GAP CLOSING

Algebraic Expressions and Equations

Intermediate / Senior
Facilitator's Guide

Topic 6

Algebraic Expressions and Equations

Diagnostic 5
Administer the diagnostic 5
Using diagnostic results to personalize interventions 5
Solutions 5

Using Intervention Materials 8
Translating into Algebraic Expressions and Equations 9
Equivalent Expressions 14
Evaluating Algebraic Expressions 19
Relating Pattern Rules to Expressions and Equations 24

The Facilitator's Guide for this entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/6-AlgebraicExp_FG_IS.pdf. The Student Book for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/6-AlgebraicExp_SB_IS.pdf.

ALGEBRAIC EXPRESSIONS AND EQUATIONS

Relevant Learning Expectations for Grade 9

MPM1D

Number Sense and Algebra

- substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases)
- add and subtract polynomials with up to two variables, using a variety of tools
- multiply a polynomial by a monomial involving the same variable, using a variety of tools
- expand and simplify polynomial expressions involving one variable, using a variety of tools

Linear Relations

- construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools, for linearly related and non-linearly related data collected from a variety of sources

Analytic Geometry

- express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$

MFM1P

Number Sense and Algebra

- substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases)
- add and subtract polynomials involving the same variable up to degree three, using a variety of tools
- multiply a polynomial by a monomial involving the same variable to give results up to degree three, using a variety of tools

Linear Relations

- construct tables of values, scatter plots, and lines or curves of best fit as appropriate, using a variety of tools, for linearly related and non-linearly related data collected from a variety of sources

Analytic Geometry

- express a linear relation as an equation in two variables, using the rate of change and the initial value

Possible reasons why a student might struggle when working with algebraic expressions and equations

Students may struggle when working with algebraic expressions and equations.

Some of the problems include:

- depending too heavily on key words when attempting to translate verbal expressions into algebraic form
- lack of understanding of the function of a variable
- not being comfortable with certain conventions, e.g., that $2b$ means two times b
- not recognizing that any algebraic expression can be described in many different ways
- not recognizing that an equality sometimes describes a fact related to a single value (e.g., $n + 4 = 9$ is only true for $n = 5$); sometimes describes an "identity" true for all values [e.g., $2n – 5 = (n – 4) + (n – 1)$], and sometimes describes a relationship between two quantities, e.g., $m = n + 1$
- lack of fluency with integer operations, which makes simplification of expressions difficult
- not recognizing the rules for substitution, e.g., not realizing that the same value must be substituted for all instances of the same variable or that either the same or different values can be substituted for different variables
- lack of fluency with BEDMAS rules when substituting
- not recognizing the relationship between the way a pattern grows and the algebraic expression describing its general term

DIAGNOSTIC

Administer the diagnostic

Using the diagnostic results to personalize interventions

Materials
- square tiles (optional)
- algebra tiles (optional)

Intervention materials are included on each of these topics:
- Translating into algebraic expressions and equations
- Equivalent expressions
- Evaluating algebraic expressions
- Relating pattern rules to expressions and equations

You may use all or only part of these sets of materials, based on student performance with the diagnostic. If students need help in understanding the intent of a question in the diagnostic, you are encouraged to clarify that intent.

Evaluating Diagnostic Results	Suggested Intervention Materials
If students struggle with Questions 1–3	use *Translating into Algebraic Expressions and Equations*
If students struggle with Questions 4–6	use *Equivalent Expressions*
If students struggle with Questions 7–9	use *Evaluating Algebraic Expressions*
If students struggle with Questions 10–12	use *Relating Pattern Rules to Expressions and Equations*

Solutions

1. **b)** e.g., subtract a number (j) from 8 or how much less than 8 a number (j) is
c) e.g., multiply a number (j) by 4 and add 8
d) e.g., double a number (j) and then subtract it from 20 and the answer is 10

2. **a)** e.g., $3n + 2$
b) e.g., $30 - 4n$
c) e.g., $2m + 3 = 85$
d) e.g., $2n - 4 = m$

3. **a)** e.g., $4s$
b) e.g., $2l + 2w$
c) e.g., $5f + 2t$

4. e.g., Adding –3 of something is like taking away those 3. If you have 4 of something and take away 3 of them, you are left with 1 of them. *Note: Substitution of* a *single value of a is not sufficient to be correct.*

5. e.g., If you have 5 of something but take away 1 of something else, you still have the 5 of the something.

6. **a)** $7a + 12$
b) $a - 15$
c) e.g., $9t - 8s + 5$

7. **a)** 29
b) 26
c) 21
d) 48

8. e.g., $3t + 26$ and $5t^2$

9. **a)** e.g., If m is positive, then $3m > 2m$. If you subtract the same amount (20) from a greater number, you have more left.
b) e.g., If t is negative, $2t$ is more than $3t$. If you take away more, you end up with less.

10. **a)** $2f + 1$
b) $3f + 2$

11. **a)** $3n$
b) $5n + 2$
c) $52 - 2n$

12. e.g., The general term is $4x + 4$ and asking where it is 120 is a way of figuring out which term x has that value.

Diagnostic

1. Describe what each expression or equation tells you to do with a number represented by the letter "*j*." The first one is modelled for you.

a) 2*j* *Double the number j represents*

b) 8 – *j* ______________

c) 4*j* + 8 ______________

d) 20 – 2*j* = 10 ______________

2. Use an algebraic expressionto say the same thing.

a) triple a number and then add 2 ______________

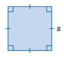
b) multiply a number by 4 and then subtract the product from 30 ______________

c) three more than twice a number is 85 ______________

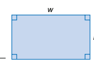
d) one number is four less than twice another number ______________

3. Use an algebraic expression to describe each of the following:

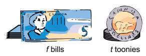
a) the perimeter of the square ______________



b) the perimeter of the rectangle ______________



c) the total value of the money ______________



4. Explain why 4*a* + (–3*a*) = *a*.

5. Explain why 5*a* – 1 is not 4*a*.

Diagnostic (Continued)

6. Write the simplest form for each expression.

a) 2*a* + 4 + 5*a* + 8 ______________

b) –2*a* + (–7) + 3*a* – 8 ______________

c) 9*t* + (–5) + (–8*s*) + 10 ______________

7. Evaluate the following expressions for the given values.

a) 4k –3, if *k* = 8

b) 20 – 3*k*, if *k* = –2

c) 6 + *m* + 2m^2, if *m* = –3

d) 3a^2, if *a* = 4

8. Two algebraic expressions involving the variable *t* have the value 20 when *t* = –2. What might they be?

9. Without substituting values, tell why each has to be true.

a) 3m – 20 > 2m – 20, if *m* is positive

b) 40 – 3*t* > 40 – 2*t*, if *t* is negative

10. Write a pattern rule for the number of tiles in each pattern using the variable *f*, where *f* is the figure number.

a)

