to 53.

3. e.g., $15x + 18 = 320$; $15x - 100 = 196$; $600 - 15x = 320$.

4. **a)** e.g., 17; I need an extra 56 and each time x goes up by 1, $8x$ goes up by 8, so I want x to go up by 7.
b) e.g., 26; I need an extra 72 and each time m goes up by 1, $12m$ goes up by 12; if m goes up by 6, then $12m$ goes up by 72.
c) e.g., 89; I need to subtract 44 less. Each time c goes down by 1, $4c$ goes down by 4; if c goes down by 11, then $4c$ goes down by 44.

5. e.g., In the first equation 6x + 18 is only 138, so you need *x* to be bigger to make 6x + 18 bigger. In the second equation, you are taking away the *x*'s. 180 – 100 = 80, so you took away too much and have to take away less.

6. **a)** $t = 23$
b) $m = 34$
c) $j = 23$
d) $c = 9.5$
e) $k = 138.5$
f) $k = 53$

7. e.g., $7k - 1.5 = 21.026$. Seeing thousandths would make me wonder whether guess and test would be the best method.

Open Question

Using Guess and Test

Learning Goal

- selecting appropriate guesses when using a guess and test strategy to solve equations.

Open Question

Sometimes you can solve an equation by guessing a solution.

- Create five different equations involving the variable *k* where a guess of $k = 15$ would make sense as a first guess. Make sure 15 is not the actual solution.

  As you create your equations, make sure:
  - some of the equations involve addition and some involve subtraction.
  - the coefficient of *k* is never 1, e.g. the equation involves 2*k* or 3k, but not just *k*.
  - some of the coefficients of *k* are negative, e.g. the equation involves -2*k* or -*k*.
  - some equations have *k* on both sides of the equal sign.
  - some of the solutions are decimals.

- Solve each equation. Show your thinking.

Think Sheet

Using Guess and Test (Continued)

Think Sheet

You can solve an equation using Guess and Test.

Guess and Test is a 3-step process:
- Step 1: Make a reasonable guess for a solution.
-Step 2: Substitute to test.
-Step 3: Adjust your guess, if needed.

For example, to solve *3x - 8 = 19*:

Start by guessing 10. 10 makes sense since it is an easy number to work with.
3 x 10 - 8 = 22. 22 is too high. Try a lower number.
Try 9.
3 x 9 - 8 = 19. 19 works, so the solution is x = 9.

Guess and test also works if the solution is a simple decimal.

For example, to solve 5x + 3 = 29.
Start by guessing 5. This makes sense since 5 x 5 = 25 and is close to 29:
5 x 5 + 3 = 28. 28 is too low, but not very low.

Try 5.3.

5 x 5.3 + 3 = 29.5 29.5 is too high.

Try 5.2.

5 x 5.2 + 3 = 29. This is correct, so the solution is x = 5.2.

But if the solution to an equation were an infinite decimal or a decimal like 4.1286, it might take a lot of guesses or we might never get an exact solution.

1. Would a first guess of 10 make sense to solve each equation? Tell why or why not for each one.
   a) $5x - 8 = 40$
   b) $37 + 17x = 160$
   c) $300 - 12s = 200$

Using Guess and Test (Continued)

2. What would be a good first guess to solve each equation?
   Explain why.
   a) $6x + 3 = 117$
   b) $8x - 37 = 107$
   c) $56 - 3x = 14$
   d) $46 + 5x = 93$
   e) $53 = 212 - 5x$

3. You are solving an equation involving 15*x*. A first guess of $x = 20$ makes sense. List three possible equations.

4. You are solving equations using guess and test. The result of your first guess is shown. What would be a good second guess? Explain why.

a)	$8x - 36 = 100$	First guess: 10	$8 \times 10 - 36 = 44$
b)	$12m - 35 = 277$	First guess: 20	$12 \times 20 - 35 = 205$
c)	$500 - 4c = 144$	First guess: 100	$500 - 4 \times 100 = 100$

5. You guess $x = 20$ as a first guess to solve both of these equations. Why would your next guess be higher for one of them, but lower for the other?

   $6x + 18 = 150$

   $180 - 5x = 85$

Using Guess and Test (Continued)

6. Use guess and test to solve each equation.

a) $9t + 17 = 224$

b) $15m - 123 = 387$

c) $400 - 6j = 262$

d) $134 = 8c + 58$

e) $516 = 4k - 38$

f) $48 + 6k = 10k - 164$

7. Give an example of an equation where guess and test might NOT be a good method and explain why it might not.

Using a Balance Model

Learning Goal

- representing an equation with a model as a strategy to solve it.

Open Question

Materials
- pan balance
- counters/cubes
- some small light paper bags
- calculators

Questions to Ask Before Using the Open Question

◊ *Place five cubes in a light, opaque bag. Write the letter x on the bag. Place the bag and three cubes on one side of a pan balance and eight cubes on the other side. How does the balance show the equation $x + 3 = 8$?* (e.g., There is a bag named *x* and three cubes and it balances eight cubes.) *Do you know how many cubes must be in the bag?* (5) *Why might you say the solution to the equation is 5?* (e.g., 5 cubes have to be in the bag to make the pans balance.)

