

# GAP CLOSING

## Powers and Roots

Intermediate / Senior  
Student Book



# Powers and Roots

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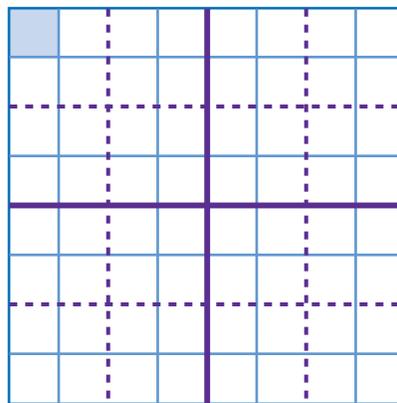
## Diagnostic

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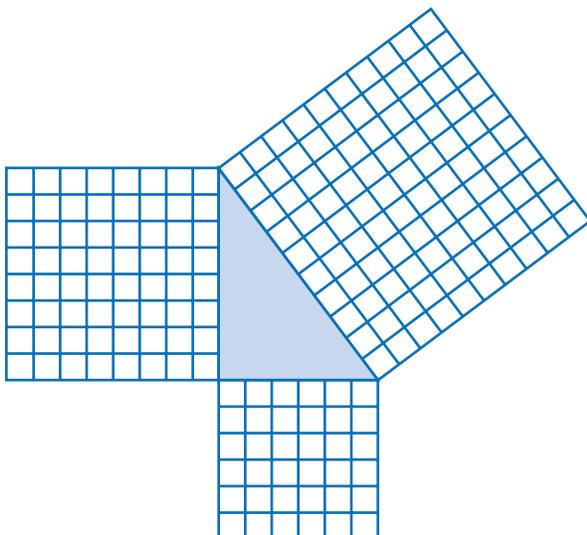
1. A perfect square is the result of multiplying a whole number by itself. For example, 64 is a perfect square since it's  $8 \times 8$ .  
List three perfect squares between 1000 and 2000.
2. The square root of a number is what you multiply by itself to get the number. For example, 8 is the square root of 64 since  $8 \times 8 = 64$ . Estimate each square root. Do NOT use a calculator.  
a)  $\sqrt{250}$                       b)  $\sqrt{88}$                       c)  $\sqrt{622}$
3. What picture would you draw to show why  $\sqrt{16}$  is 4?
4. Why does it make sense that  $\sqrt{250\,000}$  is 100 times as much as  $\sqrt{25}$ ?
5. Which does  $5^3$  mean?  
A:  $5 \times 5 \times 5$   
B:  $3 \times 3 \times 3 \times 3 \times 3$   
C:  $5 \times 3 \times 5 \times 3$   
D:  $5 \times 3$
6. What is the value of each power?  
a)  $3^4$                       b)  $4^3$                       c)  $10^3$                       d)  $(-2)^5$                       e)  $(0.2)^2$

7. Without referring to their actual values, why does it make sense that  $3^5$  might be more than  $5^3$ ?

8. Use the thicker line and dotted line divisions to help you use a power to represent the number of dark squares that could be fit into the largest square. Explain your thinking.



9. How can you use information about the squares in the picture to tell you that this triangle is a right triangle? Do NOT use a protractor.



10. The two shortest sides of a right triangle are given. Determine the length of the longest side without measuring.

a) 4 cm and 5 cm

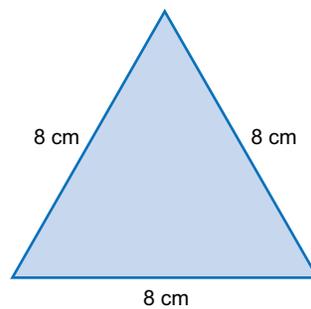
b) 5 cm and 12 cm

11. The longest side of a right triangle is 10 cm. One leg length is given. Determine the other leg length without measuring.

a) leg is 3 cm

b) leg is 5 cm

12. Determine the height of the triangle without measuring.



# Perfect Squares and Square Roots

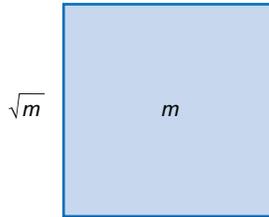
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## Learning Goal

- relating numerical and geometric descriptions of squares and square roots.

### Open Question

If we draw a square and the area has the value  $m$ , then the side length is called the **square root** of  $m$ . We write it  $\sqrt{m}$ . If  $\sqrt{m}$  is an integer, then  $m$  is called a **perfect square**.