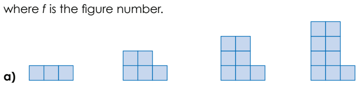


Figure 1 Figure 2 Figure 3 Figure 4

Diagnostic (Continued)

b)

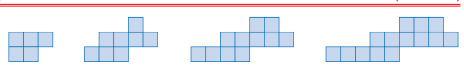


Figure 1 Figure 2 Figure 3 Figure 4

11. Use an algebraic expression or equation to write the general term of each patterm. Use the variable *n*.

a) 3, 6, 9, 12, …

b) 7, 12, 17, 22, 27, …

c) 50, 48, 46, 44, …

12. How does the equation 4*x* + 4 = 120 help you figure out where the number 120 appears in the pattern: 8, 12, 16, 20, …?

USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for working with polynomials and relating expressions and equations to linear and later, other types of relations.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate before, during, and after using your choice of approaches. The three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

Translating into Algebraic Expressions and Equations

Learning Goal

- representing numerical rules and relationships using algebraic expressions

Open Question

Questions to Ask Before Using the Open Question
◊ *Why might you use the expression 4s to describe the perimeter of a square?* (e.g., If the square has a side length called *s*, you would include 4 of them in the perimeter.) ◊ *Why might What might the expression 3t describe?* (e.g., number of sides in *t* triangles) ◊ *Why might you use the equation 25q + 5n = 400 to tell how many quarters and nickels you might have if you have $4?* (e.g., $4 is 400 cents, a quarter is 25¢ and a nickel is 5¢, so you multiply the number of nickels by 5, the number of quarters by 25 and add them and hope the total is 400.)
Using the Open Question
Make sure students understand that the same expression or equation must be used three times. By viewing or listening to student responses, note if they: • are equally able to translate in situations involving addition and subtraction • correctly use coefficients and constants to describe situations • describe reasonable situations.
Consolidating and Reflecting on the Open Question
◊ *What do all of your situations for the expression [5p] have in common?* (e.g., They all involve groups of 5.) ◊ *What do all of your situations for the equation [32 + 2d = 80] have in common?* (e.g., There is always a total of 80 of something and there are always 32 things as well as lots of groups of 2 or 2 groups of the same thing.) ◊ *What kinds of situations involve using subtraction?* (e.g., ones where you were taking something away or comparing two things)

Solutions

e.g., $5p$
- the total amount of money you earn if you earn $5 an hour and work *p* hours
- the number of fingers on *p* hands
- the number of people at *p* tables if there are 5 people at a table

e.g., $2000 – 30t$
- the amount of money you have left after *t* weeks if you start with $2000 and spend $30 a week
- the height of a ball, after *t* seconds, that starts at a height of 2000 m and drops 30 m each second
- the number of pages left to read after *t* days if a book has 2000 pages and you read 30 pages a day

e.g., $6h = 120$
- the number of tables in a room if there are 120 people and each table seats 6
- the number of rows of eggs if each row holds 6 eggs and there are 120 eggs
- the number of packages of hamburger buns if each package holds 6 buns and there are 120 buns

e.g., $2l + 2w = 600$
- the length and width of a rectangle with perimeter 600 units
- the number of boys and girls if there are a total of 600 legs
- the number of pairs of mittens and pairs of boots if there are a total of 600 mittens and boots in the lost and found

Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ *Why might you use the expression 4s to describe the perimeter of a square?* (e.g., If the square has a side length called *s*, you would include 4 of them in the perimeter.)
◊ *What might the expression 3t describe?* (e.g., number of sides in *t* triangles)
◊ *Why might you use the equation 25q + 5n = 400 to tell how many quarters and nickels you might have if you have $4?* (e.g., $4 is 400 cents, a quarter is 25¢ and a nickel is 5¢, so you multiply the number of nickels by 5, the number of quarters by 25 and add them and the total is 400.)

Using the Think Sheet

Read through the introductory box with the students or clarify any questions they might have. Make sure they recognize the difference in the verbal expressions for expressions like $n – 2$ as compared to $2 – n$.

Also make sure they understand the distinction among the three different types of equations.

Do not focus on the vocabulary, but make sure students distinguish between *coefficients* and *constants* and understand the meaning of the word *term*.

Assign the tasks.

By viewing or listening to student responses, note if they:
- recognize in a situation which numbers become coefficients in the algebraic expression and which become constants
- can “translate” algebraic expressions or equations into verbal ones and vice versa
- relate real-life situations to algebraic expressions

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How can you tell whether you will need one or two variables to describe something algebraically?* (e.g., if there is only one number or one measurement or if all of the measurements are the same, I will only need one variable.)
◊ *How can you tell whether you will need an equation or an expression to describe a situation?* (e.g., It depends whether you are just describing what to do to a number or whether you are describing a relationship.)
◊ *If an algebraic expression or equation involves the term 3m, what can you be sure is true about the situation it describes?* (e.g., There are lots of groups of 3 or else 3 equal groups.)
◊ *What kinds of situations involve using subtraction?* (e.g., ones where you were taking something away or comparing two things.)

Solutions

1.

five more than a number	$5 – n$ <e>
a number is multiplied by five and four is added	$4 + t + 5$ <d>
a number is multiplied by four and five is added	$n – 5$ <f>
four more than a number and then five more	$4n + 5$ <c>
a number is subtracted from five	$n + 5$ <a>
five less than a number	$4 – 5n$

2. **a)** e.g., add a number to its square
b) e.g., multiply a number by 4, add 3, then subtract 5
c) e.g., double a number and subtract it from 20
d) e.g., add a number to 14

3. e.g.,

a) $5p$

b) $3n + (3n + 1)$

c) $1.50m + 1.29p$

d) $d \div 5$

e) $100 + 10w$

4. e.g.,

a) $3 + m = 18$

b) $4 + 2m = 3m$

c) $d = 2n$

d) $10 - n = 4n$

5. **a)** e.g., Since there are 7 days in a week and you go up by 1 each time you move 1 space, if you add 7, you are back to the same column you started with, but in the next row.

b) e.g., It is one row up and 3 spaces back since it's 10 spaces before *d*.

6. **a)** e.g., $2a + 2b$

b) $2a + 2b = 84$

c) $b - a$

d) $b - a = 8$

e) bh

f) $bh = 42$

7. e.g., $2x - 10$

If you earn \$2 a week for *x* weeks but give your brother \$10, it is what you have left.

