GAP CLOSING

Powers and Roots

Intermediate / Senior
Facilitator Guide

Powers and Roots

The Facilitator's Guide for this entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/5-PowersRoots FG IS.pdf. The Student Book for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/5-PowersRoots_SB_IS.pdf.

POWERS AND ROOTS

Relevant Expectations for Grade 9

MPM1D

Number Sense and Algebra

- substitute into and evaluate algebraic expressions involving exponents
- describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three
- derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents
- extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents
- relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations

Measurement and Geometry

- relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$
- solve problems using the Pythagorean theorem, as required in applications

MPM1P

Number Sense and Algebra

- relate their understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations
- substitute into and evaluate algebraic expressions involving exponents

Measurement and Geometry

- relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$
- solve problems using the Pythagorean theorem, as required in applications

Possible reasons why a student may struggle when working with powers, roots, and the Pythagorean theorem

Students may struggle with powers, roots and the Pythagorean theorem.

Some of the problems include:

- mixing up what a perfect square is and what a square root is
- not recognizing the different roles of the base and the exponent in a power
- lack of understanding of what a square root means when the root is not a whole number
- inability to estimate square roots that are not whole numbers
- not recognizing the relationship between $\sqrt{n}$ and $\sqrt{n00}$
- confusion about the relative sizes of squares and square roots of proper fractions
- over-generalizing the Pythagorean theorem; applying it to non-right triangles
- inability to determine a missing leg length in a right triangle (in contrast to a missing hypotenuse length)

DIAGNOSTIC

Materials
- calculator

Administer the diagnostic

Using diagnostic results to personalize interventions

Intervention materials are included on each of these topics:
- perfect squares and square roots
- powers
- Pythagorean theorem

You may use all or only part of these sets of materials, based on student performance with the diagnostic. If students need help in understanding the intent of a question in the diagnostic, you are encouraged to clarify that intent.

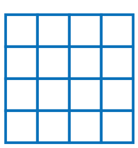
Evaluating Diagnostic Results	Suggested Intervention Materials
If students struggle with Questions 1–4	use *Perfect Squares and Square Roots*
If students struggle with Questions 5–8	use *Powers*
If students struggle with Questions 9–12	use *Pythagorean Theorem*

Solutions

1. e.g., 1296, 1600, 1764
[**Note:** Students could write 36 × 36, 40 × 40 or 42 × 42]

2. **a)** e.g., 15 **b)** e.g., 9.5 **c)** e.g., 25

3. e.g.,



4. e.g., $\sqrt{25}$ = 5 and $\sqrt{250\ 000}$ = 500 That makes sense since 250 000 = 10 000 × 25, the square root of 10 000 = 100, the square root of 25 is 5, and 500 = 5 × 100.

5. A

6. **a)** 81 **b)** 64 **c)** 1000
d) –32 **e)** 0.04

7. e.g., 3 × 3 × 3 × 3 × 3 is 9 × 9 × 3, but 5^3 is 5 × 5 × 5. Since the product of two 9s is more than the product of two 5s and one 3 is less than one 5, it makes sense that 3^5 is greater.

8. e.g., 4^3 There are 4 big sets of 4 smaller sets of 4 squares.

9. e.g., the number of squares on the two shorter sides add to the number of squares on the longest side in a right triangle.

10. **a)** $\sqrt{41}$ cm (or about 6.4 cm) **b)** 13 cm

11. **a)** $\sqrt{91}$ cm (or about 9.5 cm) **b)** $\sqrt{75}$ cm (or about 8.7 cm)

12. $\sqrt{48}$ cm (or about 6.9 cm)

Diagnostic

1. A perfect square is the result of multiplying a whole number by itself. For example, 64 is a perfect square since it's 8 x 8.
List three perfect squares between 1000 and 2000.

2. The square root of a number is what you multiply by itself to get the number. For example, 8 is the square root of 64 since 8 x 8 -= 64. Estimate each square root. Do NOT use a calculator.

a) $\sqrt{250}$ **b)** $\sqrt{88}$ **c)** $\sqrt{622}$

3. What picture would you draw to show why $\sqrt{16}$ is 4?

4. Why does it make sense that $\sqrt{250\ 000}$ is 100 times as much as $\sqrt{25}$?

5. Which does 5^3 mean?

A: 5 × 5 × 5

B: 3 × 3 × 3 × 3 × 3

C: 5 × 3 × 5 × 3

D: 5 × 3

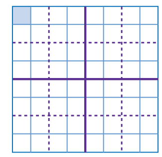
6. What is the value of each power?

a) 3^4 **b)** 4^3 **c)** 10^3 **d)** $(–2)^5$ **e)** $(0.2)^2$

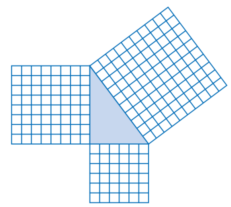
Diagnostic (Continued)

7. Without referring to their actual values, why does it make sense that 3^5 might be more than 5^3?

8. Use the thicker line and dotted liine divisions to help you use a power to represent the number of dark squares that could be fit into the largest square. Explain your thinking.



9. How can you use information about the squares in the picture to tell you that this triangle is a right triangle? Do NOT use a protractor.