- Choose eight numbers, all between 100 and 300 using the rules that follow. (Think of each number as the area of a square.)
  - Three numbers are perfect squares and five are not perfect squares.
  - At least one of the numbers is the double of another number.
  - At least one number has many factors and at least one number does not have many factors.

## Perfect Squares and Square Roots

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- Choose four numbers between 0 and 1 using the rules that follow. (Think of each number as the area of a square.)
  - Make sure that two values are close to 0.
  - Make sure that two values are close to 1.
  - At least one should be a fraction and at least one should be a decimal.
  - One should be the square of a fraction or decimal.
  
- For both groups of values:
  - For each number that is the square of another whole number, fraction, or decimal, tell the value of the square root.
  - For each other number, estimate the value of the square root and explain your estimate.

Think Sheet

A **perfect square** is the product of two identical whole numbers.

For example, 16 is a perfect square since  $4 \times 4 = 16$ . We can say **4 squared** is 16.

Other perfect squares are listed below:

1  $1 \times 1$

4  $2 \times 2$

9  $3 \times 3$

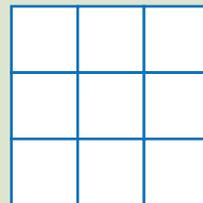
25  $5 \times 5$

36  $6 \times 6$

...

We can model perfect squares as square shapes.

For example,  $3 \times 3$  looks like this:

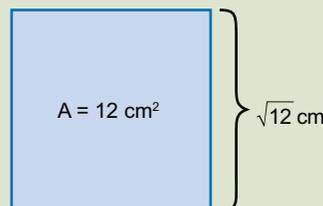


Notice that perfect squares get farther and farther apart. For example, 1 and 4 are only three apart, but 4 and 9 are five apart and 9 and 16 are seven apart.

The **square root** of a number is the number that we multiply by itself to result in that original number. The square roots of perfect squares are whole numbers. For example, the square root of 16 is 4.

Square roots do not have to be whole numbers. We write *the square root of* using the symbol  $\sqrt{\quad}$ . The square root of 8 is written as  $\sqrt{8}$ . It is between 2 and 3 since  $2 \times 2 = 4$  and  $3 \times 3 = 9$  and 8 is between 4 and 9.

One way to model a square root is to think of it as the side length of a square with a given area. For example, to show  $\sqrt{12}$ , think of a square with area 12. The square root of 12 is the side length.



We can use the  $\sqrt{\quad}$  button on a calculator to get the value of a square root.

- To estimate the square root of a number, we could start by relating the square to known perfect squares.

For example, since 125 is between 121 ( $11 \times 11$ ) and 144 ( $12 \times 12$ ),  $\sqrt{125}$  is between 11 and 12. It is probably closer to 11 since 125 is closer to 121 than to 144. It helps to know some perfect squares as shown in the table.

If we use a calculator, we learn that  $\sqrt{125}$  is about 11.18.

- We use the facts  $10 \times 10 = 100$  and  $100 \times 100 = 10\,000$  to help estimate square roots of larger numbers. For example, since  $\sqrt{15}$  is close to 4, then  $\sqrt{1500}$  is close to 40 and  $\sqrt{150\,000}$  is close to 400.

Number	Square Root
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
10 000	100
1 000 000	1000

1. a) List all of the perfect squares between 200 and 300.

b) Explain how you know you have all of them.

2. Explain how you know that 250 cannot be a perfect square.

3. a) Which of these powers of 10 are perfect squares? Explain.

100      1000      10 000      100 000

- b) Why are they not all perfect squares?

4. a) You want to multiply  $2 \times 2 \times 5 \times 3$  by a number to make a perfect square. List three possible amounts you could multiply by and prove that each is a perfect square.

- b) When you factor those perfect squares down to primes, what do you notice?

- c) What is the least number you could multiply  $2 \times 2 \times 4 \times 5$  by to make a perfect square? Explain.

5. What is the side length of each of three square gardens? (The areas are given.) How do you know?

a)  $64 \text{ m}^2$

b)  $144 \text{ m}^2$

c)  $200 \text{ m}^2$

6. The square root of a number is closer to 7 than to 8. What might the number be? How do you know?

7. Estimate each square root without using a calculator. Explain your strategy.

a)  $\sqrt{30}$

b)  $\sqrt{300}$

c)  $\sqrt{3000}$



# Powers

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## Learning Goal

- recognizing the efficiency of representing repeated multiplication using powers.