Open Question

Translating into Algebraic Expressions and Equations

Learning Goal

- representing numerical rules and relationships using algebraic expressions

Open Question

Algebraic Expressions

$2000 - 30t$	$100m - 4$	$10 + 5w$
$3n + 40$	$5p$	$2y + 1$

Algebraic Equations

$100 - 2n = 48$	$6h = 120$	$32 + 2d = 80$
$5f + 2t = 200$	$P = 3s$	$2l + 2w = 600$

- Choose at least two of the algebraic expressions and two of the algebraic equations.
- For each one, describe at least three different real-world situations that the **algebraic expression** or **algebraic equation** might describe.

Think Sheet

Translating into Algebraic Expressions and Equations (Continued)

Think Sheet

An **algebraic expression** is a combination of numbers, variables, and operations. Some examples are:

$4 - 2t$ $3n + 1$ $2x - x^2 + 4$ $2n$ $t + (t - 1)$

For the algebraic expression 4 - 2*t*:

term	each part of the expression	4 and -2t are terms
coefficient	what you multiply the variable by	-2 is the coefficient
constant	a value that does not change	4 is the constant

- When a number (coefficient) sits right next to a variable, you assume those values are multiplied. For example, 2*n* means two *n*'s; that is 2 multiplied by *n*.
- Sometimes the coefficient 1 is not written. For example, in the expression x – 3, the coefficient of x is 1.
- Algebraic expressions allow you to describe what to do with a number quickly.

The chart shows how different verbal expressions are said (or written) algebraically.

Verbal expression	2 less than a number	1 more than triple a number	The sum of two numbers in a row	The result after subtracting a number from 10
Algebraic expression	$n - 2$	$3n + 1$	$t + (t + 1)$	$10 - s$

Notice that 10 – *s* is what you write for subtracting a number from 10. But you would write *s* – 10 to subtract 10 from a number.

Translating into Algebraic Expressions and Equations (Continued)

An **algebraic equation** is a statement where two expressions, at least one of which is an algebraic expression, have the same value.

There are 3 kinds of equations:

Type of equation	Example	What it means
Ones where you figure out an unknown value	$3n + 1 = 4$ or $3n + 1 = 2n + 5$	There is a value of *n* for which 3*n* + 1 is 4. There is a value of *n* for which 3*n* +1 and 2*n* +5 have the same value.
Ones which are always true	$2t + t = 3t$	The equation shows two ways of saying the same thing.
Ones describing relationships	$y = 2x + 3$ or $A = lw$	There is a relationship between the two variables *x* and *y* and you can calculate *y* if you know *x*. There is a relationship between the length, width and area of a rectangle.

1. Match each algebraic expression, on the right, with an equivalent phrase in words. There will be one item in each column without a match.
   - **a)** five more than a number — $5 - n$
   - **b)** a number is multiplied by five and four is added — $4 + t + 5$
   - **c)** a number is multiplied by four and five is added — $n - 5$
   - **d)** four more than a number and then five more — $4n + 5$
   - **e)** a number is subtracted from five — $n + 5$
   - **f)** five less than a number — $4 - 5n$

2. Describe each algebraic expression using words.
   - **a)** $x + x^2$
   - **b)** $3 + 4t - 5$
   - **c)** $20 - 2t$
   - **d)** $14 + t$

Translating into Algebraic Expressions and Equations (Continued)

3. Write an algebraic expression to describe each situation.

a) the total cost of *p* items, if each costs $5

b) the sum of a multiple of 3 and the number one greater than that multiple of 3

c) the total cost of *m* muffins that cost $1.50 each and *p* muffins that cost $1.29 each

d) one person's share, if 5 people pay *d* dollars for something and share the cost equally

e) your total savings after *w* weeks, if you had $100 saved and are saving another $10 a week

4. Write an equation to describe each situation.

a) Three more than a number is equal to eighteen.

b) Four more than double a number is equal to three times the same number.

c) One number is always double another.

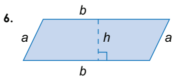
d) If a number is subtracted from 10, the result is four times the original number.

5. a) Pick any day in the first half of the calendar month. Put a d in the box. Explain why the number directly below the *d* is *d* + 7.

b) Pick a date in the last two rows of the calendar. Put a *d* in the box. Where is *d* - 10?

Translating into Algebraic Expressions and Equations (Continued)

6.



a) Write an algebraic expression to describe the perimeter of the parallelogram.

b) Write an equation that says that the perimeter is 84 cm.

c) Write an algebraic expression to describe how much longer *b* is than *a*.

d) Write an equation that says that the longer side length (*b*) is 8 cm longer than the shorter side length (*a*).

e) Write an algebraic expression to describe the area of the parallelogram.

f) Write an equation that says that the area is 42cm^2.

7. Choose one of the expressions: $x + 20$ $\quad 2x - 10$ $\quad x - 12$

Describe a real-life situation where an equation involving your expression might be used.

Equivalent Expressions

Learning Goal

- reasoning that any algebraic expression can be represented in a variety of ways

Open Question

Materials
- algebra tiles (positive and negative *x*'s and 1s)

Questions to Ask Before Using the Open Question

◊ Show a pair of positive and negative 1 tiles. *What number am I showing?* (e.g., You could call it +1 + (–1) or you could call it 0.)
◊ *Why is the expression 2t – 5 equivalent to t + t – 5?* (e.g., if you put together two separate *t*'s, you get 2*t*.)
◊ *Why might you call that a simplified expression?* (e.g., since it has fewer terms)
◊ *Give an example of an expression you could simplify using the zero principle.* (e.g., 2 + (–4) is equivalent to (–2).)

Using the Open Question

Ensure that students are comfortable with using the zero principle when variable tiles are used, as modelled on the page.

Then make sure students understand that they must create a pair of equivalent expressions to fit each condition and explain the equivalence.

Encourage them to do more than three pairs of expressions, if there is time.

By viewing or listening to student responses, note if they:
- can apply the zero principle;
- recognize which terms can be combined.

Consolidating and Reflecting on the Open Question

◊ *What makes two expressions equivalent?* (e.g., If you can combine the terms of one of them to get the other one, they are equivalent.)
◊ *What kinds of terms can you combine?* (only ones that are numbers or ones with the same variable)
◊ *Does every expression have an equivalent one?* (Yes, e.g., even if the expression were just *m*, you could add some form of 0, like 5 + –5 and it would be equivalent.)

