GAP CLOSING

Integers

Intermediate / Senior
Student Book

Topic 3

Integers

This entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/3-Integers_SB_IS.pdf. The Facilitator's Guide for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/3-Integers_FG_IS.pdf.

Diagnostic

1. Draw a number line from –10 to +10. Mark the locations of these integers:

–2, –8, 0, +5.

2. Describe three things that the number –2 might represent.

3. Order these integers from least to greatest: 6, –2, 3, –8, –20, +15, 9, –9.

Explain how you know which number is the least.

4. Explain why –2 < –1, even though +2 > +1.

[Recall that < means "less than" and > means "greater than."]

5. Add each pair of integers.

a) (–3) + (–8)

b) (–20) + (+16)

c) (+9) + (–13)

d) (+13) + (–3)

6. Use a model to show why your answer to Question 5c) makes sense. Explain the model.

7. Subtract each pair of integers.

a) 4 – (–2)

b) 8 – (+16)

c) (–9) – (–2)

d) (–11) – (–18)

8. Use a model to show why your answer to Question 7d) makes sense.

9. Multiply each pair of integers.

a) (–3) × 8

b) 9 × (–2)

c) (–5) × (–10)

d) (9) × (–7)

10. Use a model to show why your answer to Question 9b) makes sense. Explain the model.

11. Divide each pair of integers.

a) (–4) ÷ (–2)

b) (–8) ÷ 4

c) 16 ÷ (–4)

d) (+20) ÷ (–5)

12. Use a model to show why your answer to Question 11b) makes sense. Explain the model.

13. Circle the correct equation. Explain why it is right.

(–2) + 8 × (–4) = –24 or (–2) + 8 × (–4) = –34

14. Which of these expressions is greater? How much greater?

(–3) + 6 ÷ [4 – (–2)] or (–3) + 8 ÷ 4 – (–2)

Representing and Comparing Integers

Learning Goal

- selecting a strategy to represent and compare integers depending on the values of those integers

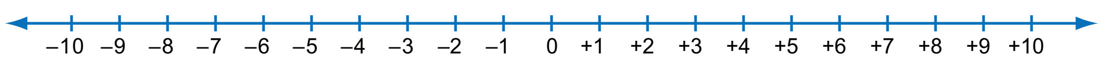
Open Question

Integers include three groups of numbers:

Positive integers: the numbers you say when you count (1, 2, 3, 4, 5, 6,…), although we often put a + sign in front of them when we are talking about them as integers (e.g., +1, +2, +3, …)

Zero: 0

Negative integers: the opposites of the counting numbers (–1, –2, –3, –4,…). Each negative integer is as far from 0 as its opposite on a number line, but on the left side of zero.



- Choose eight integers, using these rules:
  – Five of them are negative.
  – When you place the integers on the number line, no two of the negatives are the same distance apart.

- Show them on a number line and check that you have used the rules.

- Order the eight integers from least to greatest.

- Choose **two** of the negative integers. Explain how you positioned them on the number line.

- Show another way you might represent or describe those two negative integers.

Think Sheet

Representing Integers

Integers include three groups of numbers:

Positive integers: the numbers we say when you count (1, 2, 3, 4, 5, 6,…), although we often put a + sign in front of them when we are talking about them as integers

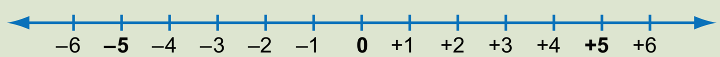
Zero: 0

Negative integers: the opposites of the counting numbers (–1, –2, –3, –4, …)

- The most familiar use of negative integers is for temperatures. For example, –3° means 3 degrees below 0.

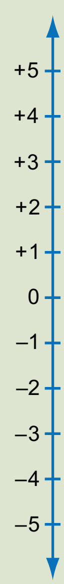
  Sometimes people use negative integers to describe debts; for example, if you owe $5, you could say that you have –5 dollars. Negative numbers are sometimes used in golf scores and hockey statistics.

- The negative integers are to the left of 0 on the **number line**. They are opposites of the positive integers; opposite means equally far from 0. So, for example (–5) is exactly the same distance from 0 to the left as (+5) is to the right.

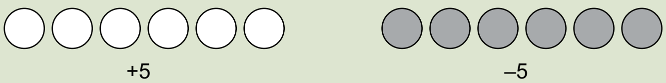


- Both positive and negative integers can be far from 0. For example, +200 is quite far from 0, but so is –200. They can also both be near to 0. For example, +1 and –1 are both very close to 0.

