

GAP CLOSING

Algebraic Expressions and Equations

**Intermediate / Senior
Student Book**

Topic 6

Algebraic Expressions and Equations

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Diagnostic

1. Describe what each expression or equation tells you to do with a number represented by the letter "j." The first one is modelled for you.

a) $2j$ Double the number j represents

b) $8 - j$ _____

c) $4j + 8$ _____

d) $20 - 2j = 10$ _____

2. Use an algebraic expression to say the same thing.

a) triple a number and then add 2 _____

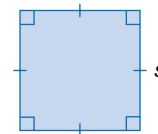
b) multiply a number by 4 and then subtract the product from 30 _____

c) three more than twice a number is 85 _____

d) one number is four less than twice another number _____

3. Use an algebraic expression to describe each of the following:

a) the perimeter of the square _____



b) the perimeter of the rectangle _____



c) the total value of the money _____



f bills



t toonies

4. Explain why $4a + (-3a) = a$.

5. Explain why $5a - 1$ is not $4a$.

6. Write the simplest form for each expression.

- a) $2a + 4 + 5a + 8$ _____
- b) $-2a + (-7) + 3a - 8$ _____
- c) $9t + (-5) + (-8s) + 10$ _____

7. Evaluate the following expressions for the given values.

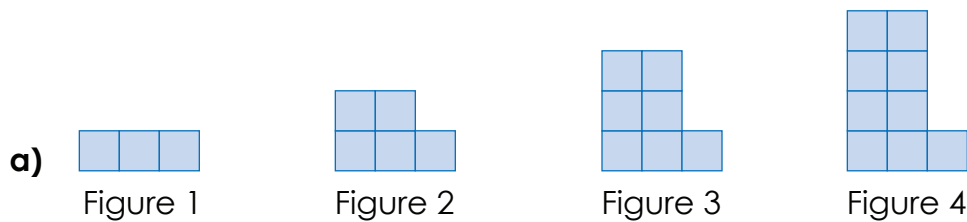
- a) $4k - 3$, if $k = 8$
- b) $20 - 3k$, if $k = -2$
- c) $6 + m + 2m^2$, if $m = -3$
- d) $3a^2$, if $a = 4$

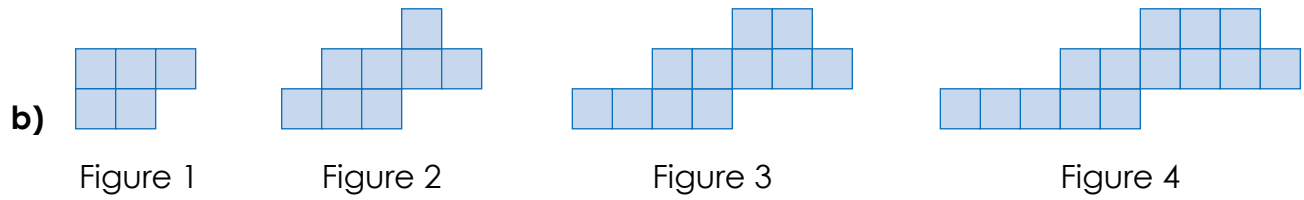
8. Two algebraic expressions involving the variable t have the value 20 when $t = -2$. What might they be?

9. Without substituting values, tell why each has to be true.

- a) $3m - 20 > 2m - 20$, if m is positive
- b) $40 - 3t > 40 - 2t$, if t is negative

10. Write a pattern rule for the number of tiles in each pattern using the variable f , where f is the figure number.





11. Use an algebraic expression or equation to write the general term of each pattern. Use the variable n .

a) 3, 6, 9, 12, ...

b) 7, 12, 17, 22, 27, ...

c) 50, 48, 46, 44, ...

12. How does the equation $4x + 4 = 120$ help you figure out where the number 120 appears in the pattern: 8, 12, 16, 20, ...?