### Open Question

The way to shorten a repeated multiplication is to use a power.

For example,  $2^4$  means  $2 \times 2 \times 2 \times 2$ ;

2 is multiplied by itself 4 times. It is a power since it is the product of a number multiplied by itself.

The 2 is the **base**. The 4 is the **exponent** and  $2^4$  is the **power**.



The diagram shows the expression  $2^4$ . A red arrow points from the word "base" to the number 2. Another red arrow points from the word "exponent" to the number 4.

- Use the digits 1, 2, 3, 4, and 5 as bases and exponents. Also use the decimal 0.5 and the integer  $-2$  as a base. List all the powers you can and calculate their values.

- Tell what you notice about the powers.

## Think Sheet

Multiplication is a short way to record repeated addition. For example, it is quicker to write  $4 \times 5$  than  $5 + 5 + 5 + 5$ .

The way to shorten a repeated multiplication is to use a power.

For example,  $2^4$  means  $2 \times 2 \times 2 \times 2$ ;

2 is multiplied by itself 4 times.

The 2 is the **base**. The 4 is the **exponent** and  $2^4$  is the **power**.

base  $\longrightarrow$   $2^4$   $\longleftarrow$  exponent

- There are special names if the exponent is 2 or 3. For example,  $3^2$  is read *three squared*. If the exponent is 3, we use the word *cubed*; e.g.,  $5^3$  is read *five cubed*.

Otherwise, we use **ordinal words**, e.g., we read  $6^5$  as *six to the fifth (power)*.

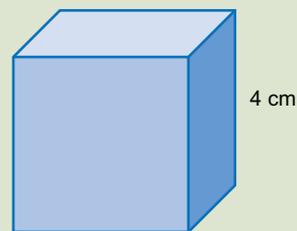
- We relate a square (or a number to the second power) to the area of a square. That helps us understand why we use the unit  $\text{cm}^2$  or  $\text{m}^2$  for area.

$9 (3^2) \text{ cm}^2$  is the area of this square.



We relate a cube (or a number to the third power) to the volume of a cube. That helps explain why we use the unit  $\text{cm}^3$  or  $\text{m}^3$  for volume.

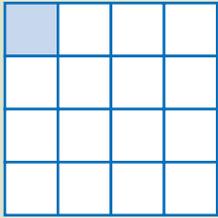
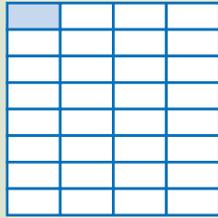
$64 (4^3) \text{ cm}^3$  is the volume of this cube.



- Powers of whole numbers grow very quickly. For example,  
 $3^4 = 81$ , but  
 $3^5 = 243$  and  
 $3^6 = 729$ .

- Powers of fractions or decimals less than 1 shrink as the exponent increases. For example,

$$\left(\frac{1}{2}\right)^4 = \frac{1}{16}, \quad \text{but} \quad \left(\frac{1}{2}\right)^5 = \frac{1}{32}.$$

 $\frac{1}{16}$  $\frac{1}{32}$ 

- Powers of negative numbers can be positive. If the exponent is even, the power is positive. If it is odd, the power is negative.

For example,  $(-3)^3 = -27$ , but  $(-3)^4 = +81$ .

1. Write each power as a multiplication.

**a)**  $3^4$

**b)**  $3^6$

**c)**  $4^6$

**d)**  $2^{10}$

**e)**  $(-3)^4$

2. Draw a picture that shows the meaning of each power.

**a)**  $5^3$

**b)**  $8^2$

3. A large box holds 4 small ones.

Each small box holds 4 smaller boxes.

Each of the smaller boxes holds 4 tiny boxes.

Use a power to tell how many tiny boxes would fit in the large box.

Explain your thinking.

4. Tell a story (as in Question 3) that might describe  $6^4$ .

5.  $(-2)^{\square}$  is a positive number. What could  $\square$  be?

6. a) Order from least to greatest:

$$\left(\frac{1}{2}\right)^3 \quad \left(\frac{1}{2}\right)^4 \quad \left(\frac{1}{2}\right)^2 \quad \left(\frac{1}{2}\right)^5$$

- b) Order from least to greatest:

$$2^3 \quad 2^4 \quad 2^2 \quad 2^5$$

- c) What do you notice if you compare the answers to parts a) and b)?