Diagnostic (Continued)

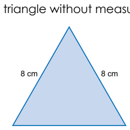
10. The two shortest sides of a right triangle are given. Determine the length of the longest side without measuring.

a) 4 cm and 5 cm **b)** 5 cm and 12 cm

11. The longest side of a right triangle is 10 cm. One leg length is given. Determine the other leg length without measuring.

a) leg is 3 cm **b)** leg is 5 cm

12. Determine the height of the triangle without measuring.



USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for successfully working with powers, whether positive, negative, or rational, with roots, and with applications of the Pythagorean theorem.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

Perfect Squares and Square Roots

Learning Goal

- relating numerical and geometric descriptions of squares and square roots.

Open Question

Materials
- grid paper
- calculators

Questions to Ask Before Using the Open Question

◊ *A square has a side length of 10 centimetres. What is its area?* (100 cm^2) *How do you know?* (e.g., I multiplied 10 × 10.)
◊ *Suppose you know that the area of a square is* 110 cm^2*; what do you know about the side length?* (e.g., It has to be a little more than 10, but not a lot more.) *Why a little more than 10?* (e.g., 110 is more than 10 × 10.)
◊ *If one square's area is more than another square's area, do you think that the side length would be more?* (e.g., Yes, an area of 25 is more than 16 and 5 is more than 4.)

Using the Open Question

Encourage students to choose numbers spread out between 100 and 300.

By viewing or listening to student responses, note if they:
- recognize perfect squares;
- know whether numbers have many factors;
- can estimate square roots;
- recognize that square roots of proper fractions are greater than the fractions.

Consolidating and Reflecting on the Open Question

◊ *How did you know that for example, 121 was a perfect square?* (I knew I should multiply numbers more than 10 to have an area of more than 100. I tried 11; 11 × 11 = 121.)
◊ *Which of your estimated side lengths do you think was closest to the actual value? Why?* (e.g., I think I was really close with the square root of 120 since 120 is so close to 121.)
◊ *If you estimated* $\sqrt{280}$ *as 16.8, how could you tell how close you were?* (e.g., I would square 16.8. The result is around 282, so that it is quite close.)
◊ *What did you notice about the square roots of the fractions?* (e.g., They are actually more than the fractions.) *Was that what you expected?* (e.g., No, I think of the area as more than the side length.)

Solutions

e.g.,

Between 100 and 300:
Perfect squares: 121 225 169

Double: 242

Lots of factors: 120, 240, 280
Not many factors: 101

$\sqrt{121} = 11$
$\sqrt{225} = 15$
$\sqrt{169} = 13$

$\sqrt{242}$ is about 15 since 242 is not that much more than 225
$\sqrt{240}$ is about 15 since 240 is not that much more than 225
$\sqrt{120}$ is about 11 since 120 is not that much less than 121
$\sqrt{280}$ is about 17 since 17 × 17 = 289
$\sqrt{101}$ is about 10 since 101 is almost 100

Between 0 and 1:
$\sqrt{\frac{4}{25}} = \frac{2}{5}$ since $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$
$\sqrt{0.01}$ = 0.1 since 0.1 × 0.1 = 0.01
$\sqrt{0.8}$ is about 0.9 since 0.9 × 0.9 = 0.81 which is really close to 0.8
$\sqrt{\frac{2}{3}}$ is about $\frac{5}{6}$ since $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. That is close to $\frac{24}{36}$, which is $\frac{2}{3}$.

Think Sheet

Materials
- grid paper
- calculators

Questions to Ask Before Assigning the Think Sheet

◊ *A square has a side length of 10 cm. What is its area?* (100 cm^2) *How do you know?* (I multiplied 10 × 10.)
◊ *Suppose you know that the area of a square is 110 cm^2; what do you know about the side length?* (e.g., It has to be a little more than 10, but not a lot more.) *Why a little more than 10?* (e.g., 110 is more than 10 × 10.)
◊ *If one square's area is more than another square's area, do you think that the side length would be more?* (e.g., Yes, for example an area of 25 is more than 16 and 5 is more than 4.)

Using the Think Sheet

Read through the introductory box with the students and make sure they understand the material explained in the instructional box.

Provide calculators and grid paper for students to visualize square roots.

Ensure that students correctly distinguish between perfect squares and square roots.

Assign the tasks.

By viewing or listening to student responses, note if they:
- recognize perfect squares;
- realize that numbers between perfect squares do not have whole number square roots;
- recognize how the factored form of a number can help them create perfect squares;
- can estimate square roots;
- recognize the relationship between $\sqrt{n}$ and $\sqrt{100 \times n}$;
- recognize that square roots of proper fractions are greater than the fractions.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How did you know that, for example, 225 was a perfect square?* (I squared numbers in the teens and 15 × 15 = 225.)
◊ *Why is 2 × 2 × 5 × 3 not a perfect square?* (It is 60 and it is not a perfect square.)
◊ *Why can you multiply 2 × 2 × 5 × 3 by 5 × 3 to get a perfect square?* (e.g., You would have 2 × 5 × 3 multiplied by itself so the square is the square of 2 × 5 × 3.)
◊ *Why is $\sqrt{65}$ closer to 8 than $\sqrt{70}$?* (e.g., 65 is closer to 64 than 70.)
◊ *What did you notice about $\sqrt{30}$ and $\sqrt{3000}$?* (e.g., It is one tenth as much.) *Why could you have predicted that?* (e.g., 3000 = 30 × 100 and the square root of 100 is 10.)
◊ *What did you notice about the square roots of the fractions?* (e.g., They were actually more than the fractions.) *Was that what you expected?* (e.g., No, I always think of the area as more than the side length.)
◊ *Why did your answer to Question 12b have to be $\frac{1}{9}$?* (e.g., I know that if $\frac{1}{3}$ times a number is the square of the number, the number is $\frac{1}{3}$ and the square is $\frac{1}{9}$.)