Solutions

e.g., $5t + 32 - 3t - 18 + 2m + 3n = 2t + 14 + 2m + 3n$; These are equivalent since, if you have 5*t* and take away 3 of them, there are only 2*t* left; also 32 – 18 = 14. You could not combine different variables or variables with numbers.

e.g., $5t + 32 - 3t - 18 + 4t - 8 = 6t + 6$; These are equivalent since if you have 5*t* and take away 3 of them, there are only 2*t* left. If you add 4*t* more, you are back up to 6*t*. Also 32 – 18 = 14 and 14 – 8 = 6. You could not combine variables with numbers.

e.g., $4m + 3n - 8 - 4m = 3n - 8$, These are equivalent since if you have *4m and* take them away, there are no *m*'s left. You could not combine variables with numbers.

Think Sheet

Materials
- algebra tiles (positive and negative x's , y's and 1s)

Questions to Ask Before Assigning the Think Sheet

◊ Show a pair of positive and negative 1 tiles. *What number am I showing?* (e.g., You could call it +1 + (–1) or you could call it 0.)
◊ *Why is the expression 2t – 5 equivalent to t + t – 5?* (e.g., If you put together two separate *t*'s, you get 2*t*.)
◊ *Why might you call that a simplified expression?* (e.g., It has fewer terms.)
◊ *Give an example of an expression you could simplify using the zero principle.* (e.g., 2 + (–4) is equivalent to (–2).)

Using the Think Sheet

Read through the introductory box with the students or clarify any questions they might have.

Assign the tasks.

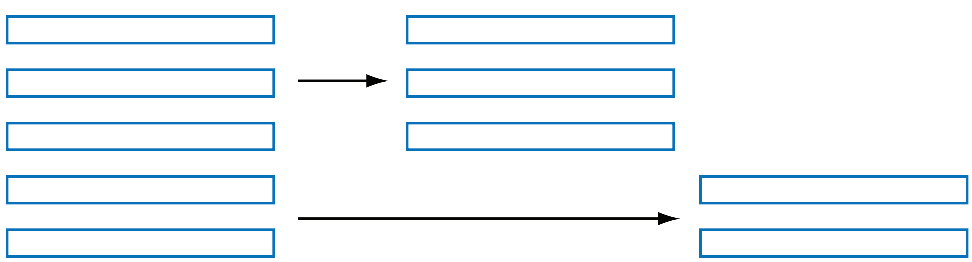
By viewing or listening to student responses, note if they can:
- use models of algebraic expressions to simplify them;
- use the zero principle with and without models to simplify algebraic expressions;
- create equivalent expressions by using multiplication and division as well as addition and subtraction;
- relate expressions to real situations.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

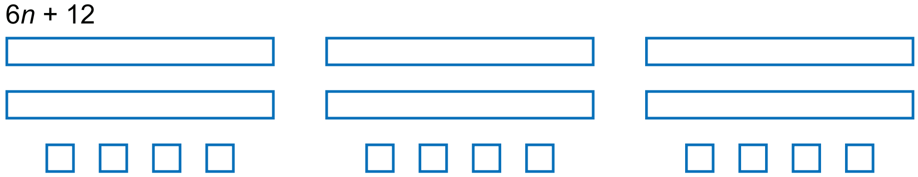
◊ *What makes two expressions equivalent?* (e.g., If you can combine the terms of one of them to get the other one, they are equivalent.)
◊ *How do models help you determine equivalent expressions?* (e.g., You can either use the zero principle to take away opposites or you can just combine tiles that are alike.)
◊ *What kinds of terms can you combine?* (only ones that are numbers or ones with the same variable)
◊ *Does every expression have an equivalent one?* (Yes, e.g., even if the expression were just m, you could add some form of 0, like 5 + –5 and it would be equivalent.)

Solutions

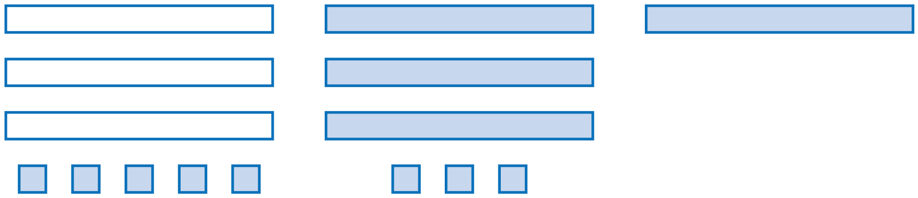
1.



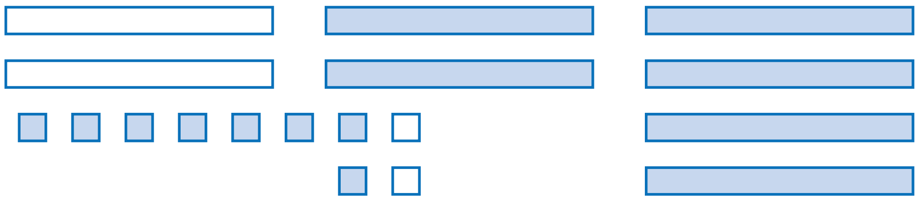
2. a) 6*n* + 12



b) $-n - 8$



c) $-4n - 6$



d) usually fewer tiles since I used the zero principle to get rid of some tiles.

3. **a)** e.g., $2s - 3s + 5 + 3$
b) e.g., $3s - s - 6y + 2y + 3$
c) e.g., $-9 - 1 - 2x + x$

4. **a)** Since there are 10 columns and the numbers go up one at a time, you are ten higher when you get back to the same column.
b) e.g., $s + (s + 10) + (s + 20)$
c) e.g., $3s + 30$
d) e.g., $s + s + s + 30$
e) e.g., $s + (s - 1) + (s + 1)$ or $3s$

5. e.g., $2l + 2w$ or $l + l + w + w$

6. **a)** e.g., If you take $\frac{1}{5}$ of n, you are dividing by 5 so if you also multiply by 5, it is like not doing anything since they are opposite operations.
b) e.g., $n + 2 - 2$ since, if you add 2 and then take it away, it is like not doing anything since they are opposite operations; $\frac{3 \times n}{3}$ since you are dividing by 3 so, if you also multiply by 3, it is like not doing anything since they are opposite operations.

7. $n + (n + 3) + (n - 2)$ and $3n + 1$

8. e.g., $n + (n + 3) + n - m$ and $3n + 3 - m$

Open Question

Equivalent Expressions

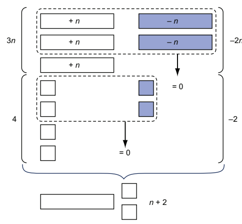
Learning Goal

- reasoning that any algebraic expression can be represented in a variety of ways

Open Question

Gemma wants to find an equivalent expression to $3n + 4 + (-2n) + (-2)$.