◊ *How does the diagram show the equation $2x + 6 = 21$?*

x	*x*	6
21		

(e.g., The total length of $2x + 6$ matches the length 21.)

◊ *How do you know that x must be worth 7.5?* (e.g., If you take the 6 part away from the 21, there is 15 left that has to be split into two *x*'s, so each one must be 7.5)

Using the Open Question

Make sure students understand that they must both create and model the five equations to fit the conditions.

They must also describe each solution method based on the model selected and then solve each equation.

By viewing or listening to student responses, note if they:
- can translate between a model and an equation and vice versa;
- recognize that the same amount can be added or subtracted to both pans or both lengths;
- use an appropriate operation to make it easier to figure out what *x* is worth;
- can solve equations whether or not there are variables on both sides of the equation.

Consolidating and Reflecting on the Open Question

◊ *How could you model an equation of the form ___x + ___ = ___?* (e.g., I would model the right number of *x*'s and some counters on one side and the right number of counters on the other side.)

◊ *What would be different for the model of an equation of the form ___x – ___ = ___?* (e.g., I would have the balance unbalanced and the left side would show an *x* but since something had to be subtracted to equal the other side, it would be heavier.)

◊ *One of your equations was $3x + 2 = 2x + 7$. What options did you have for your first move?* (e.g., I could have taken two *x*'s from both sides or maybe the 2.)

◊ *Which of your equations did you choose to model with the lengths? Why did you make that choice?* (e.g., I modelled $18 – 3x = 9$ with lengths, since I thought it was easier to show the subtraction.)

Solutions

e.g.

- On the balance, there are 4 x-boxes on one side and 14 positive counters on the other side. I know the left side is heavier, so I have to remove some to balance the right side. Since the equation says –2, I would have to remove 2.

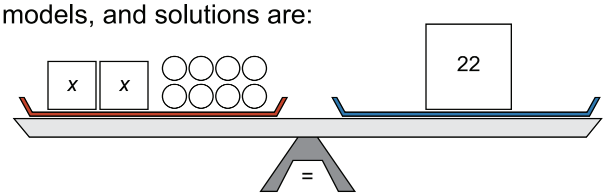
Using the rectangles, there are 4 x-sections and there is a 2 covering the last part of the last x, so it is $4x - 2$. And the total length without the 2 matches the length of 14.

- With the pan balance, I would add 2 positive counters to both sides. Then I would organize the 16 counters on the right into 4 groups to balance the 4 x-boxes. Each x-box would be worth 4 positive counters, so $x = 4$.

With the rectangles, I would add a length of 2 to the 14 and also add 2 to the top, which is like removing the 2 taken away from the last x. Then I would divide the 16 into 4 equal sections and each section would match one x. Each section would be worth 4.

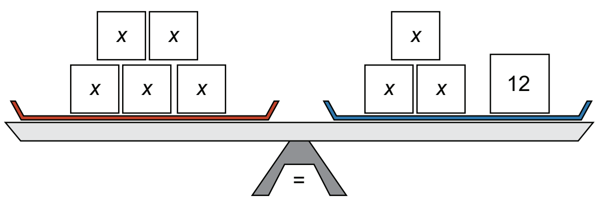
My equations, models, and solutions are:

- $2x + 8 = 22$



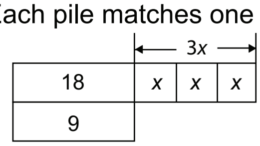
  I would remove 8 counters from both sides. I would organize the 14 counters on the right into 2 piles. Each pile matches one x-box, so $x = 7$.

- $5x = 3x + 12$



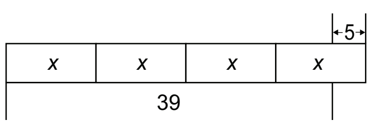
  I would remove 3 x-boxes from each side. I would organize the 12 counters on the right into 2 piles. Each pile matches one x-box, so $x = 6$.

- $18 - 3x = 9$



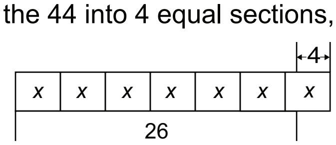
  I would add 3 x-boxes to the bottom and top. That means the top becomes just 18. Then I would see that $3x + 9$ matches 18. Then I would remove 9 from both lines. That would leave 3 x-sections matching 9. I would split the 9 into 3 equal parts and each part would be worth 3, so $x = 3$.

- $4x - 5 = 39$



  I would add 5 to the bottom row and top row. That makes the top $4x$ and the bottom 44. I would divide the 44 into 4 equal sections, so each would be 11, so $x = 11$

- $6x - 4 = 26$



  I would add 4 to the bottom row and top row. That makes the top $6x$ and the bottom 30. I would divide the 30 into 6 equal sections, so each would be 5, so $x = 5$.