- We can use an up-down number line, more like a thermometer, instead of a horizontal number line, to model the integers. Here the positive numbers are above the negative ones.



- We can also use **counters** to represent integers. We can use one colour to represent positive integers and a different colour to represent negative integers. For example, +5 could be modelled as 5 light counters and –5 could be modelled as 5 dark ones.

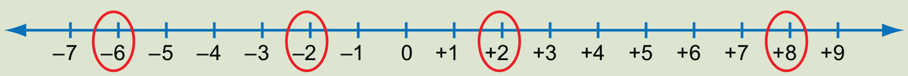


Comparing Integers

- One integer is greater than another, if it is farther to the right on a horizontal **number line** or higher on a vertical number line.

  For example, +8 > +2 and –2 > –6.

  [Recall that > means "greater than" and < means "less than."}



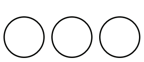
- It is difficult to compare integers with **counters**; number lines make the most sense to use for comparison.

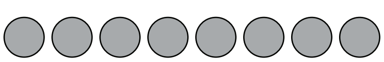
1. a) Draw a number line from –10 to +10. Mark these integers with dark dots on your number line:

+2, –6, –8, +7.

b) Mark each of their opposites with an X.

2. What integers are represented? (Remember that positive is light.)

a) 

b) 

c) Draw the opposites of each of the integers in parts a) and b) and tell what the new integers are.

3. Name two integers to meet each requirement:

a) 5 units from –2 on a number line

b) 12 units from +4 on a number line

c) 5 units from –8 on a number line

4. Two opposite integers are 16 apart on a number line. What could they be?

5. Describe something that one of the integers you named in Question 3a might represent.

6. Do you think that 0 has an opposite? If so, what is it?

7. Replace the ⬚ with a greater than (>) or less than (<) sign to make these expressions true.

a) –2 ⬚ +2

b) +8 ⬚ –12

c) –12 ⬚ –8

8. Put these integers in order from least to greatest:

–2, +8, +2, –6, –10, –1

9. List two integers to fit each description.

a) between –4 and 3

b) between –4 and –10

c) between –12 and +1

d) a little greater than –4

e) a little less than –9

10. Fill in the blanks so that these temperatures are in order from coldest to warmest:

a) ☐°, –3°, ☐°, ☐°, +1°

b) –12°, ☐°, –10°, ☐°, ☐°, –5°

c) ☐°, –5°, +2°, ☐°

11. List four possible values to make the statement true or explain why it is not possible.

a) an integer greater than –2 and greater than –8

b) an integer greater than –12 and less than –2

c) an integer greater than –2 and less than –12

12. Why is any negative integer less than any positive one?

Adding and Subtracting Integers

Learning Goal

- selecting a strategy to add or subtract integers depending on the values of those integers

Open Question

The sum of two integers is between –20 and –4. If you subtract these two integers, the difference is a negative number close to 0.

- List four possible pairs of integers.

- Explain how you know they are correct.

- Make up two other rules describing a sum and difference of two integers; either the sum or the difference or both must be negative.

Choose four possible pairs of integers to satisfy those rules. Show that they satisfy the rules you made up.

Think Sheet

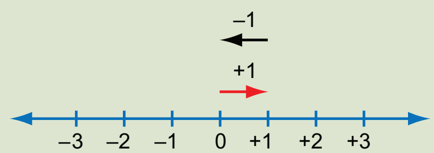
Adding

When we add two integers, we combine them, just as when we add whole numbers.

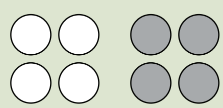
- We must also remember the **Zero Principle: –1 + 1 = 0.**

  For example: If a boy had $1 (+1) in his wallet and a debt of $1 (–1), it is as if he had $0 (or no money).

  Also moving forward one step on a number line from 0 in the direction of +1 and then moving one step in the direction of –1 puts you back at 0.



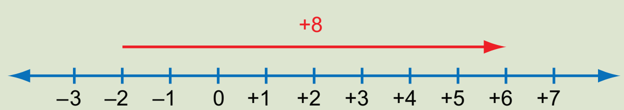
  When you have counters, the zero principle allows you to model 0 with any equal number of dark and light counters since any pair of light and dark counters is 0.



0

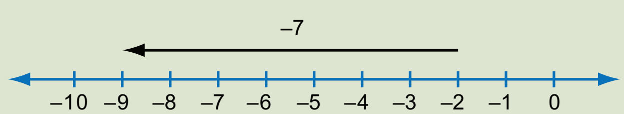
- If we add two numbers on a **number line**, we start at the first number and move the appropriate number of steps in the direction from 0 of the second number. The sum is the final location.