She used the zero principle [that $(+1) + (-1) = 0$ and $(+n) + (-n) = 0$] to figure that out.



Similarly, the expression $3m - 2 + 4t - 5 - 3t$ is equivalent to the expression $3m + t - 7$.

- Create at least three different algebraic expressions and their equivalent expressions to meet each of these conditions:
  - One expression has 6 terms (6 separate parts) and the equivalent one has 4 terms.
  - One expression has 6 terms and the equivalent one has 2 terms.
  - One expression involves two variables, but the equivalent expression only involves one variable. You might need to create your own "model" for the second variable.

Explain why each of your pairs of expressions is equivalent.

Think Sheet

Equivalent Expressions (Continued)

Think Sheet

Just like 6 x 2 is a quicker way to write 2 + 2 + 2 + 2 + 2 + 2, there are sometimes quicker ways to write algebraic expressions. The two ways to write the expressions are **equivalent**; they mean the same thing.

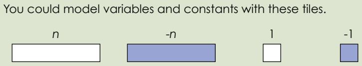
For example,

$n + 1 + 1 + 1$ is equivalent to $n + 3$.

$p + p$ is equivalent to $2p$.

- You can use algebra tiles to help you write simpler equivalent expressions.

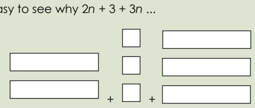
You could model variables and constants with these tiles.



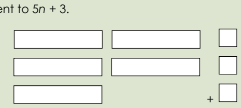
For example :

Expression	n	$n + 2$	$2n - 3$	$-3n + 1$
Model				

Then it is easy to see why $2n + 3 + 3n$ ...



is equivalent to $5n + 3$.

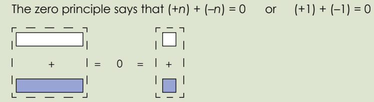


Even though the letter n is used here, the tile could be used to represent any variable.

Equivalent Expressions (Continued)

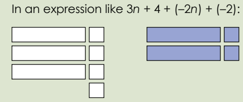
- Sometimes there are positive and negative tiles and you can use the **zero principle** to simplify.

The zero principle says that $(+n) + (-n) = 0$ or $(+1) + (-1) = 0$

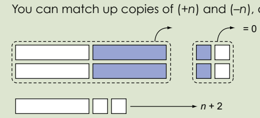


When you add or subtract zeroes, the amount does not change, so you can get rid of extra zeroes to simplify expressions.

In an expression like $3n + 4 + (-2n) + (-2)$:



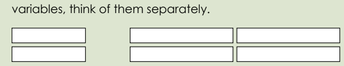
You can match up copies of $(+n)$ and $(-n)$, and $(+1)$ and (-1) to eliminate zeroes.



So an equivalent expression for $3n + 4 + (-2n) + (-2) = n + 2$.

Even without the models, we might have rearranged $3n + 4 + (-2n) + (-2)$ as $3n + (-2n) + 4 + (-2)$. Altogether there would be $1n$ (since $3 + (-2) = 1$) and +2 (since $4 + (-2) = +2$.

- You can combine identical variables, **like terms**, (e.g., $(-5t)$ and $(+4t)$ or $(+2n)$ and $(+3n)$ or numbers), but you can't mix them together. If there are two different variables, think of them separately.



1. Use a model to show why $5q = 3q + 2q$.

2. Model each. Then write an equivalent expression.
   a) $2n + 4 + 4n + 8$
   b) $3n + (-5) + (-4n) + (-3)$
   c) $2n - 8 + (-6n) + 2$
   d) Do your equivalent expression use more or fewer tiles than the original expressions? Why does that make sense?

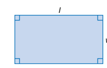
3. Write an equivalent expression for each, using two more terms. NOTE: If you use models, use different sizes for *s* and *y*.
   a) $-s + 8$
   b) $2s - 4y + 3$
   c) $-10 - x$

4. Choose a number on the hundreds chart and replace the value with the variable *s*.
   a) Why does the expression $s + 10$ describe the number directly below *s*?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

   b) Write the two numbers below *s* in terms of *s*.
   c) What algebraic expression would result from adding *s* to the two numbers directly below it?
   d) Write an equivalent expression for your answer to part c).
   e) Write two equivalent expressions for adding *s* to the two numbers on each side of that square.

5. Write two equivalent expressions for the perimeter of the rectangle.



6. a) Why does it make sense that $5 \times \frac{n}{5}$ is equivalent to *n*?
   b) Write several other expressions equivalent to *n* and tell why each one is equivalent.

Evaluating Algebraic Expressions

Learning Goal

- reasoning about how values of an algebraic expression will change when different values are substituted

Open Question

Questions to Ask Before Using the Open Question

◊ *What is the value of m + 1 when m = 1?* (2)
◊ *What about 2m + 1 – m?* (e.g., It has to be the same since it is equivalent.)
◊ *What other expression involving m could you create that has the value of 2 when m = 1?* (e.g., 2*m*)

Using the Open Question

Make sure students understand that they must create expressions involving the variable *m* and substitute 4 for the value of *m* in each one. The resulting value must be –2.

Encourage them to create as many expressions as possible, ideally using different operations.

By viewing or listening to student responses, note if they:
- substitute the same value for each instance of *m*;
- correctly use order of operations to determine the value of an expression after substitution;
- recognize the relationships between at least some of their expressions.

Consolidating and Reflecting on the Open Question

◊ *Why do you use –14 with 3m?* (e.g., 3*m* is 12 and you want to get down to –2, so you have to subtract 14.)
◊ *Why do you subtract more when you use 4m?* (e.g., 4*m* is higher than 3*m* but you have to end up at the same place, –2, so you have to take away more.)
◊ *Can you use all four operations to create an expression?* (Yes, e.g., (2*m* + 8) ÷ (2 – 10)) *Where was the multiplication?* (when you multiplied *m* by 2)

Solutions

e.g.,
$3m - 14$
$5m - 18 - m$
$5m - 22$
$6m - 26$
$6m - 30 + m$
$-m + 2$
$2m^2 - 34$
$\frac{m}{(-2)}$

Think Sheet

Questions to Ask Before Assigning the Think Sheet
◊ *What is the value of m + 1 when m = 1?* (2) ◊ *What about 2m + 1 – m?* (e.g., It has to be the same since it is equivalent.) ◊ *What other expression involving m could you create that has the value of 2 when m = 1?* (e.g., 2*m*)
Using the Think Sheet
Read through the introductory box with the students or clarify any questions they might have. Assign the tasks. By viewing or listening to student responses, note if they can: • substitute and evaluate correctly; • predict how the values of two related expressions will compare; • predict how the values of an expression for two different values of the variable will compare; • predict characteristics of the values of an expression when different values are substituted.
Consolidating and Reflecting: Questions to Ask After Using the Think Sheet
◊ *If you substitute –2 into 3m + 1 will the value be positive or negative? Why?* (negative, since 3*m* is less than –1) ◊ *Suppose you were substituting into the expression 5j + 10. How do you know the numbers will get bigger when j gets bigger?* (If you add a positive number 5 times, you get more if you are adding a bigger number, so the 5*j* part grows. The 10 stays the same, but altogether, there is more.) ◊ *How do you know the numbers will all be multiples of 5?* (e.g., 5*j* is a multiple of 5 and so is 10, so if you put together lots of groups of 5, you have a multiple of 5.)