Translating into Algebraic Expressions and Equations

Learning Goal

- representing numerical rules and relationships using algebraic expressions

Open Question

Algebraic Expressions

$$2000 - 30t$$

$$100m - 4$$

$$10 + 5w$$

$$3n + 40$$

$$5p$$

$$2y + 1$$

Algebraic Equations

$$100 - 2n = 48$$

$$6h = 120$$

$$32 + 2d = 80$$

$$5f + 2t = 200$$

$$P = 3s$$

$$2l + 2w = 600$$

- Choose at least two of the algebraic expressions and two of the algebraic equations.
- For each one, describe at least three different real-world situations that the **algebraic expression** or **algebraic equation** might describe.

Think Sheet

An **algebraic expression** is a combination of numbers, variables, and operations. Some examples are:

$$4 - 2t \quad 3n + 1 \quad 2x - x^2 + 4 \quad 2n \quad t + (t - 1)$$

For the algebraic expression $4 - 2t$:

term	each part of the expression	4 and $-2t$ are terms
coefficient	what you multiply the variable by	-2 is the coefficient
constant	a value that does not change	4 is the constant

- When a number (coefficient) sits right next to a variable, you assume those values are multiplied. For example, $2n$ means two n 's; that is 2 multiplied by n .
- Sometimes the coefficient 1 is not written. For example, in the expression $x - 3$, the coefficient of x is 1.
- Algebraic expressions allow you to describe what to do with a number quickly.

The chart shows how different verbal expressions are said (or written) algebraically.

Verbal expression	2 less than a number	1 more than triple a number	The sum of two numbers in a row	The result after subtracting a number from 10
Algebraic expression	$n - 2$	$3n + 1$	$t + (t + 1)$	$10 - s$

Notice that $10 - s$ is what you write for subtracting a number from 10. But you would write $s - 10$ to subtract 10 from a number.

An **algebraic equation** is a statement where two expressions, at least one of which is an algebraic expression, have the same value.

There are 3 kinds of equations:

Type of equation	Example	What it means
Ones where you figure out an unknown value	$3n + 1 = 4$	There is a value of n for which $3n + 1$ is 4.
	or $3n + 1 = 2n + 5$	There is a value of n for which $3n + 1$ and $2n + 5$ have the same value.
Ones which are always true	$2t + t = 3t$	The equation shows two ways of saying the same thing.
Ones describing relationships	$y = 2x + 3$	There is a relationship between the two variables x and y and you can calculate y if you know x .
	or $A = lw$	There is a relationship between the length, width and area of a rectangle.

1. Match each algebraic expression, on the right, with an equivalent phrase in words. There will be one item in each column without a match.

- | | |
|---|-------------|
| a) five more than a number | $5 - n$ |
| b) a number is multiplied by five and four is added | $4 + t + 5$ |
| c) a number is multiplied by four and five is added | $n - 5$ |
| d) four more than a number and then five more | $4n + 5$ |
| e) a number is subtracted from five | $n + 5$ |
| f) five less than a number | $4 - 5n$ |

2. Describe each algebraic expression using words.

- a) $x + x^2$
- b) $3 + 4t - 5$
- c) $20 - 2t$
- d) $14 + t$

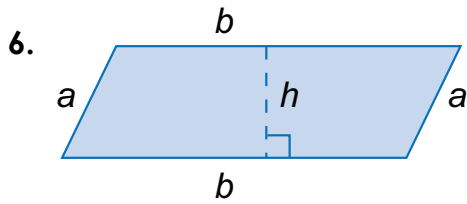
3. Write an algebraic expression to describe each situation.
- a) the total cost of p items, if each costs \$5
 - b) the sum of a multiple of 3 and the number one greater than that multiple of 3
 - c) the total cost of m muffins that cost \$1.50 each and p muffins that cost \$1.29 each
 - d) one person's share, if 5 people pay d dollars for something and share the cost equally
 - e) your total savings after w weeks, if you had \$100 saved and are saving another \$10 a week

4. Write an equation to describe each situation.
- a) Three more than a number is equal to eighteen.
 - b) Four more than double a number is equal to three times the same number.
 - c) One number is always double another.
 - d) If a number is subtracted from 10, the result is four times the original number.

5. a) Pick any day in the first half of the calendar month. Put a d in the box. Explain why the number directly below the d is $d + 7$.

September						
S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

- b) Pick a date in the last two rows of the calendar. Put a d in the box. Where is $d - 10$?