7. A certain power has a base that is one less than its exponent. The value of the power is about 1000. What could the power be?
8. Choose values to make these statements true:
- a)  $\square^4 = 9^2$                       b)  $6^8 = 36\square$                       c)  $25^3 = \square^6$
9. Which value is greater each time? How do you know?
- a)  $2^3$  or  $2^5$                       b)  $10^3$  or  $9^3$                       c)  $10^3$  or  $100^2$
10. To replace the boxes below, you can repeat numbers or use different numbers. Tell what the numbers might be if:
- a)  $\square^3 < \square^2$
- b)  $\square^3$  is at least 100 more than  $\square^2$
- c)  $\square^3$  is less than  $\frac{1}{5}$  of  $\square^2$ .
11. Explain why  $20^5$  is 400 times as much as  $20^3$ .
12. How would you write each of these as a single power?
- a)  $3 \times 3 \times 5 \times 5$                       b)  $6 \times 6 \times 3 \times 3 \times 6 \times 3$
- c)  $2 \times 2 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4$
13. Why is it useful to write a number as a power?



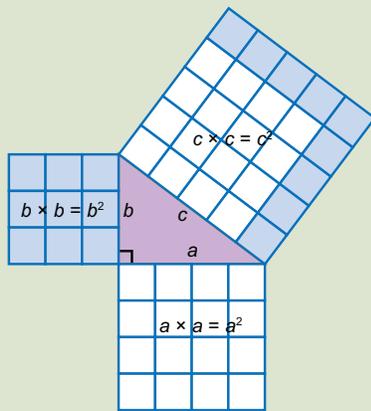
Think Sheet

Right triangles are special.

When we know two of the side lengths and which two lengths they are, we automatically know the third one. [This is not true for just any triangle.] It is true in a right triangle because the total area of the two squares we can build on the smaller sides (the **legs**) of a right triangle is the same as the area of the square we can build on the longest side, the **hypotenuse** (the side across from the right angle).

Since the area of the square is the square of the

side length each time, we write  $a^2 + b^2 = c^2$  if  $a$ ,  $b$  and  $c$  are the side lengths of the triangles, and  $c$  is the hypotenuse.



This is called the **Pythagorean theorem**.

If we know a triangle is a right triangle, we can use the equation above to figure out the third side. For example, if the hypotenuse is 12 units and one leg is 4 units, then

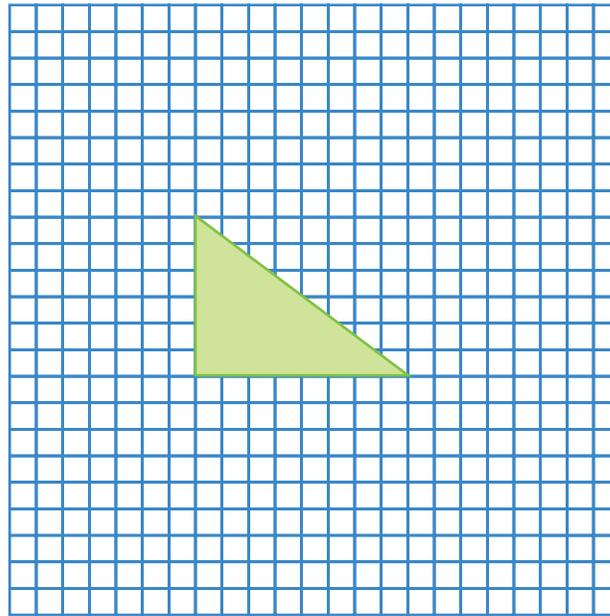
$$a^2 = 12^2 - 4^2 = 128; \text{ that means } a = \sqrt{128}, \text{ or about } 11.3 \text{ units.}$$

- Since this relationship is only true for right triangles, it is a way to test whether a triangle is a right triangle without drawing it.