Think Sheet

Materials
- pan balance
- counters
- some small paper bags
- calculators

Questions to Ask Before Assigning the Think Sheet

◊ *Place five cubes in an opaque paper bag. Write the letter x on the bag. Place the bag and three counters on one side of a pan balance and eight counters on the other side.* How does the balance show the equation x + 3 = 8? (e.g., There is a bag named *x* and three counters and it balances eight counters.) Do you know how many cubes must be in the bag? (5) Why might you say the solution to the equation is 5? (e.g., 5 cubes have to be in the bag to make the pans balance.)

◊ *How does the diagram show the equation 2x + 6 = 21?*

x	*x*	6
21		

(e.g., The total length of 2x + 6 matches the length 21.)

◊ *How do you know that x must be worth 7.5?* (e.g., If you take the 6 part away from the 21, there is 15 left that has to be split into two *x*'s, so each one must be 7.5.)

Using the Think Sheet

Make sure students understand the material explained in the instructional box.

Then assign the tasks on the page.

By viewing or listening to student responses, note if they:
- can translate between a model and an equation and vice versa;
- recognize that the same amount can be added or subtracted to both pans or both lengths;
- relate the operations performed on the pan balance or with the rectangles to the coefficients and constants in the equation;
- can solve equations whether or not there are variables on both sides of the equation;
- recognize that the real pan balance model fails unless the solution is a whole number.

Consolidating and Reflecting on the Open Question

◊ *How could you model an equation of the form ___x + ___ = ___?* (e.g., I would model the right number of *x*'s and some counters on one side and the right number of counters on the other side.)

◊ *What would be different for the model of an equation of the form ___x – ___ = ___?* (e.g., I would start with one side being heavier than the other; then I would add cubes to the light side to balance it, before I continued.)

◊ *Which of the equations in Question 5 did you choose to model with the lengths? Why did you make that choice?* (e.g., I modelled 300 – 9*j* = 192 with lengths, since I thought it was easier to show the subtraction.)

◊ *Why would you not be able to use an actual pan balance to solve the equation 3x + 2 = 10?* (e.g., Each bag would have to have $\frac{8}{3}$ counters in it and that is not possible.)

Note: When using the rectangle model, remember that a constant is added to a group of variables to match the full length of *c* if the equation is of the form *ax* + *b* = *c*. If the equation is of the form *ax* – *b* = *c*, then an amount *b* is removed from the last of the *x* boxes in one row of the model. The *c* matches the part that is NOT removed.

Solutions

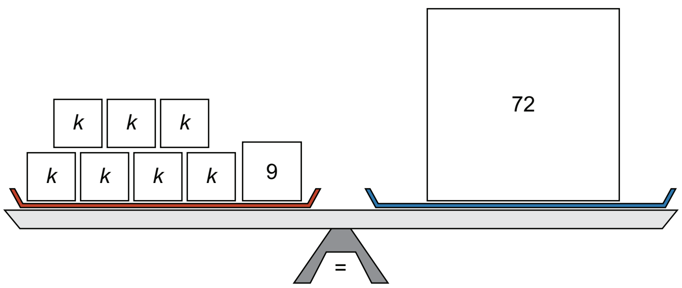
1. a) $2x + 1 = 19$
   b) $4x + 9 = 77$
   c) $5t + 8 = 43$
   d) $3x + 12 = 66$
   e) $4x - 11 = 69$

2. a) e.g., Then you would have just three boxes on the left and you could match them up with one of three equal groups on the right.
   b) $3s = 21$
   c) $s = 7$

3. a) e.g., Take eight counters from each side.
   b) e.g., Take two counters from both sides.
   c) I would split the 48 into 30 and 18; the 18 would match the bottom 18.

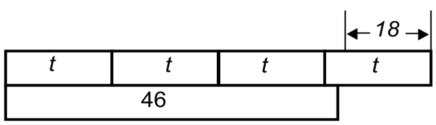
4. e.g., $6x - 8 = 16$ **or** $6x - 8 = 22$ or $6x - 8 = 28$

5. a) e.g.,



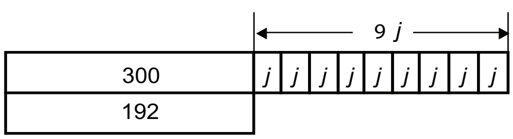
$k = 9$

b) e.g.