- a) Write an algebraic expression to describe the perimeter of the parallelogram.
- b) Write an equation that says that the perimeter is 84 cm.
- c) Write an algebraic expression to describe how much longer b is than a .
- d) Write an equation that says that the longer side length (b) is 8 cm longer than the shorter side length (a).
- e) Write an algebraic expression to describe the area of the parallelogram.
- f) Write an equation that says that the area is 42cm^2 .

7. Choose one of the expressions: $x + 20$ $2x - 10$ $x - 12$

Describe a real-life situation where an equation involving your expression might be used.

Equivalent Expressions

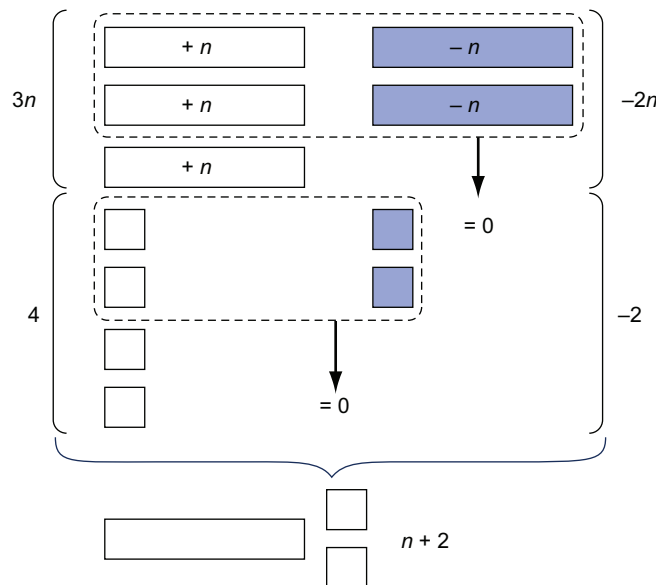
Learning Goal

- reasoning that any algebraic expression can be represented in a variety of ways

Open Question

Gemma wants to find an equivalent expression to $3n + 4 + (-2n) + (-2)$.

She used the zero principle [that $(+1) + (-1) = 0$ and $(+n) + (-n) = 0$] to figure that out.



Similarly, the expression $3m - 2 + 4t - 5 - 3t$ is equivalent to the expression $3m + t - 7$.

- Create at least three different algebraic expressions and their equivalent expressions to meet each of these conditions:
 - One expression has 6 terms (6 separate parts) and the equivalent one has 4 terms.
 - One expression has 6 terms and the equivalent one has 2 terms.
 - One expression involves two variables, but the equivalent expression only involves one variable. You might need to create your own “model” for the second variable.

Explain why each of your pairs of expressions is equivalent.

Think Sheet

Just like 6×2 is a quicker way to write $2 + 2 + 2 + 2 + 2 + 2$, there are sometimes quicker ways to write algebraic expressions. The two ways to write the expressions are **equivalent**; they mean the same thing.

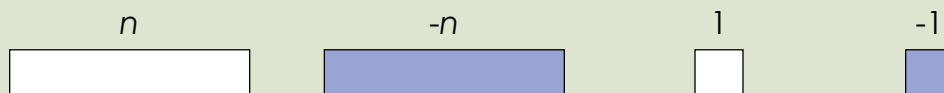
For example,

$n + 1 + 1 + 1$ is equivalent to $n + 3$.

$p + p$ is equivalent to $2p$.

- You can use algebra tiles to help you write simpler equivalent expressions.

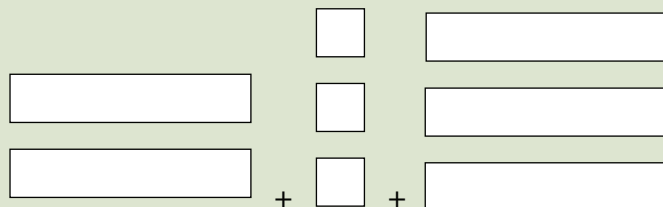
You could model variables and constants with these tiles.