Solutions

1. **a)** 225, 256, 289
b) e.g., 14 × 14 is less than 200 and 18 × 18 is more than 300.

2. e.g., It is between 225 and 256 and there are no integers between 15 and 16.

3. **a)** $100 = 10^2$ $10\ 000 = 100^2$
b) 1000 is between 31 × 31, which is 961 and 32 × 32, which is 1024. 100 000 is between 316 × 316 which is 99 856 and 317 × 317 which is 100 489.

4. **a)** e.g., 15 or 60 or 240
b) There is an even number of each prime.
c) 5; There are two pairs of 2s (since 4 = 2 × 2), so I needed another 5 so that 5 was multiplied by itself, too.

5. **a)** 8 m since 8 × 8 = 64 **b)** 12 m since 12 × 12 = 144
c) about 14.14 m since I figured out $\sqrt{200}$

6. e.g., 50 since 7 × 7 = 49. It is a lot closer to 50 than 8 × 8, which is 64

7. **a)** e.g., 5.5 since 30 is about halfway between 25 and 36.
b) e.g., 17.3 since 300 is between 17 × 17, which is 289, and 18 × 18, which is 324 but it is a little closer to 289.
c) e.g., 55 since $55^2 = 3025$.

8. **a)** e.g., one is 10 times as much as the other.
b) e.g., 3000 = 30 × 100 and $\sqrt{100} = 10$.

9. e.g., I know that 8 × 8 = 64, 6 × 6 = 36 and 5 × 5 = 25, so I would multiply 8 × 6 × 5.

10. **a)** 2-digits, e.g., a 3-digit number is between 99 and 1000; $\sqrt{100} = 10$ and $\sqrt{1000}$ is just a bit more than 30, so the square root has two digits
b) 2-digits; a 4-digit number is between 1000 and 10 000; $\sqrt{1000}$ is a 2-digit number; $\sqrt{10\ 000}$ is 100, which is the lowest 3-digit number, but $\sqrt{9999}$ is less.

11. e.g., When you multiply fractions less than 1, the products are smaller than the numbers you multiply. So, if you multiply a fraction less than one by itself, its square would be less than the fraction.

12. **a)** 25 **b)** $\frac{1}{9}$ **c)** 100 **d)** $\frac{1}{25}$

Open Question

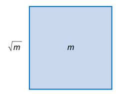
Perfect Squares and Square Roots

Learning Goal

- relating numerical and geometric descriptions of squares and square roots.

Open Question

If we draw a square and the area has the value m, then the side length is called the **square root** of m. We write it $\sqrt{m}$. If $\sqrt{m}$ is an integer, then m is called a **perfect square.**



- Choose eight numbers, all between 100 and 300 using the rules that follow. (Think of each number as the area of a square.)
  – Three numbers are perfect squares and five are not perfect squares.
  – At least one of the numbers is the double of another number.
  – At least one number has many factors and at least one number does not have many factors.

Perfect Squares and Square Roots

- Choose four numbers between 0 and 1 using the rules that follow. (Think of each number as the area of a square.)
  – Make sure that two values are close to 0.
  – Make sure that two values are close to 1.
  – At least one should be a fraction and at least one should be a decimal.
  – One should be the square of a fraction or decimal.

- For both groups of values:
  – For each number that is the square of another whole number, fraction, or decimal, tell the value of the square root.
  – For each other number, estimate the value of the square root and explain your estimate.

Think Sheet

Perfect Squares and Square Roots (Continued)

Think Sheet

A **perfect square** is the product of two identical whole numbers.

For example, 16 is a perfect square since 4 × 4 = 16. We can say **4 squared** is 16.

Other perfect squares are listed below:

1 1 × 1

4 2 × 2

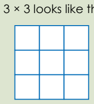
9 3 × 3

25 5 × 5

36 6 × 6

…

We can model perfect squares as square shapes. For example, 3 × 3 looks like this:

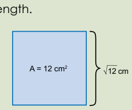


Notice that perfect squares get farther and farther apart. For example, 1 and 4 are only three apart, but 4 and 9 are five apart and 9 and 16 are seven apart.

The **square root** of a number is the number that we multiply by itself to result in that original number. The square roots of perfect squares are whole numbers. For example, the square root of 16 is 4.

Square roots do not have to be whole numbers. We write *the square root of* using the symbol $\sqrt{\ }$. The square root of 8 is written as $\sqrt{8}$. It is between 2 and 3 since 2 × 2 = 4 and 3 × 3 = 9 and 8 is between 4 and 9.

One way to model a square root is to think of it as the side length of a square with a given area. For example, to show $\sqrt{12}$, think of a square with area 12. The square root of 12 is the side length.



Perfect Squares and Square Roots (Continued)

We can use the $\sqrt{\ }$ button on a calculator to get the value of a square root.

- To estimate the square root of a number, we could start by relating the square to known perfect squares.

  For example, since 125 is between 121 (11 × 11) and 144 (12 × 12), $\sqrt{125}$ is between 11 and 12. It is probably closer to 11 since 125 is closer to 121 than to 144. It helps to know some perfect squares as shown in the table.

  If we use a calculator, we learn that $\sqrt{125}$ is about 11.18.

- We use the facts 10 × 10 = 100 and 100 × 100 = 10 000 to help estimate square roots of larger numbers. For example, since $\sqrt{15}$ is close to 4, then $\sqrt{1500}$ is close to 40 and $\sqrt{150\ 000}$ is close to 400.