Solutions

1. **a)** –9
b) 7
c) 36
d) 68
e) 9
f) $\frac{1}{5}$

2. **a)** –4, –1, +2, +5, +8
b) 6, 9, 12, 15, 18
c) –8, +2, +12, +22, + 32
d) 9, 14, 19, 24, 29
e) –2, –3, –4, –5, –6
f) 4, 1, –2, –5, –8

3. **a)** e.g., If *n* = 1, 3*n* = 3.
If *n* = 2, 3*n* = 6.
If *n* = 3, 3*n* = 9
b) e.g., They are all in the three times table.
c) e.g., 3*n* means multiply a number by 3, so it will always be a multiple of 3.

4. **a)** if $m = 8$. If you add $3m$, you want m bigger. ($28 > –20$)
   b) if $t = 1$. If you subtract $8t$, you want t smaller. ($22 > –50$)
   c) if $t = 3$. If t is –3, you will be adding a negative to –9. If t is +3, you will be adding a positive to –9, so it will be greater. ($3 > –21$)

5. **a)** e.g., If m increases, then $4m$ increases and $4m – 2$ increases. Since $4m – 2$ is positive when m is 1, it has to be positive when m is 10.
   b) e.g., 4 times a number is even and so is 2 and if you subtract an even from an even, you get an even
   c) e.g., If m is little, $6m$ is not that big either, so subtracting 200 will get into the negative numbers.
   d) e.g., If m is big, $6m$ is big and if you take a really big number (more than 200) from 200, you are into the negative numbers

6. **a)** e.g., $2p$
   b) e.g., $2p$
   c) e.g., $200 + p$

Open Question

Equivalent Expressions (Continued)

7. Use equivalent representations to describe this sum: A number is added to three more than it and two less than it.

8. Write an algebraic expression with 5 terms that is equivalent to an algebraic expression with 3 terms.

Think Sheet

Evaluating Algebraic Expressions

Learning Goal

- reasoning about how values of an algebraic expression will change when different values are substituted

Open Question

An algebraic expression involving the variable m has the value –2 when m = +4.

One example is m – 6 since 4 – 6 = –2.

- What else could the algebraic expression be?

- List as many possibilities as you can think of, including some where the variable m appears more than once in the expression. You may want to use different operations in the different expressions.

Evaluating Algebraic Expressions (Continued)

Think Sheet

When an expression involves a variable, you can **substitute** values for the variable to **evaluate** the expression.

To evaluate $2m$ when m = 4, substitiute 4 for m, and calculate 2 x 4 = 8.

- If the same variable appears more than once in an equation, or expression, you must use the same value in each of those places.

  For example, to evaluate $3p – 2 + p$ when p = –1, calculate 3 x (–1) – 2 + (–1).

- Equivalent expressions always have the same value when the same substitution is made.

  For example, $3n + 2 = n + 1 + 2n + 1$. If n = 5, it is true that 3 x 5 + 2 = 5 + 1 + 2 x 5 + 1.

- Non-equivalent expressions might have the same value when the same substitution is made or might not.

  For example, $25 – n = 4n$, but only when n = 5 and not for other values of n.

- If an expression involves more than one variable, these variables can be substituted with either different values or the same values.

For example:

Substitute p = 6 and s = 3 into
$5p – 7s$
= 5 x 6 – 7 x 3
= 30 – 21
= 9

Substitute p = 4 and s = 4 into
$5p – 7s$
= 5 x 4 – 7 x 4
= 20 – 28
= (–8)

When evaluating expressions, the normal order of operations (BEDMAS) rules apply. For example:

Substitute m = 6 into
$3 + 2m$
= 3 + 2 × 6
= 15

1. Evaluate each expression.
   a) $3 – 4m$, when m = 3
   b) $15 + 8m$, when m = –1
   c) $j + 2j^2$, when j = 4

Evaluating Algebraic Expressions (Continued)

d) $j + (2j)^2$, when $j = 4$

e) $15 – 3p$, when $p = 2$

f) $\frac{3n + 2}{10 – n}$, when $n = 0$

2. Substitute the values of $m = 0$, then 1, then 2, then 3, then 4 into each expression.

	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$
a) $3m – 4$					
b) $3m + 6$					
c) $10m – 8$					
d) $5m + 9$					
e) $–m – 2$					
f) $–3m + 4$					

3. a) Evaluate the expression $3n$ for several different whole number values of n.

b) What is true each time?

c) How could you have predicted that?

4. Predict which value will be greater. Then test your prediction.

a) $4 + 3m$ if $m = –8$ OR if $m = 8$

b) $30 – 8t$ if $t = 1$ OR if $t = 10$

c) $4t – t^2$ if $t = –3$ OR if $t = 3$

Relating Pattern Rules to Expressions and Equations

Learning Goal

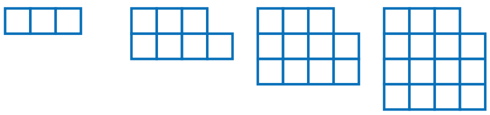
- representing certain algebraic expressions and equations using linear patterns

Open Question

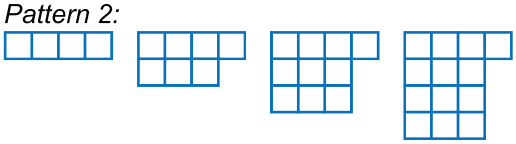
Materials
- square tiles or linking cubes

Questions to Ask Before Using the Open Question

◊ *Model the two patterns shown with square tiles or linking cubes.*
Pattern 1:



Pattern 2:



◊ *How is it easy to tell that the numbers go up by 4 in the first pattern but by 3 in the second one?* (e.g., Because there is an extra row of 4 each time in the first pattern, but an extra row of 3 each time in the second one.)