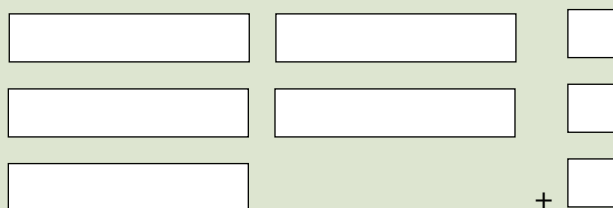
For example :

Expression	n	$n + 2$	$2n - 3$	$-3n + 1$
Model				

Then it is easy to see why $2n + 3 + 3n \dots$



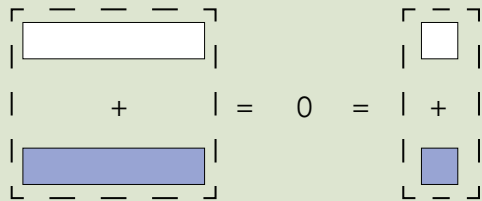
is equivalent to $5n + 3$.



Even though the letter n is used here, the tile could be used to represent any variable.

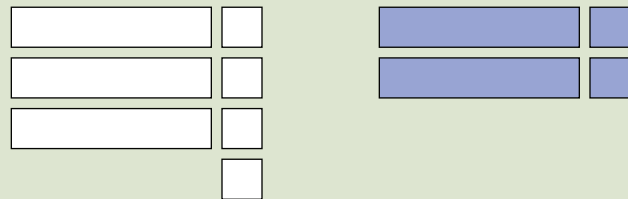
- Sometimes there are positive and negative tiles and you can use the **zero principle** to simplify.

The zero principle says that $(+n) + (-n) = 0$ or $(+1) + (-1) = 0$

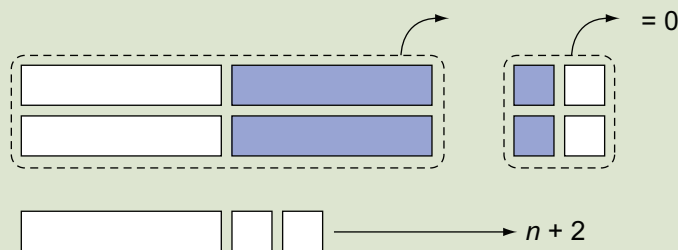


When you add or subtract zeroes, the amount does not change, so you can get rid of extra zeroes to simplify expressions.

In an expression like $3n + 4 + (-2n) + (-2)$:



You can match up copies of $(+n)$ and $(-n)$, and $(+1)$ and (-1) to eliminate zeroes.



So an equivalent expression for $3n + 4 + (-2n) + (-2) = n + 2$.

Even without the models, we might have rearranged $3n + 4 + (-2n) + (-2)$ as $3n + (-2n) + 4 + (-2)$. Altogether there would be $1n$ (since $3 + (-2) = 1$) and $+2$ (since $4 + (-2) = +2$).

- You can combine identical variables, **like terms**, (e.g., $(-5t)$ and $(+4t)$ or $(+2n)$ and $(+3n)$ or numbers), but you can't mix them together. If there are two different variables, think of them separately.

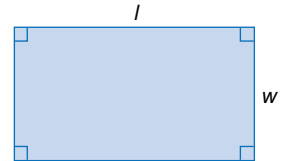


- Use a model to show why $5q = 3q + 2q$.
- Model each. Then write an equivalent expression.
 - $2n + 4 + 4n + 8$
 - $3n + (-5) + (-4n) + (-3)$
 - $2n - 8 + (-6n) + 2$
 - Do your equivalent expression use more or fewer tiles than the original expressions? Why does that make sense?
- Write an equivalent expression for each, using two more terms. NOTE: If you use models, use different sizes for s and y .
 - $-s + 8$
 - $2s - 4y + 3$
 - $-10 - x$
- Choose a number on the hundreds chart and replace the value with the variable s .
 - Why does the expression $s + 10$ describe the number directly below s ?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	18	17	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

- b) Write the two numbers below s in terms of s .
- c) What algebraic expression would result from adding s to the two numbers directly below it?
- d) Write an equivalent expression for your answer to part c).
- e) Write two equivalent expressions for adding s to the two numbers on each side of that square.