Number	Square Root
1	1
4	2
9	3
16	4
25	5
36	6
49	7
64	8
81	9
100	10
121	11
144	12
10 000	100
1 000 000	1000

1. a) List all of the perfect squares between 200 and 300.

b) Explain how you know you have all of them.

2. Explain how you know that 250 cannot be a perfect square.

Perfect Squares and Square Roots (Continued)

3. a) Which of these powers of 10 are perfect squares? Explain.
   100 1000 10 000 100 000

   b) Why are they not all perfect squares?

4. a) You want to multiply $2 \times 2 \times 5 \times 3$ by a number to make a perfect square. List three possible amounts you could multiply by and prove that each is a perfect square.

   b) When you factor those perfect squares down to primes, what do you notice?

   c) What is the least number you could multiply $2 \times 2 \times 4 \times 5$ by to make a perfect square? Explain.

5. What is the side length of each of three square gardens? (The areas are given.) How do you know?
   a) 64 m^2 b) 144 m^2 c) 200 m^2

6. The square root of a number is closer to 7 than to 8. What might the number be? How do you know?

7. Estimate each square root without using a calculator. Explain your strategy.
   a) $\sqrt{30}$ b) $\sqrt{300}$ c) $\sqrt{3000}$

Perfect Squares and Square Roots (Continued)

8. a) What did you notice about the answers to Questions 7a) and 7c)?

   b) Why does that make sense?

9. How could factoring 57 600 as $64 \times 36 \times 25$ help you figure out its square root?

10. How many digits could the whole number part of the square root of these whole numbers have? Explain your thinking.
    a) a 3-digit number b) a 4-digit number

11. Why does it make sense that $\sqrt{\frac{1}{4}}$ is more than $\frac{1}{4}$?

12. A number is related to its square root as indicated. What is the number?
    a) The number is 5 times its square root.
    b) The number is $\frac{1}{3}$ of its square root.
    c) The number is 90 more than its square root.
    d) The number is $\frac{4}{25}$ less than its square root.

Powers

Learning Goal

- recognizing the efficiency of representing repeated multiplication using powers.

Open Question

Materials
- calculators

Questions to Ask Before Using the Open Question

◊ *Which do you think is more: 3 × 3 × 3 or 4 × 4 × 4?* (The second one is more since the numbers you are multiplying are greater.)
◊ *Which do you think is more: 3 × 3 × 3 or 3 × 3 × 3 × 3?* (The second one is more since you are multiplying by an extra 3.)
◊ *Which do you think would probably be greater: multiplying the number 3 by itself 10 times or multiplying the number 10 by itself 3 times?* (e.g., I think if you multiplied a lot of 3s it would get a lot bigger than multiplying just a few 10s. For example, 3 × 3 × 3 × 3 is already 81 and if you multiply by 3 × 3 again, it is already over 700. If you use more 3s, it would be greater than 10^3, which is 1000.)

Using the Open Question

Encourage students to make as many observations about the numbers as they can.

By viewing or listening to student responses, note if they:
- can evaluate simple powers;
- recognize that a number to the first power is just that number;
- recognize that if you increase the power by 1, you simply multiply once more by the base;
- recognize that powers of greater whole number bases grow faster than powers of lower whole number bases;
- observe that 2 and 4 share some powers;
- recognize that powers of numbers between 0 and 1 shrink as the power increases;
- recognize that powers of negatives can be positive or negative.

Consolidating and Reflecting on the Open Question

◊ *Why does it make sense that $6^1 = 6$?* (e.g., It means you are only writing down one six to multiply – there is not any multiplication to do.)
◊ *Once you had figured out 3^4, how did you figure out 3^5?* (I multiplied by another 3.)
◊ *Why did it make sense that powers of 2 were even?* (e.g., You keep multiplying evens by evens, so the products are also even.)
◊ *Why did it make sense that powers of 4 were also powers of 2?* (e.g., 4 is 2 × 2, so lots of 4s is really the product of lots of 2s.)
◊ *Which of your powers was greatest? Were you surprised?* (e.g., It was 5^5. That makes sense since it was the biggest base and the biggest exponent.)
◊ *What was interesting about the powers of 0.5?* (e.g., They were like the powers of 5, but a lot smaller. They also kept getting smaller instead of larger.)
◊ *What was interesting about the powers of –2?* (e.g., They were like the powers of 2, but they flipped between being positive and negative.)

Solutions

e.g.,

$1^1 = 1$	$1^2 = 1$	$1^3 = 1$	$1^4 = 1$	$1^5 = 1$
$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
$4^1 = 4$	$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1024$

$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$
$0.5^1 = 0.5$	$0.5^2 = 0.25$	$0.5^3 = 0.125$	$0.5^4 = 0.0625$	$0.5^5 = 0.03125$
$(-2)^1 = -2$	$(-2)^2 = 4$	$(-2)^3 = -8$	$(-2)^4 = 16$	$(-2)^5 = -32$

I noticed that 1 raised to any exponent is still 1.

I notice that if you raise a number to the first power, it is that number.

I notice that when you have a bigger base, the powers get bigger faster.

I notice that the powers of 2 and 4 are all even and the powers of 1, 3, and 5 are all odd.

I noticed that the powers of 4 were also powers of 2.

I noticed that the powers of 0.5 had a lot of the same digits as the powers of 5.

I noticed that the powers of 0.5 kept getting smaller instead of larger.