Using the Open Question

◊ *Provide square tiles or linking cubes to allow students to build the model shown on the page. Help them notice that it makes sense if a rule is 6f – 4 that if f is one more there is an extra 6 squares. What would a pattern for 5f – 4 look like?* (e.g., the same except there are rows of 5 instead of 6)

Make sure students understand that they must build the required pattern models in such a way that the pattern rule is as obvious to see as possible.

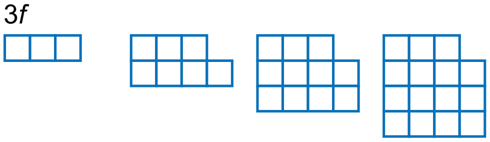
By viewing or listening to student responses, note if they can:
- create a number of patterns where 30 is a term;
- build models to make the change from one term to the next very easy to see
- use an equation to find out where a number is in a pattern

Consolidating and Reflecting on the Open Question

◊ *How could you make it clear that a pattern goes up by 4 each time?* (e.g., arrange the pattern in rows of 4 with whatever adjustments you need to in the first row.) *What do you know about the rule for that pattern?* (e.g., It will have a 4*f* in it.)
◊ *How would the models for the patterns for 4n + 3 and 3n + 4 be alike and different?* (e.g., One would have rows of 4 with 3 extra in one row and the other with have rows of 3 with 4 extra in one row; they both have rows and they both have 7 in the first row.)
◊ *How could you figure out where the number 82 is in the pattern with the rule 2n + 30?* (Solve 2*n* + 30 = 82.)

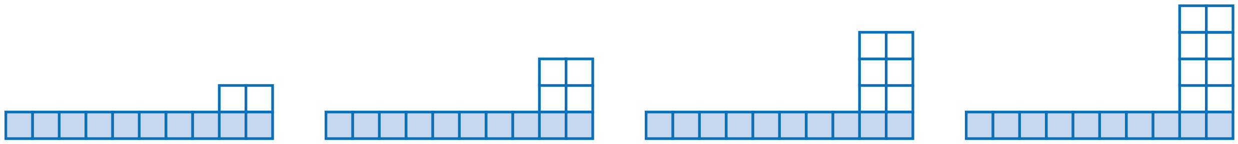
Solutions

e.g.,
3*f*



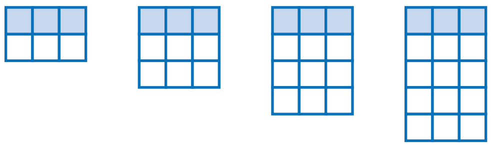
3, 6, 9, 12
3*f* = 30

$2f + 10$



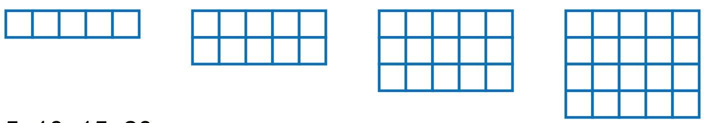
12, 14, 16, 18
$2f + 10 = 30$

$3f + 3$



6, 9, 12, 15
$3f + 3 = 30$

$5f$



5, 10, 15, 20
$5f = 30$

Think Sheet

Materials
- square tiles or linking cubes

Questions to Ask Before Assigning the Think Sheet

◊ *How would I continue the pattern 5, 8, 11, ...?* (e.g., 14, 17, …) *Why?* (It goes up by 3s.)
◊ *Build the pattern for 3m + 2 for four terms by creating rows of 3 with 2 extra tiles on the first row. How does the arrangement show that the pattern goes up by 3?* (e.g., There is an extra row of 3 each time.) *Why does the pattern rule 3n + 2 make sense?* (e.g., Because there are 5, then 8, and then 11 squares—they go up by 3s.)

Using the Think Sheet

Read through the introductory box with the students or clarify any questions they might have.

Assign the tasks.

By viewing or listening to student responses, note if they can:
- use a pattern model to determine the related numerical pattern;
- reorganize a pattern model to demonstrate two different equivalent pattern rules;
- predict a pattern rule based on how certain patterns grow;
- use equations to determine where a number is in a linear growing pattern.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How could you make it clear that a pattern goes up by 4 each time?* (e.g., Arrange the pattern in rows of 4 with whatever adjustments you need to in the first row.) *What do you know about the rule for that pattern?* (e.g., It will have a $4f$ in it.)
◊ *How could you arrange a model to show why 2n + 5 and n + (n + 5) are equivalent pattern rules?* (e.g., You could colour the squares differently – one would have one colour for double the figure number and another colour for 5 squares; and the second would have one colour for the figure number and another colour for the rest of the squares.)
◊ *How could you figure out where the number 82 is in the pattern with the rule 2n + 30?* (Solve $2n + 30 = 82$.)

Solutions

1. a) With $3n + 1$, you get bigger by 3 each time like you should and the first number is 4, like it is.

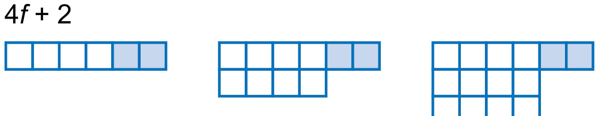
   b) With 2n + 4, you get bigger by 2 each time like you should and the first number is 6, like it is.

2. a) The 1 is the blue square and the $2f$ are the f columns of 2.

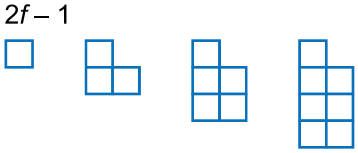
   b) The bottom row is the f and the top row is the $f + 1$.

   c) e.g., $f + f + 1 + 3$ or $f + 2 + f + 2$

3. a) $4f + 2$



   b) $2f – 1$



4. **a)** $2p$
b) $5p – 1$
c) $4p + 24$
d) $204 – 2p$
e) $64 – 3p$

5. e.g., They both involve a $5n$, but one has addition and one has a subtraction.

6. **a)** e.g., 8, 16, 24, 32, … or 9, 17, 25, 33, …
b) The numbers would go up by 8 or down by 8.

7. **a)** $2n = 100$
b) $3n + 1 = 100$
c) $4n + 8 = 100$
d) $122 – n = 100$
e) $310 – 10n = 100$

8. **a)** 11, 14, 17, 20, 23
b) e.g., It does make sense since you get to 35 in four more terms.

9. e.g., $20n – 12$
a) 8, 28, 48, 68, …
b) e.g., $20n – 12 = 208$.