5. Write two equivalent expressions for the perimeter of the rectangle.



6. a) Why does it make sense that $5 \times \frac{n}{5}$ is equivalent to n ?
- b) Write several other expressions equivalent to n and tell why each one is equivalent.

Think Sheet

When an expression involves a variable, you can **substitute** values for the variable to **evaluate** the expression.

To evaluate $2m$ when $m = 4$, substitute 4 for m , and calculate $2 \times 4 = 8$.

- If the same variable appears more than once in an equation, or expression, you must use the same value in each of those places.

For example, to evaluate $3p - 2 + p$ when $p = -1$, calculate $3 \times (-1) - 2 + (-1)$.

- Equivalent expressions always have the same value when the same substitution is made.

For example, $3n + 2 = n + 1 + 2n + 1$. If $n = 5$, it is true that $3 \times 5 + 2 = 5 + 1 + 2 \times 5 + 1$.

- Non-equivalent expressions might have the same value when the same substitution is made or might not.

For example, $25 - n = 4n$, but only when $n = 5$ and not for other values of n .

- If an expression involves more than one variable, these variables can be substituted with either different values or the same values.

For example:

Substitute $p = 6$ and $s = 3$ into

$$\begin{aligned} &5p - 7s \\ &= 5 \times 6 - 7 \times 3 \\ &= 30 - 21 \\ &= 9 \end{aligned}$$

Substitute $p = 4$ and $s = 4$ into

$$\begin{aligned} &5p - 7s \\ &= 5 \times 4 - 7 \times 4 \\ &= 20 - 28 \\ &= (-8) \end{aligned}$$

When evaluating expressions, the normal order of operations (BEDMAS) rules apply. For example:

Substitute $m = 6$ into

$$\begin{aligned} &3 + 2m \\ &= 3 + 2 \times 6 \\ &= 15 \end{aligned}$$

1. Evaluate each expression.

- $3 - 4m$, when $m = 3$
- $15 + 8m$, when $m = -1$
- $j + 2j^2$, when $j = 4$

d) $j + (2j)^2$, when $j = 4$

e) $15 - 3p$, when $p = 2$

f) $\frac{3n + 2}{10 - n}$, when $n = 0$

2. Substitute the values of $m = 0$, then 1, then 2, then 3, then 4 into each expression.

	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$
a) $3m - 4$					
b) $3m + 6$					
c) $10m - 8$					
d) $5m + 9$					
e) $-m - 2$					
f) $-3m + 4$					

3. a) Evaluate the expression $3n$ for several different whole number values of n .

b) What is true each time?

c) How could you have predicted that?

4. Predict which value will be greater. Then test your prediction.

a) $4 + 3m$ if $m = -8$ OR if $m = 8$

b) $30 - 8t$ if $t = 1$ OR if $t = 10$

c) $4t - t^2$ if $t = -3$ OR if $t = 3$

5. How could you predict that each is true even before you substitute?
- a) If $m = 10$, then $4m - 2$ has to be positive.

 - b) If $m = 10$, then $4m - 2$ has to be even.

 - c) $6m - 200$ is negative for small values of m

 - d) $200 - 6m$ is negative for large values of m
6. For each part, create an expression involving p to meet the condition.
- a) It is even when p is 4.

 - b) It is a multiple of 10 when $p = 5$.

 - c) It is greater than 100 when $p = -4$.

Relating Pattern Rules to Expressions and Equations

Learning Goal

- representing certain algebraic expressions and equations using linear patterns

Open Question

The pattern 2, 8, 14, 20, ... can be modelled as shown below. Notice that the numbers go up by 6 and so there is an extra row of 6 each time.



Figure 1

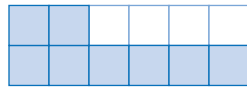


Figure 2

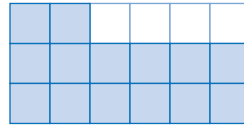


Figure 3

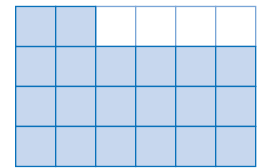


Figure 4

- The pattern rule for all the squares shown (including the faint ones) is $6f$ since if f is the figure number, there are f rows of 6.
- The pattern rule for the dark squares is $6f - 4$ since there are 4 squares not counted each time.