I noticed that the powers of –2 are like the powers of 2, but they switch between being negative and positive.

Think Sheet

Materials
- calculators

Questions to Ask Before Assigning the Think Sheet

◊ *Which do you think is more: 3 × 3 × 3 or 4 × 4 × 4?* (The second one is more since the numbers you are multiplying are greater.)
◊ *Which do you think is more: 3 × 3 × 3 or 3 × 3 × 3 × 3?* (The second one is more since you are multiplying by an extra 3.)
◊ *Which do you think would probably be greater: multiplying the number 3 by itself 10 times or multiplying the number 10 by itself 3 times?* (e.g., I think if you multiplied a lot of 3s it would get a lot bigger than multiplying just a few 10s. For example, 3 × 3 × 3 × 3 is already 81 and if you multiply by 3 × 3 again, it is already over 700. So it is more.)

Using the Think Sheet

Read through the introductory box with the students and make sure they understand the material explained in the instructional box.

Assign the tasks.

By viewing or listening to student responses, note if they:
- can evaluate powers;
- relate squares and cubes to geometric models;
- relate powers to appropriate contexts;
- relate powers of two different bases;
- relate different powers of the same base;
- recognize that powers of numbers between 0 and 1 shrink as the power increases;
- recognize that powers of negatives can be positive or negative;
- can use factored forms of numbers to help represent them as powers.

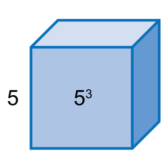
Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *Once you figure out 3^4, how would you figure out 3^6?* (I would multiply by another 3 × 3 or 9.)
◊ *How were your pictures in Question 2 different? Why were they different?* (e.g., Squares are about area but cubes are about volume.)
◊ *What was interesting about the powers of $\frac{1}{2}$?* (e.g., They kept getting smaller instead of larger.)
◊ *Why did it make sense that powers of 9 were also powers of 3?* (e.g., 9 is 3 × 3, so lots of 9s is also the product of lots of 3s.)
◊ *If you listed all of the powers of –3, what would you notice?* (e.g., They are like the powers of 3, but they flip between being positive and negative.)
◊ *How did you know that 10^3 was less than 100^2?* (e.g., 100^2 is 10^4 and that is more than 10^3.)

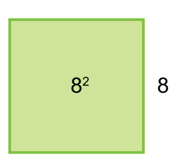
Solutions

1. **a)** 3 × 3 × 3 × 3
b) 3 × 3 × 3 × 3 × 3 × 3
c) 4 × 4 × 4 × 4 × 4 × 4
d) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
e) (–3) × (–3) × (–3) × (–3)

2. **a)**



b)



3. 4^3; e.g., If the large box holds 4 small ones and the small ones hold 4 smaller ones, that would be 4×4 (or 4^2) boxes. But since each smaller box holds four tiny ones, that would be 4×4^2, or 4^3, tiny boxes.

4. e.g., A huge box holds 6 large boxes.
Each large box holds 6 medium size boxes.
Each medium box holds 6 small boxes.
Each small box holds 6 tiny boxes.

5. e.g., 2 (or any even number)

6. **a)** $\left(\frac{1}{2}\right)^5$ $\left(\frac{1}{2}\right)^4$ $\left(\frac{1}{2}\right)^3$ $\left(\frac{1}{2}\right)^2$
b) 2^2 2^3 2^4 2^5
c) e.g., bigger powers of whole numbers are bigger, but bigger powers of fractions get smaller.

7. 4^5

8. **a)** 3 b) 4 c) 5

9. **a)** 2^5 since you are multiplying more 2s.
b) 10^3 since you are multiplying 3 greater numbers.
c) 100^2 since it is the same as 10^4, and that is more than 10^3.

10. **a)** e.g., 2 and 3 **b)** e.g., 10 **c)** e.g., $\frac{1}{10}$

11. $20^5 = 20 \times 20 \times 20 \times 20 \times 20$; the first three 20s make 20^3 and there is still $20 \times 20 = 400$ to mulitiply.

12. **a)** 15^2 **b)** 18^3 **c)** e.g., 2^{12}

13. e.g., It takes a lot less space than writing the same factor over and over.

Open Question

Powers

Learning Goal

- recognizing the efficiency of representing repeated multiplication using powers.

Open Question

The way to shorten a repeated multiplication is to use a power.

For example, 2^4 means $2 \times 2 \times 2 \times 2$;
2 is multiplied by itself 4 times. It is a power since it is the product of a number multiplied by itself.

The 2 is the **base**. The 4 is the **exponent** and 2^4 is the **power**.

base → 2^4 ← exponent

- Use the digits 1, 2, 3, 4, and 5 as bases and exponents. Also use the decimal 0.5 and the integer –2 as a base. List all the powers you can and calculate their values.

- Tell what you notice about the powers.

Think Sheet

Powers (Continued)

Think Sheet

Multiplication is a short way to record repeated addition. For example, it is quicker to write 4×5 than $5 + 5 + 5 + 5$.

The way to shorten a repeated multiplication is to use a power.

For example, 2^4 means $2 \times 2 \times 2 \times 2$;
2 is multiplied by itself 4 times.

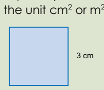
The 2 is the **base**. The 4 is the **exponent** and 2^4 is the **power**.

base → 2^4 ← exponent

- There are special names if the exponent is 2 or 3. For example, 3^2 is read *three squared*. If the exponent is 3, we use the word *cubed*; e.g., 5^3 is read *five cubed*.