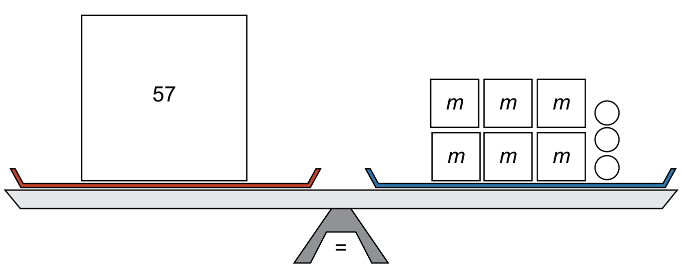
$t = 16$

c) e.g.,



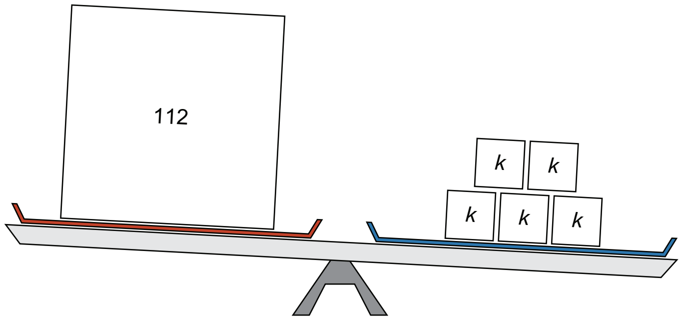
$j = 12$

d) e.g.,



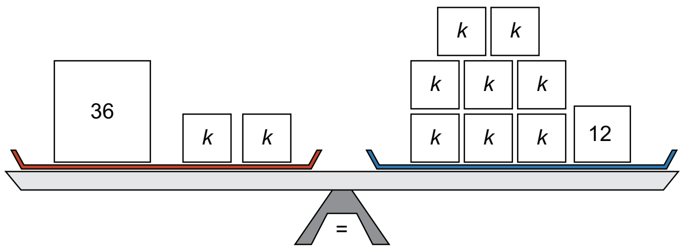
$m = 9$

e) e.g.,



k = 40

f) e.g.,



k = 4

6. e.g.,

a) $2k + 6 = 2$.

b) No, because you cannot have negative counters in a box.

c) e.g.,

If I take away 6 from top and bottom, I have 2*k* matching - 4, so each *k* is -2.

<table>
<tr><td>k</td><td>k</td><td>6</td></tr>
<tr><td colspan="3">2</td></tr>
</table>

Open Question

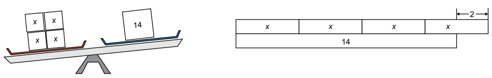
Using a Balance Model

Learning Goal

- representing an equation with a model as a strategy to solve it.

Open Question

The two models below can be used to represent the equation $4x - 2 = 14$.



- Explain why both of them represent the equation.
- Describe how to use the models to solve the equation to determine the value of x.
- Create and model five equations. Use either or both models.
  - Make sure that some of the equations involve subtraction.
  - Make sure that the coefficient of the variable is never 1.
  - Make sure that some equations have the variable on both sides of the equal sign.
- Describe how to use your models to solve each of your equations.

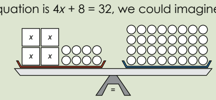
Think Sheet

Using a Balance Model (Continued)

Think Sheet

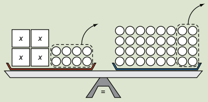
The two sides of an equation are equal; they balance. We can think of modelling the equation on a pan balance to figure out the unknown value that would make the two sides equal.

For example, if the equation is $4x + 8 = 32$, we could imagine

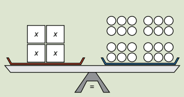


We could remove 8 ones from each side of the balance.

This is like subtracting 8 from each side of the equation: $4x = 32 - 8 = 24$.



We can divide the 24 counters on the right into four equal groups — one group to match each x box. That means that we can divide both sides by 4 to see that $x = 24 \div 4$ or 6.

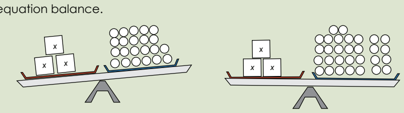


Note: We can draw a box that is marked 24 instead of drawing 24 counters.

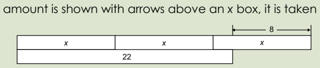
Using a Balance Model (Continued)

If the equation involved subtracting from both sides, the model would have to be different. In this case, we might start with an unbalanced model and figure out what to do to make it balance.

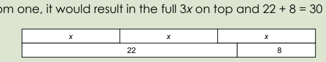
For example, if the equation were $3x - 8 = 22$, we would imagine that one side shows $3x$ and the other shows 22, but you must remove 8 from the $3x$ side to make the equation balance.



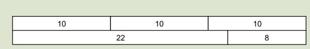
It is hard to visualize how to take the 8 away, so another model might be more helpful. If an amount is shown with arrows above an x box, it is taken away.



Notice that a section 8 units long is cut from $3x$ and the remaining section balances 22. If the 8 unit section were not removed from the top row and added to the bottom one, it would result in the full $3x$ on top and $22 + 8 = 30$ on bottom.

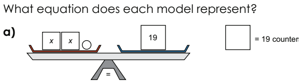


Since the 30 is made up of three sections labeled x, each section must be worth 10.