- Create at least four other algebraic expressions that could be pattern rules. Each pattern should include the number 30 somewhere.

- Show the first four terms of the pattern with pictures and then show the first four terms using numbers. Try to arrange your pictures to make the pattern rule easy to see.

- Write an equation that would help you figure out where the number 30 is in each pattern.

Think Sheet

You can think of an algebraic expression as a pattern rule. For example, $p + 2$ is the rule for this table of values or the associated picture.

Position	Value
1	3
2	4
3	5
4	6



Figure 1



Figure 2

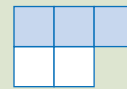


Figure 3

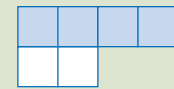


Figure 4

Notice that in the picture, the number of squares is always 2 more than the Figure number. The number of white tiles is the Figure number. The number of dark tiles is the 2.

To model the algebraic expression $3p + 2$, you can use this table of values which relates p to $3p + 2$ or you can use the shape pattern shown.

Position	Value
1	5
2	8
3	11
4	14
5	17

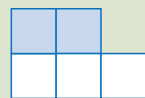


Figure 1

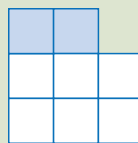


Figure 2

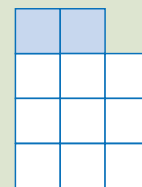


Figure 3

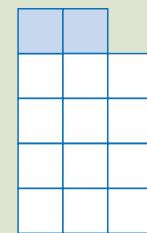


Figure 4

Notice that the way the pattern was coloured made it easier to see why the coefficient of p was 3. There were 3 times as many squares as the figure number.

It was easy to see the $+ 2$ of $3p + 2$ using the 2 shaded tiles.

Sometimes you can focus on the columns and other times you can focus on the rows to help figure out the rule.

The pattern below has the rule $2f + 1$ since there are twice as many white squares as the figure number and 1 extra shaded square.



Figure 1

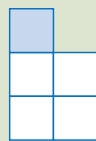


Figure 2

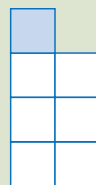


Figure 3

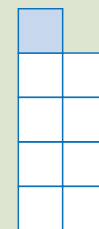
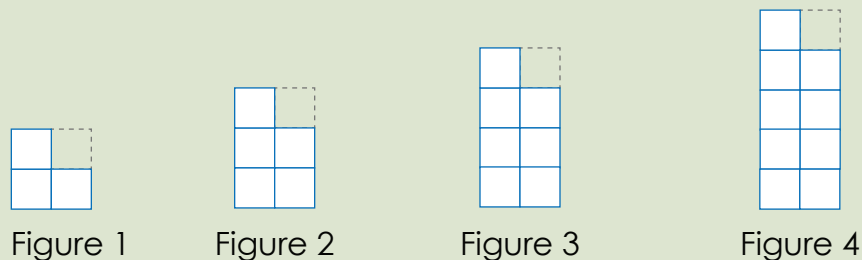


Figure 4

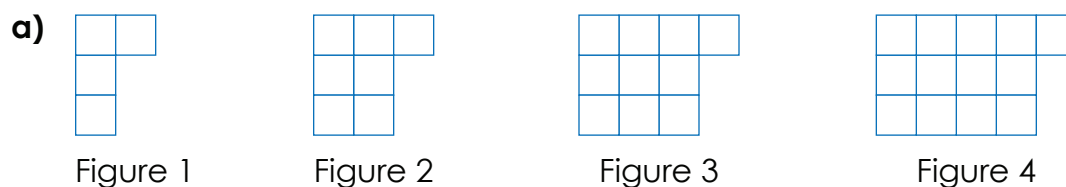
If you colour the figures differently, you might see an equivalent expression $2(f + 1) - 1$. The -1 is shown by fading out one square. The $2(f + 1)$ is shown by having two columns, each with $f + 1$ squares in it.