Open Question

Evaluating Algebraic Expressions (Continued)

5. How could you predict that each is true even before you substitute?

a) If $m = 10$, then $4m - 2$ has to be positive.

b) If $m = 10$, then $4m - 2$ has to be even.

c) $6m - 200$ is negative for small values of m

d) $200 - 6m$ is negative for large values of m

6. For each part, create an expression involving p to meet the condition.

a) It is even when p is 4.

b) It is a multiple of 10 when $p = 5$.

c) It is greater than 100 when $p = -4$.

Think Sheet

Relating Pattern Rules to Expressions and Equations

Learning Goal

- representing certain algebraic expressions and equations using linear patterns

Open Question

The pattern 2, 8, 14, 20, …. can be modelled as shown below. Notice that the numbers go up by 6 and so there is an extra row of 6 each time.

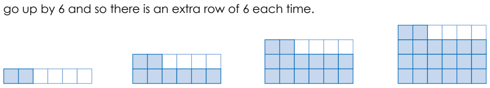


Figure 1 Figure 2 Figure 3 Figure 4

– The pattern rule for all the squares shown (including the faint ones) is $6f$ since if f is the figure number, there are f rows of 6.

– The pattern rule for the dark squares is $6f - 4$ since there are 4 squares not counted each time.

- Create at least four other algebraic expressions that could be pattern rules. Each pattern should include the number 30 somewhere.

- Show the first four terms of the pattern with pictures and then show the first four terms using numbers. Try to arrange your pictures to make the pattern rule easy to see.

- Write an equation that would help you figure out where the number 30 is in each pattern.

Relating Pattern Rules to Expressions and Equations (Continued)

Think Sheet

You can think of an algebraic expression as a pattern rule. For example, $p + 2$ is the rule for this table of values or the associated picture.

Position	Value
1	3
2	4
3	5
4	6

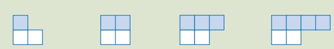


Figure 1 Figure 2 Figure 3 Figure 4

Notice that in the picture, the number of squares is always 2 more than the Figure number. The number of white tiles is the Figure number. The number of dark tiles is the 2.

To model the algebraic expression $3p + 2$, you can use this table of values which relates p to $3p + 2$ or you can use the shape pattern shown.

Position	Value
1	5
2	8
3	11
4	14
5	17

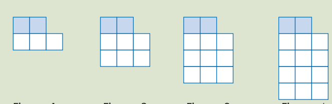


Figure 1 Figure 2 Figure 3 Figure 4

Notice that the way the pattern was coloured made it easier to see why the coefficient of p was 3. There were 3 times as many squares as the figure number.

It was easy to see the + 2 of $3p + 2$ using the 2 shaded tiles.

Sometimes you can focus on the columns and other times you can focus on the rows to help figure out the rule.

The pattern below has the rule $2f + 1$ since there are twice as many white squares as the figure number and 1 extra shaded square.

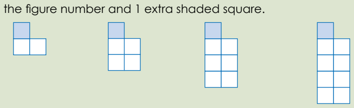


Figure 1 Figure 2 Figure 3 Figure 4

Relating Pattern Rules to Expressions and Equations (Continued)

If you colour the figures differently, you might see an equivalent expression 2(*f* + 1) – 1. The –1 is shown by fading out one square. The 2(*f* + 1) is shown by having two columns, each with *f* + 1 squares in it.

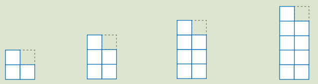


Figure 1 Figure 2 Figure 3 Figure 4

You can think of an equation as a way of asking which figure in a pattern has a certain number of squares.

For example, 2f + 1 = 31 is solved by figuring out which Figure in the last pattern shown has exactly 31 squares.

1. Tell why the pattern rule describes each pattern.

   a)

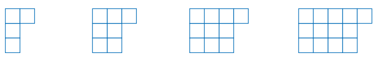


Figure 1 Figure 2 Figure 3 Figure 4

Pattern rule 3*n* + 1

   b)

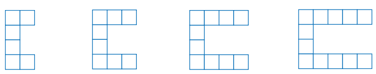


Figure 1 Figure 2 Figure 3 Figure 4

Pattern rule: 2*n* + 4

Relating Pattern Rules to Expressions and Equations (Continued)

c) Create two equivalent rules for this pattern.

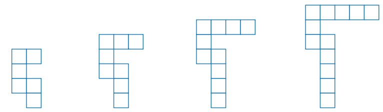


Figure 1 Figure 2 Figure 3 Figure 4

2. The pattern below can be described by the equivalent rules 2*f* + 1 or *f* + (*f* + 1).

Figure 1 Figure 2 Figure 3 Figure 4

   a) How does this way of colouring help you see the pattern rule 2*f* + 1?

Figure 1 Figure 2 Figure 3 Figure 4

   b) How does this way of colouring help you see the pattern rule *f* + (*f* + 1)?

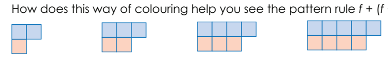


Figure 1 Figure 2 Figure 3 Figure 4

Relating Pattern Rules to Expressions and Equations (Continued)

3. Draw the first four figures of the pattern below. Shade or arrange squares in your figures to help make the rule as obvious as you can.
   a) Pattern: 4*f* + 2
   b) Pattern: 2*f* – 1
4. Describe a pattern rule for each pattern using the variable *p*. The rule should tell what the value in the pattern is based on its position *p* in the pattern.
   a) 2, 4, 6, 8, …
   b) 4, 9, 14, 19, 24, 29, …
   c) 28, 32, 36, 40, 44, …
   d) 202, 200, 198, 196, …
   e) 61, 58, 55, 52, …
5. How are the pattern rules for these patterns alike? How are they different?

   Pattern 1: 6, 11, 16, 21, 26, 31, …

   Pattern 2: 200, 195, 190, 185, 180, …

Relating Pattern Rules to Expressions and Equations (Continued)

6. A pattern rule includes the term 8*n*.
   a) List two possible patterns.
   b) What would have to be true about any pattern you listed?
7. What equation would you use to find out the position in the pattern of the number 100?
   a) 2, 4, 6, 8, …
   b) 4, 7,10, …
   c) 12, 16, 20, 24, …
   d) 121, 120, 119,118, …
   e) 300, 290, 280, …
8. a) Describe the first five terms of the pattern you might be thinking of when you write the equation 3*n* + 8 = 35.
   b) Check if the equation actually does make sense for your pattern.
9. Create your own algebraic expression.
   a) List the first few terms of the pattern for which it is a pattern rule.
   b) Create an equation that would make sense for your pattern rule.