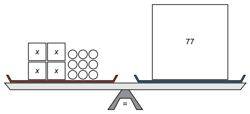
1. What equation does each model represent?

a)

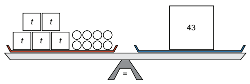


Using a Balance Model (Continued)

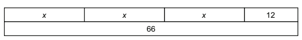
b)



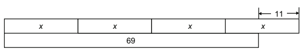
c)



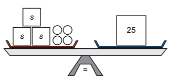
d)



e)



2.



a) Why might it make sense to take 4 counters off of each side of the balance?

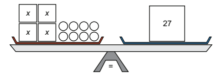
b) What is the resulting equation?

c) Solve the equation to determine the value of *s*.

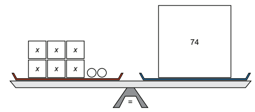
Using a Balance Model (Continued)

3. What would be your first step to solve each of these?

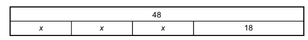
a)



b)



c)



4. You are solving an equation modelled on a pan balance. Your first step is to take 8 counters off of each side and your next step is to separate the counters on the right side into 6 equal groups. List three possible equations.

Using a Balance Model (Continued)

5. Sketch a model (pan balance or rectangles of equal lengths) you could use to help you to solve each equation. Then solve it.

a) $7k + 9 = 72$

b) $4t - 18 = 46$

c) $300 - 9j = 192$

d) $57 = 6m + 3$

e) $112 = 5k - 88$

f) $36 + 2k = 12 + 8k$

6. a) Create an equation with a solution of $k = -2$.

b) Explain whether you would be able to use a real pan balance to solve it.

c) Explain how to use a model to solve it.

Using Opposite Operations

Learning Goal

- working backwards to solve an equation.

Open Question

Questions to Ask Before Using the Open Question

◊ *What makes two fractions equivalent?* (They are equal.)
◊ *What do you think makes two equations equivalent?* (e.g., They are equal.) *What might that mean?* (e.g., They have the same solution.) Indicate that the equations not only have the same solution but provide exactly the same information in a different form.
◊ *Why are addition and subtraction opposite operations?* (e.g., If you add a number to something, you can undo the addition by subtracting that number from the sum.)
◊ *What is the opposite operation to multiplication?* (division) If desired, introduce the phrase isolate the variable, i.e., the idea that, if the variable has a coefficient of 1 on one side of the equation and there is a value on the other side, then there is nothing left to do to figure out the value of the variable.

Using the Open Question

Make sure students understand why the two equations at the top of the page are equivalent. Ensure that they realize that they must create at least 7 equations or more, if they wish to meet the required conditions.

By viewing or listening to student responses, note if they:
- realize that if adding is used, the equation involves a subtraction or vice versa;
- realize that if dividing is used, the equation involves a multiplication or vice versa;
- realize that although we often add or subtract before we multiply or divide, it is not required;
- divide all terms of the equation through when dividing first;
- solve equations using opposite operations.

Consolidating and Reflecting on the Open Question

◊ *What did you notice about the equations where you divided?* (e.g., The coefficient of the variable was not 1.)
◊ *What did you notice about the equations where you added?* (e.g., The constant on the same side of the equation as the variable was negative.)
◊ *What kind of equation did you use when you divided first?* (e.g., One where every coefficient and constant was a multiple of the same number.)
◊ *Could you divide first and then add to solve the equation $4x - 2 = 12$?* (e.g., Yes) *What would the equivalent equation be?* (e.g., $2x - 1 = 6$.)

Solutions

e.g.

- If you can add 20 to a number to get 68, the number must be 20 less than 68. The number would be 6*s*.
- If you subtract 20, you get a simpler equation: $6s = 48$ and if you divide you can find the value of one *s*.

– adding and then dividing:	$6s - 20 = 28$	$s = 8$
– only subtracting:	$s + 20 = 50$	$s = 30$
– only dividing:	$4s = 40$	$s = 10$
– subtracting twice:	$3s + 3 = s + 19$	$s = 8$
– dividing and then subtracting:	$4s + 8 = 40$	$s = 8$
– dividing and then adding:	$4s - 8 = 40$	$s = 12$
– adding and then multiplying:	$\frac{s}{3} - 20 = 6$	$s = 78$

Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ *What makes two fractions equivalent?* (They are equal.)
◊ *What do you think makes two equations equivalent?* (e.g., They are equal.) *What might that mean?* (e.g., They have the same solution.) Indicate that the equations not only have the same solution but provide exactly the same information in a different form.
◊ *Why are addition and subtraction opposite operations?* (e.g., If you add a number to something, you can undo the addition by subtracting that number from the sum.)
◊ *What is the opposite operation to multiplication?* (division) If desired, introduce the phrase isolate the variable, i.e., the idea that, if the variable has a coefficient of 1 on one side of the equation and there is a value on the other side, there is nothing left to do to figure out the value of the variable.

Using the Think Sheet

Work through the introductory section with students or make sure they understand what they have read.

Assign the tasks.