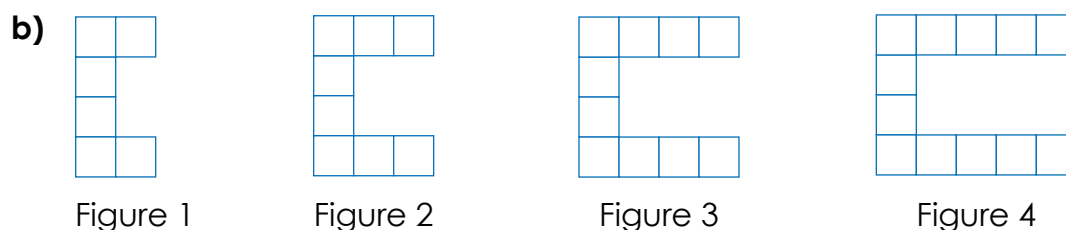
You can think of an equation as a way of asking which figure in a pattern has a certain number of squares.

For example, $2f + 1 = 31$ is solved by figuring out which Figure in the last pattern shown has exactly 31 squares.

1. Tell why the pattern rule describes each pattern.



Pattern rule $3n + 1$



Pattern rule: $2n + 4$

c) Create two equivalent rules for this pattern.

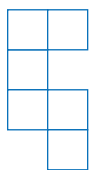


Figure 1

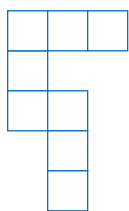


Figure 2

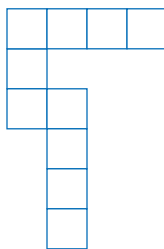


Figure 3

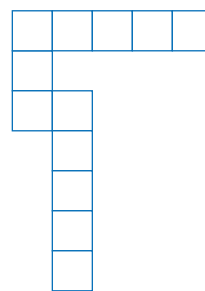


Figure 4

2. The pattern below can be described by the equivalent rules $2f + 1$ or $f + (f + 1)$.



Figure 1

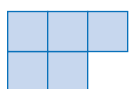


Figure 2

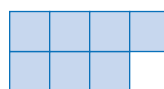


Figure 3

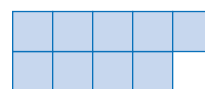


Figure 4

a) How does this way of colouring help you see the pattern rule $2f + 1$?



Figure 1

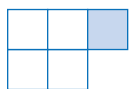


Figure 2

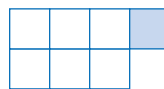


Figure 3



Figure 4

b) How does this way of colouring help you see the pattern rule $f + (f + 1)$?



Figure 1

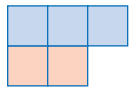


Figure 2

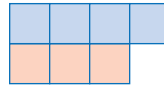


Figure 3

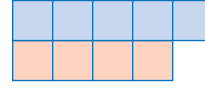


Figure 4

3. Draw the first four figures of the pattern below. Shade or arrange squares in your figures to help make the rule as obvious as you can.

a) Pattern: $4f + 2$

b) Pattern: $2f - 1$

4. Describe a pattern rule for each pattern using the variable p . The rule should tell what the value in the pattern is based on its position p in the pattern.

a) 2, 4, 6, 8, ...

b) 4, 9, 14, 19, 24, 29, ...

c) 28, 32, 36, 40, 44, ...

d) 202, 200, 198, 196, ...

e) 61, 58, 55, 52, ...

5. How are the pattern rules for these patterns alike? How are they different?

Pattern 1: 6, 11, 16, 21, 26, 31, ...

Pattern 2: 200, 195, 190, 185, 180, ...

6. A pattern rule includes the term $8n$.
- a) List two possible patterns.

 - b) What would have to be true about any pattern you listed?
7. What equation would you use to find out the position in the pattern of the number 100?
- a) 2, 4, 6, 8, ...

 - b) 4, 7, 10, ...

 - c) 12, 16, 20, 24, ...

 - d) 121, 120, 119, 118, ...

 - e) 300, 290, 280, ...
8. a) Describe the first five terms of the pattern you might be thinking of when you write the equation $3n + 8 = 35$.
- b) Check if the equation actually does make sense for your pattern.
9. Create your own algebraic expression.
- a) List the first few terms of the pattern for which it is a pattern rule.

 - b) Create an equation that would make sense for your pattern rule.