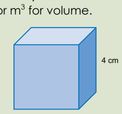
  Otherwise, we use **ordinal words**, e.g., we read 6^5 as *six to the fifth (power)*.
- We relate a square (or a number to the second power) to the area of a square. That helps us understand why we use the unit cm^2 or m^2 for area.

  9 (3^2) cm^2 is the area of this square.



  We relate a cube (or a number to the third power) to the volume of a cube. That helps explain why we use the unit cm^3 or m^3 for volume.

  64 (4^3) cm^3 is the volume of this cube.

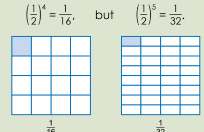


- Powers of whole numbers grow very quickly. For example,
  $3^4 = 81$, but
  $3^5 = 243$ and
  $3^6 = 729$.

Powers (Continued)

- Powers of fractions or decimals less than 1 shrink as the exponent increases. For example,

  $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$, but $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$.



- Powers of negative numbers can be positive. If the exponent is even, the power is positive. If it is odd, the power is negative.

  For example, $(-3)^3 = -27$, but $(-3)^4 = +81$.

1. Write each power as a multiplication.

a) 3^4 **b)** 3^6 **c)** 4^6

d) 2^{10} **e)** $(-3)^4$

2. Draw a picture that shows the meaning of each power.

a) 5^3 **b)** 8^2

Powers (Continued)

3. A large box holds 4 small ones.
Each small box holds 4 smaller boxes.
Each of the smaller boxes holds 4 tiny boxes.
Use a power to tell how many tiny boxes would fit in the large box.
Explain your thinking.

4. Tell a story (as in Question 3) that might describe 6^4.

5. $(-2)^{\square}$ is a positive number. What could $\square$ be?

6. **a)** Order from least to greatest:

$\left(\frac{1}{2}\right)^3 \quad \left(\frac{1}{2}\right)^4 \quad \left(\frac{1}{2}\right)^2 \quad \left(\frac{1}{2}\right)^5$

b) Order from least to greatest:

$2^3 \quad 2^4 \quad 2^2 \quad 2^5$

c) What do you notice if you compare the answers to parts a) and b)?

Powers (Continued)

7. A certain power has a base that is one less than its exponent. The value of the power is about 1000. What could the power be?

8. Choose values to make these statements true:

a) $\square^4 = 9^2$ **b)** $6^8 = 36^{\square}$ **c)** $25^3 = \square^6$

9. Which value is greater each time? How do you know?

a) 2^3 or 2^5 **b)** 10^3 or 9^3 **c)** 10^3 or 100^2

10. To replace the boxes below, you can repeat numbers or use different numbers. Tell what the numbers might be if:

a) $\square^3 < \square^2$

b) $\square^3$ is at least 100 more than $\square^2$

c) $\square^3$ is less than $\frac{1}{5}$ of $\square^2$.

11. Explain why 20^5 is 400 times as much as 20^3.

12. How would you write each of these as a single power?

a) $3 \times 3 \times 5 \times 5$ b) $6 \times 6 \times 3 \times 3 \times 6 \times 3$

c) $2 \times 2 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4$

13. Why is it useful to write a number as a power?

Pythagorean Theorem

Learning Goal

- relating numerical and geometric descriptions of the Pythagorean theorem and applying the theorem to solve problems.

Open Question

Materials
- calculators
- rulers
- grid paper

Questions to Ask Before Using the Open Question

◊ *If I drew a triangle and told you it was a right triangle, how would you check?* (I would use my protractor.)
◊ *How else might you check?* (e.g., I could see if it fits on a square corner of a grid.)
◊ *Suppose you had to place a square on a side of a triangle. How would you do that?* (I would measure the side length; then I would make a square corner at the edge of the side and go out the same distance as the side length. I would do it on the other corner too and then connect the tops to make a square.)

Using the Open Question

Students might use the grid paper to help them create their right triangles. They might use rulers to help them measure side lengths of slanted squares.

By viewing or listening to student responses, note if they:
- recognize the Pythagorean theorem as applying to right triangles;
- recognize that the Pythagorean theorem does not apply to non-right triangles;
- recognize the utility of the Pythagorean theorem.

Consolidating and Reflecting on the Open Question

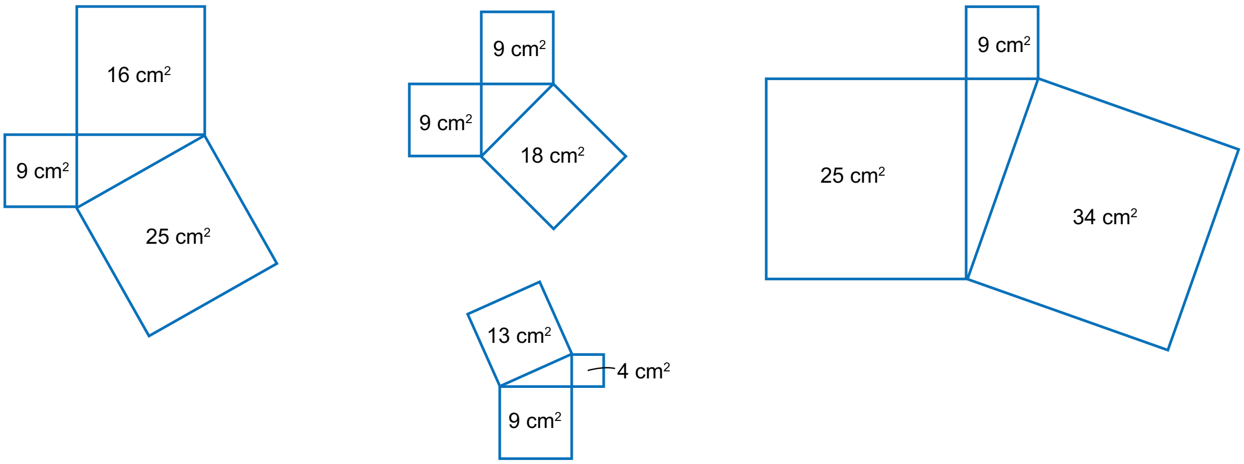
◊ *Why was the biggest square on the hypotenuse?* (e.g., The side length was the longest, so the square had the most area.)
◊ *How were the right triangles different from the other triangles?* (The two smaller areas added to the larger one.)