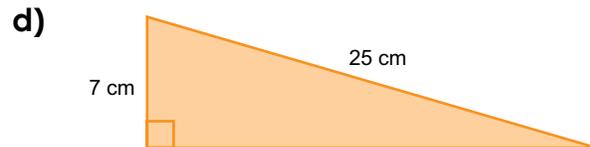
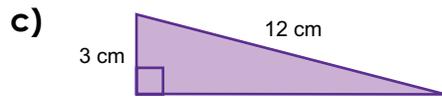
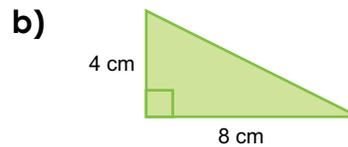
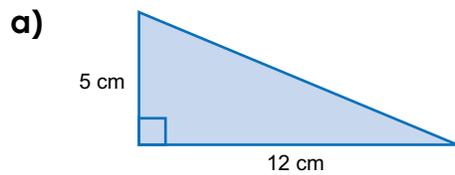
For example, if a triangle has side lengths: 9, 12 and 15; it is a right triangle, since  $9^2 + 12^2 = 225 = 15^2$ .

If a triangle has side lengths: 5, 7, and 9, it is not a right triangle since  $5^2 + 7^2 = 74$  and not  $9^2$ .

1. Draw a picture to show the squares on each side length of this right triangle. Tell how the Pythagorean theorem is shown.



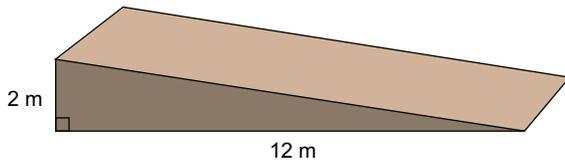
2. For each right triangle, calculate the missing side length.



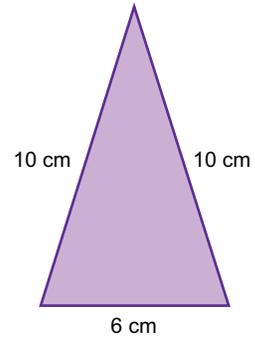
3. Decide whether these are the side lengths of a right triangle. Explain your thinking for part c)

- a)** 5 cm, 8 cm, 11 cm
- b)** 24 cm, 32 cm, 40 cm
- c)** 10 cm, 24 cm, 26 cm

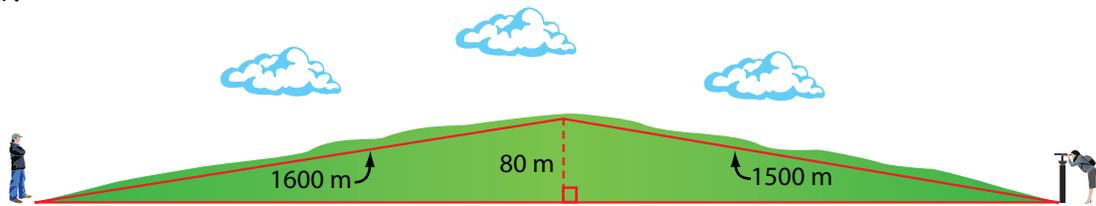
4. A ramp rises 2 metres from a point 12 metres away. How long is the ramp?



5. Determine the height of this triangle without measuring.

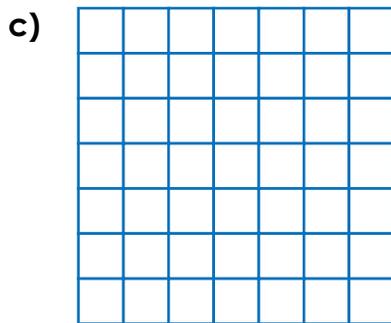
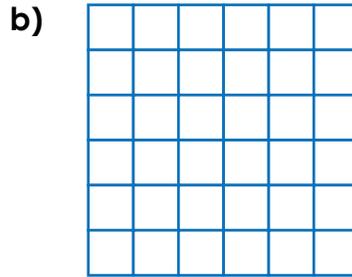
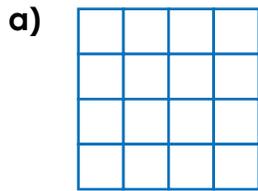


6. A hill is 80 metres high. What is the distance between the two points for viewing the hill?



7. Three rectangles with different side lengths all have a perimeter of 100 centimetres. Sketch and label the side lengths of the rectangles and figure out their diagonal lengths.

8. Calculate the lengths of the diagonals of each square. Divide by the side length. What do you notice?



9. List both leg lengths of three different right triangles, each with a hypotenuse of 10 cm.

10. Is there only one right triangle with one side length of 3 units and another of 5 units? Explain.

11. A certain triangle is almost, but not quite, a right triangle. What could the side lengths be? How do you know?