By viewing or listening to student responses, note if they:
- realize that if the equation involves a subtraction, adding makes sense to isolate the variable;
- realize that if the equation involves a multiplication, dividing makes sense to isolate the variable;
- distinguish between equations where one opposite operation is required to isolate the variable and those where more than one operation is required;
- realize that although we often add or subtract before we multiply or divide to solve an equation, it is not required;
- solve equations using opposite operations.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *For what sort of equation might you add 8 first?* (e.g., an equation of the form ___$x - 8 =$ ___)
◊ *For what sort of equation might you divide by 4?* (e.g., an equation involving $4x$)
◊ *How are equations where you have to do two opposite operations different from ones where you only do one opposite operation?* (e.g., If you need two opposite operations, there is usually a coefficient of the variable that is not 1 and there is also an addition or subtraction in the equation.)
◊ *Why might you divide first rather than subtract for 3x - 6 = 9?* (e.g.because 3,6 and 9 are all multiples of 3 and the equation $x - 2 = 3$ is simpler to work with.)

Solutions

1. a) and c).
e.g., For a), I would add first to get $3x = 30$ to make it easier to solve.
For c), I would add $9t$ to both sides first and then I would subtract 18 from both sides. That way I would know how much $9t$ is and just divide the amount by 9.

2. a) e.g., Subtract to undo the addition of the +8.
b) e.g., Add to undo the subtraction of the 18.
c) e.g., Add $8s$ to figure out how much $8s$ would be.
d) e.g., Subtract 52 to take away the 52.
e) e.g., Add $3m$ to figure out how much $3m$ would be.

3. **a)** e.g., $8x + 12 = 92$ or $8x + 10 = 90$ or $8x + 100 = 180$
b) After subtracting, I would divide by 8.

4. e.g., $3x - 12 = 18$ $3x - 10 = 50$ $3x - 40 = 80$

5. **a)** e.g., Because there is only one operation in the first equation, but two in the second.
b) One operation – addition
c) e.g., $3x = 30$ $6x = -42$ $x - 8 = 10$ $x + 12 = 40$
$x = 10$ $x = 7$ $x = 18$ $x = 22$

d) e.g. $3x - 2 = 22$, $x = 8$

$4x - 8 = 32$, $x = 10$

$20 - 3x = 11$, $x = 3$

$54 - 10x = 4$, $x = 5$

6. **a)** $m = 4$
b) $t = 20$
c) $k = 17$
d) $c = 12$
e) $k = 21$
f) $m = 8$

7. e.g., Both of them are right. Kyla would get $4x = 36$, which is the same as $x = 9$ and Eric would get $x + 5 = 14$, which also happens when $x = 9$.

Open Question

Using Opposite Operations

Learning Goal

- working backwards to solve an equation.

Open Question

$6s + 20 = 68$ and

$6s = 68 – 20$ are equivalent equations.

- Explain how they give the same information.

- Why might someone think that the way you solve $6s + 20 = 68$ involves subtracting and then dividing?

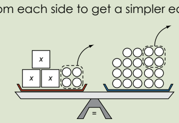
- Create and then solve equations that might be solved by performing each of these operations:
  - adding and then dividing
  - only subtracting
  - only dividing
  - subtracting twice
  - dividing and then subtracting
  - dividing and then adding
  - adding and then multiplying

Think Sheet

Using Opposite Operations (Continued)

Think Sheet

If an equation such as $3x + 4 = 19$, was modelled on a pan balance, we would subtract four counters from each side to get a simpler equivalent equation.



This means that
3x + 4 = 19 and
3x = 19 -4 are equivalent equations.

The equation was simplified to 3x = 15 by performing an opposite operation.
4 had been added to 3x, so now we subtract it.

If 3x = 15, you multiplied x by 3.
The opposite operation is division.

So if
3x = 15, then
3x ÷ 3 = 15 ÷ 3
x = 5 is a very simple equivalent equation.

This is exactly what you would have done on the pan balance, without using the balance.

When using opposite operations, it is useful to think of the order of operations (BEDMAS) in reverse.

If you add and subtract last, it might be easier to do the opposite first.

To solve 3x + 4 = 19, first subtract and then divide.

You can divide first, but you have to divide every term.

3x + 4 = 19 is equivalent to x + 4/3 = 19/3.

Using Opposite Operations (Continued)

1. For which of these equations would you add something first to solve using opposite operations? Tell why.
   a) $3x – 9 = 21$
   b) $62 = 5x + 22$
   c) $120 – 9t = 18$

2. What is the first operation you would perform in solving each equation? Why?
   a) $7x + 8 = 71$
   b) $9s – 18 = 63$
   c) $120 – 8s = 48$
   d) $52 + 9t = 151$
   e) $73 = 115 – 3m$

3. You are solving an equation involving the term $8x$. Each time, your first step is to subtract.
   a) List three possible equations.
   b) Tell what your next step would be and why.

Using Opposite Operations (Continued)

4. You solve an equation by first adding something and then dividing by 3.
List three possible equations.

5. a) Why would you use only one opposite operation to solve $4x = 20$ but two opposite operations to solve $4x + 9 = 29$?

b) How many and what opposite operations would you use to solve $x - 8 = 12$?

c) List four equations that could be solved using only one opposite operation. Solve the equations.

d) Create four equations that require two operations to solve them. Solve the equations.