Note: Inform students that this is called the Pythagorean theorem and record it for them in the usual form of $a^2 + b^2 = c^2$. Indicate that a, b, and c represent the three side lengths (c is the hypotenuse).

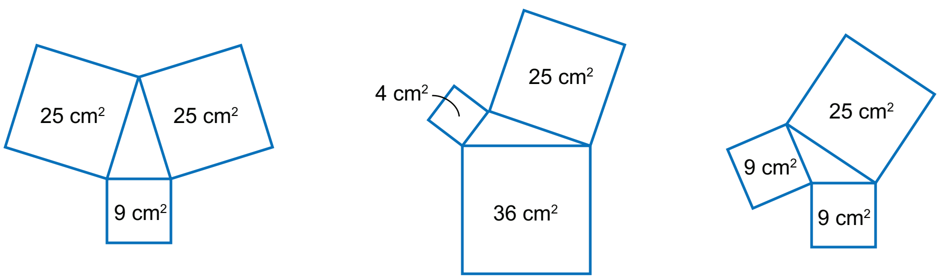
◊ *How did what you noticed describe the Pythagorean theorem?* (e.g., The squares are the areas of the squares on the side lengths, so it says that the sum of the areas of the two smaller squares is the area of the larger square.)
◊ *If you knew that one side length of a right triangle was 4 units and another was 3 units, how could the Pythagorean theorem help you figure out the hypotenuse length?* (e.g., You could add 16 and 9 to get 25. Since that is the area of the square on the hypotenuse, take the square root to get the hypotenuse length.)
◊ *How could you use the Pythagorean theorem to prove that a triangle is not a right triangle?* (e.g., If the sum of the two smaller squares is not the square of the longest side.)

Solutions

e.g., Right triangles



For the right triangles, the total areas of the smaller squares is the same as the area of the bigger square.



- The two smaller areas do not total the larger areas on non-right triangles
- You could test to see if a triangle is right if the two little areas add to the big one.
- If you know two side lengths, you would know the areas of the squares. You could add them, if they were the short sides, to get the other area and take the square root to get the side length. If one was the long side, you could subtract them to get the other area and take the square root to get the side length.

Think Sheet

Materials
- grid paper
- calculators
- rulers

Questions to Ask Before Assigning the Think Sheet

◊ *Suppose I drew a triangle and told you it was a right triangle, how would you check?* (I would use my protractor.)
◊ *How else might you check?* (e.g., I could see if it fits on a square corner of a grid.)
◊ *If I give you the three side lengths of a triangle, can you tell whether or not it is a right triangle?* (Sometimes, e.g., if the values are 4-4-4, I know it is equilateral and not right.)

Using the Think Sheet

Read through the introductory box with the students and make sure they understand the material explained in the instructional box.

Provide required materials.

Assign the tasks.

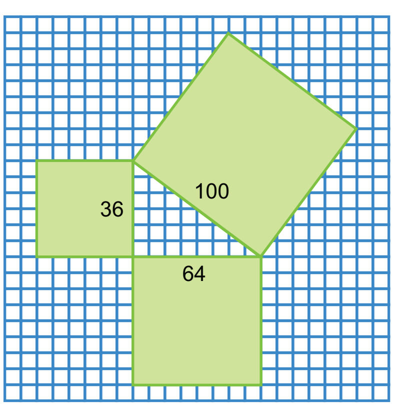
By viewing or listening to student responses, note if they:
- can correctly interpret the Pythagorean theorem geometrically;
- can use the Pythagorean theorem to determine a missing side length in a right triangle;
- can use the Pythagorean theorem to decide whether a triangle is a right triangle;
- recognize that different right triangles can have the same hypotenuse;
- recognize that you can not apply the Pythagorean theorem to determine a missing side unless you know whether the missing side is a hypotenuse or a leg.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How did your picture in Question 1 show the Pythagorean theorem?* (e.g., The areas of 36 and 64 add to 100, which is the area of the square on the hypotenuse.)
◊ *How was the Pythagorean theorem useful in Question 2?* (e.g., I used the numbers that were there for *a*, *b* or *c* and then used the theorem to get the missing value.)
◊ *How was the Pythagorean theorem useful in Question 3?* (e.g., I added the squares of the two smaller numbers to check to see if it was the same as the square of the greatest number.)
◊ *What right triangle did you use in Question 5?* (e.g., I drew a height in the triangle.)
◊ *Why did it make sense that the diagonals in Question 8 had different lengths?* (e.g., The diagonal is longer if the rectangle is narrower.)
◊ *What do you need to know beside the lengths of two sides of a right triangle to figure out the length of the third side?* (I need to know whether one of the sides is a hypotenuse or not.)