6. Use opposite operations to solve each of these equations.

a) $11m + 23 = 67$ b) $12t - 182 = 58$

c) $100 - 4k = 32$ d) $215 = 8c + 119$

e) $46 = 5k - 59$ f) $24 + 6m = 8m + 8$

Using Opposite Operations (Continued)

7. Kyla said that to solve $4x + 20 = 56$, you have to subtract 20 first.
Eric said you could divide by 4 first as long as you divide everything by 4.
With whom do you agree? Why?

Rearranging Equations and Formulas

Learning Goal

- representing an equation or formula in a different way to make it easier to solve.

Open Question

Questions to Ask Before Using the Open Question

◊ *If the area of a rectangle was 20 square units and the length was 2 units, how would you figure out the width?* (e.g., Divide 20 by 2.) *Why?* (Since you multiply two times the length to get the area and the opposite operation is division.)
◊ *Why is what you just said the same as saying that if* $A = l \times w$*, then* $w = A \div l$*?* (e.g., You are dividing the area by the length, just as we did with 20 and 2.)
◊ *How is that like solving the equation* $A = l \times w$ *for* w*?* (e.g., If the equation were $4x = 8$, you would divide 8 by 4. So it is like pretending that $A = 8$ and $l = 4$.)

Using the Open Question

Encourage students to use a combination of formulas and situations that can be described with two or more variables.

By viewing or listening to student responses, note if they:
- recognize situations involving more than one variable;
- can use inverse operations to isolate any variable in a simple equation.

Consolidating and Reflecting on the Open Question

◊ *When might an equation involve more than one variable?* (e.g., When it is describing a relationship between two amounts.)
◊ *Choose one of your equations. Explain how you solved for one of the variables in terms of the other?* (e.g., I used $P = 35 + 45h$ to describe the pay for an electrician for h hours. To solve for h, I realized you would have to take 35 from P and then divide by 45.)

Solutions

e.g.,

a) $A = bh$ (formula for the area of a parallelogram)
$h = \frac{A}{b}$ and $b = \frac{A}{h}$

b) $V = l \times w \times h$ (formula for the volume of a rectangular prism)
$l = \frac{V}{(w \times h)}$ $W = \frac{V}{(l \times h)}$ $h = \frac{V}{(w \times l)}$

c) There are some tables with four people at them and some tables with five people at them. All the tables are full and there are 100 people seated. How many of each kind of table might there be?
$100 = 4a + 5b$ $a = \frac{(100 - 5b)}{4}$ $b = \frac{(100 - 4a)}{5}$

d) An electrician charges $35 for a visit + $45 an hour. How much might you pay
$P = 35 + 45h$ $h = \frac{(P - 35)}{45}$

e) $P = 2l + 2w$ (formula for the perimeter of a rectangle)
$l = \frac{P - 2w}{2}$ $w = \frac{P - 2l}{2}$

Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ *If the area of a rectangle was 20 square units and the length was 2 units, how would you figure out the width?* (e.g., Divide 20 by 2.) *Why?* (Since you multiply two times the length to get the area and the opposite operation is division.)
◊ *Why is what you just said the same as saying that if $A = l \times w$, then $w = A \div l$?* (e.g., You are dividing the area by the length, just as we did with 20 and 2.)
◊ *How is that like solving the equation $A = l \times w$ for w?* (e.g., If the equation were $4x = 8$, you would divide 8 by 4. It is like pretending that $A = 8$ and $l = 4$.)

Using the Think Sheet

Read through the introductory box with the students or clarify any questions they might have.

Assign the tasks.

By viewing or listening to student responses, note if they:
- relate strategies for solving equations with two variables (or more) to those they learned for solving equations involving one variable;
- use opposite operations to solve equations for one variable in terms of another;
- recognize the value of using a general formula rearrangement when many substitutions are required.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How is solving $2x - 4 = 8$ like solving $2x - 4 = y$ for x?* (e.g., You really do just the same thing except you can not actually combine 4 and *y* the same way you can combine 4 and 8. You just write $y + 4$.)
◊ *What operations will you use on the formula for the circumference of a circle ($C = 2\pi r$) to solve for the radius when you know the circumference?* (e.g., I would divide, since it is the opposite of multiplying *r* by 2π.)
◊ *If you have to find lots of different values of x for different values of y when $y = 3x - 14$, why might it be better to solve for x in terms of y than to substitute each value of y in the original equation?* (e.g., If you substitute in the original equation, you have to remember each time to add the 14 and divide by 3. If you solve for *x*, you only have to remember once and then just do the substitutions.)

Solutions

1. **a)** and **c)** or **a), c)** and **d)**
   For a), the opposite of subtracting 8 is adding 8.
   For c), the opposite of subtracting 12 is adding 12.
   For d), you might add $2x$ to both sides since the opposite of subtracting it is adding it.