Solutions

1.



If you add 36 to 64, you get 100 and that is what the Pythagorean theorem is about.

2. **a)** 13 cm **b)** about 8.94 cm
c) about 11.62 cm **d)** 24 cm

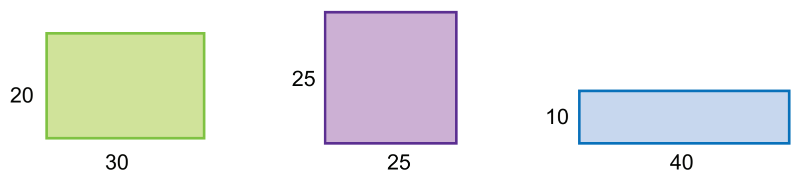
3. **a)** NOT a right triangle **b)** a right triangle
c) right triangle since $10^2 + 576 = 676$ and the square root is 26

4. about 12.17 m

5. about 9.54 cm

6. 1598 m + 1498 m = about 3096 m

7.



Diagonals are: about 36 m, about 35 m and about 41 m

8. **a)** Diagonal = 5.66 cm **b)** Diagonal = 8.49 cm
c) Diagonal = 9.9 cm

The quotients are always about 1.41.

9. e.g., 6 – 8 – 10; 7.07 – 7.07 – 10 and 7.75 – 6.3 – 10

10. No, since the 5 could be a leg or a hypotenuse.

11. e.g., 3-4-5.2 is almost 3-4-5, but not quite and I know that 9 + 16 = 25

Open Question

Pythagorean Theorem

Learning Goal

- relating numerical and geometric descriptions of the Pythagorean theorem and applying the theorem to solve problems.

Open Question

- Draw four right triangles and three non-right triangles. For each triangle, draw a square on each side of the triangle so that the side of the triangle is the full base of the square.

- Compare the total area of the two smallest squares with the area of the largest square for each triangle.

- What do you notice? How could that be useful if you knew two side lengths of a right triangle and wanted to know the third side length?

Think Sheet

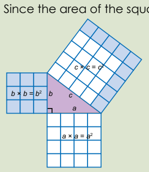
Pythagorean Theorem (Continued)

Think Sheet

Right triangles are special.

When we know two of the side lengths and which two lengths they are, we automatically know the third one. [This is not true for just any triangle.] It is true in a right triangle because the total area of the two squares we can build on the smaller sides (the **legs**) of a right triangle is the same as the area of the square we can build on the longest side, the **hypotenuse** (the side across from the right angle).

Since the area of the square is the square of the side length each time, we write $a^2 + b^2 = c^2$ if a, b and c are the side lengths of the triangles, and c is the hypotenuse.



This is called the **Pythagorean theorem**.

If we know a triangle is a right triangle, we can use the equation above to figure out the third side. For example, if the hypotenuse is 12 units and one leg is 4 units, then

$a^2 = 12^2 - 4^2 = 128$; that means $a = \sqrt{128}$, or about 11.3 units.

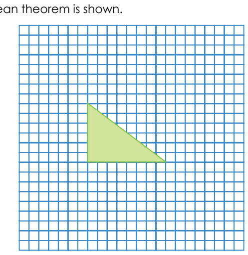
- Since this relationship is only true for right triangles, it is a way to test whether a triangle is a right triangle without drawing it.

  For example, if a triangle has side lengths: 9, 12 and 15; it is a right triangle, since $9^2 + 12^2 = 225 = 15^2$.

  If a triangle has side lengths: 5, 7, and 9, it is not a right triangle since $5^2 + 7^2 = 74$ and not 9^2.

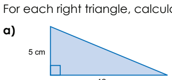
Pythagorean Theorem (Continued)

1. Draw a picture to show the squares on each side length of this right triangle. Tell how the Pythagorean theorem is shown.



2. For each right triangle, calculate the missing side length.

a)



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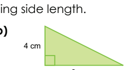
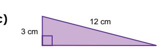
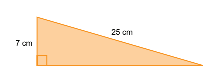


c)



d)



3. Decide whether these are the side lengths of a right triangle. Explain your thinking for part c)

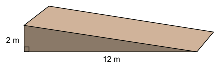
a) 5 cm, 8 cm, 11 cm

b) 24 cm, 32 cm, 40 cm

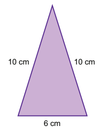
c) 10 cm, 24 cm, 26 cm

Pythagorean Theorem (Continued)

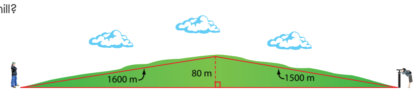
4. A ramp rises 2 metres from a point 12 metres away. How long is the ramp?



5. Determine the height of this triangle without measuring.



6. A hill is 80 metres high. What is the distance between the two points for viewing the hill?

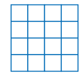


7. Three rectangles with different side lengths all have a perimeter of 100 centimetres. Sketch and label the side lengths of the rectangles and figure out their diagonal lengths.

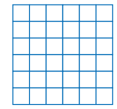
Pythagorean Theorem (Continued)

8. Calculate the lengths of the diagonals of each square. Divide by the side length. What do you notice?

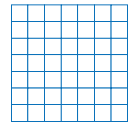
a)



b)



c)



9. List both leg lengths of three different right triangles, each with a hypotenuse of 10 cm.

10. Is there only one right triangle with one side length of 3 units and another of 5 units? Explain.

11. A certain triangle is almost, but not quite, a right triangle. What could the side lengths be? How do you know?