2. **a)** e.g., Add 8 to both sides.
   b) e.g., Add $3m$ to both sides.
   c) e.g., Add 2 to both sides.
   d) e.g., Add $4m$ to both sides.
   e) e.g., Subtract 18 from both sides.

3. **a**) 5 **b**) 10 **c**) 15 **d**) $25 \div \pi$ **e**) $50 \div \pi$

4. **a**) I would divide A by *b*. **b**) I would divide A by *h* **c**) I used the same idea.

5. **a**) To solve for *h*, I would multiply *A* by 2 and then divide by ($b + 5$); **b**) to solve for *b*, I would multiply *A* by 2, divide by *h* and then subtract 5.

6. **a)** $h = V \div 12$
   b) $w = (P - 12) \div 2$
   c) $w = (SA - 30) \div 16$
   d) $x = \frac{(y-3)}{2}$
   e) $x = \frac{y-10}{2}$
   f) $x = \frac{(18-y)}{3}$

7. **a)** e.g., I would choose Strategy A, since it is less work.
   b) e.g., I would choose Strategy B, since I could use the same formula over and over.

Open Question

Rearranging Equations and Formulas

Learning Goal

- representing an equation or formula in a different way to make it easier to solve.

Open Question

Sometimes equations describe the relationship between different variables.

Some examples are:

$E = 15h$ (the total earnings for h hours if you earn \$15 an hour)

$A = lw$ (the formula for the area of a rectangle)

$2t + 5f = 100$ (the total value of t toonies and f \$5 bills)

Sometimes you know one of the variables and want to solve for the other.

For example, for the first equation, you might know the number of hours and want to figure out the earnings or you might know the amount earned and try to figure out how many hours.

For the second equation, you might know the area and length and want to figure out the width or you might know the length and width and want to figure out the area.

- Think of at least five formulas you know or create situations that you could describe with two or more variables.
  - Make sure that some of the equations involve coefficients other than 1.
  - Make sure that different operations are used in different equations.
  - Do not use the equations or situations from the background section above.

- Show how you would solve for each variable in terms of all of the others.

Think Sheet

When an equation involves two or more variables, you can solve for any one of them using the other(s).

For example, if $y = x + 2$, then y is 2 more than x.
That means that x is 2 less than y.
You can use opposite operations to see this:

$y = x + 2$, so

$y - 2 = x + 2 - 2$

$y - 2 = x$

In a triangle, $A = bh \div 2$.

You can use opposite operations to solve for h if you know b and A.

$A = bh \div 2$, so

$2A = bh$, so

$2A \div b = h$

If you know actual values to substitute, you can substitute and then solve. For example, if the area of a triangle is 12 square units and the base is 4 units, you could write:

$A = bh \div 2$, so

$12 = 4h \div 2$, so

$2 \times 12 = 4h$, so

$24 \div 4 = h$.

Think Sheet

1. For which of these equations would you add something first to solve for x in terms of y? Why?

a) $y = 4x - 8$

b) $y = 3x + 12$

c) $6x - 12 = y$

d) $500 - 2x = y$

2. What is the first step you would perform to solve for m in terms of p?

a) $p = 6m - 8$

b) $p = 15 - 3m$

c) $p = 3m - 2$

d) $2p = 5 - 4m$

e) $10p = 18 + 3m$

3. Complete this table to determine the values of r if $C = 2\pi r$.

	C	r
a	10π	
b	20π	
c	30π	
d	50	
e	100	

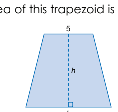
4. The formula for the area of a paralllelogram is $A = bh$

a) How would you solve for h in terms of A and b?

b) How would you solve for b in terms of A and h?

c) What do you notice about your two strategies?

5. The formula for the area of this trapezoid is $A = \left[\frac{b+5}{2}\right]h$.



a) How would you solve for h if you knew A and b?

b) How would you solve for b if you knew A and h?

6. For each equation, solve for the bolded variable in terms of the non-bolded variable.

a) $V = 12\mathbf{h}$

b) $P = 12 + 2\mathbf{w}$

c) $SA = 30 + 6\mathbf{w} + 10\mathbf{w}$

d) $4y = 8\mathbf{x} + 12$

e) $2y = 4(\mathbf{x} + 5)$

f) $3y + 9\mathbf{x} = 54$

Think Sheet

Rearranging Equations and Formulas (Continued)

7. The formula for the perimeter of a rectangle is $P = 2l + 2w$.

a) To solve for w if you know that $P = 20$ and $l = 4$, which of the following strategies could you choose?

Strategy A: Write $20 = 2 \times 4 + 2w$ and solve the equation $20 = 8 + 2w$.

Strategy B: Solve for w in terms of P and l. $w = (P - 2l) \div 2$. Then substitute for P and l, to get $w = (20 - 2 \times 4) \div 2$.

b) If the perimeter were 20 but you had to solve for a lot of values of l for a lot of different values of w, what strategy would you choose?