  For example, –2 + (+8) means: Start at –2. Move 8 steps to the right (direction of +8 from 0). The landing spot is +6.



(–2) + (+8) = (+6)

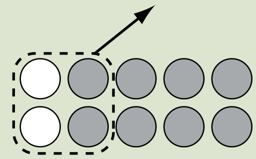
  (–2) + (–7) means: Start at –2 and move 7 steps to the left (direction of –7 from 0). The landing spot is –9.



(–2) + (–7) = (–9)

- If we add integers using **counters**, we represent both numbers and combine them. We can ignore any pairs of counters that make 0, since 0 does not affect a sum.

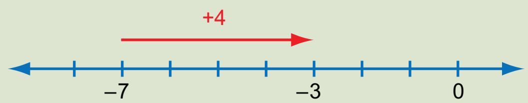
  For example: +2 + (–8) is shown at the right:



+2 + (–8) = –6

Subtracting

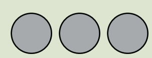
- On a **number line**, it is useful to think of subtracting by thinking of what to add to one number to get the total. For example, –3 – (–7) asks: *What do I have to add to –7 to get to –3?* The result is the distance and direction travelled.



–3 – (–7) = +4

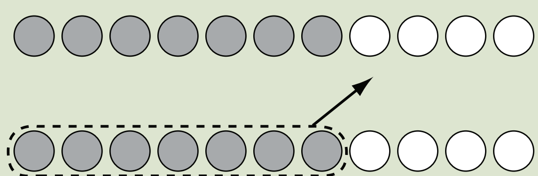
- Using **counters**, it is useful to think of subtraction as take away.

  To model –3 – (–7), we show three dark counters, but we want to take away 7 dark counters.



–3

  Since there are not seven dark counters to take away, we add zeroes to the three dark counters until we have enough to take away. The result is what is left after the seven dark counters are removed.



–3 – (–7) = +4

- If numbers are far from 0, we imagine the counters or number line.

  For example, on a number line (–45) – (+8) asks what to add to 8 to go to –45. We start at 8. We go 8 back to get to 0 and another 45 back to get to –45, so the result is –53.

  Or if we have 45 dark counters and want to remove eight light ones, we need to add 0 as eight pairs of dark and light counters. We remove the eight light ones and are left with 53 dark counters: (–45) + (–8) = –53.

  Some people think of *a* – *b* as *a* + (–*b*) and think of *a* + (–*b*) as *a* – *b*.

1. Model and solve each addition.
   a) (–7) + (+7)
   b) (+3) + (–4)
   c) (–3) + (–4)
   d) (–3) + (+4)
   e) (–8) + (+7)
   f) (–20) + (+19)

2. You add a number to (–3) and the sum is negative. List four possible numbers you might have added and four you could not have added.

3. You add two integers and the sum is –4.
   a) List two possible negative integers.
   b) List two possible integers that are not both negative.

4. Katie added all the integers from –20 to +20 in her head. Explain how could she have done that.

5. Is the statement sometimes true, always true, or never true? Explain.
   a) The sum of two positive integers is positive.
   b) The sum of two negative integers is negative.
   c) The sum of a positive and negative integer is positive.

6. Solve each subtraction. Model at least three of them.

a) (–7) – (+7)

b) (+7) – (–7)

c) 8 – (–4)

d) 4 – 8

e) 4 – (–8)

f) 34 – (–19)

g) 19 – (+34)

h) 19 – (–34)

7. How can you use a number line to show why 5 – (–4) is the opposite of (–4) – 5?

8. You subtract two integers and the difference is –8. What could those integers be?

9. Use a model to explain why any number – (–7) is the same as that number + 7.

10. Complete the statement to make it true.

a) If you subtract a positive integer from a negative one, the result

______________________.

b) If you subtract a negative integer from a positive one, the result

_______________________.

c) If you subtract a negative integer from a negative one, the result is negative if _______________________.

d) If you subtract a positive integer from a positive one, the result is negative if

__________________________.

Multiplying and Dividing Integers

Learning Goal

- justifying the value and sign of a product or quotient of two integers

Open Question

The product of two integers is between –100 and –20. If you divide the same two integers, the quotient is an integer close to 0.

- List four possible pairs of integers. Make sure some of the divisors are positive, some are negative, and the quotients are all different.

- Show that your pairs satisfy the rules.

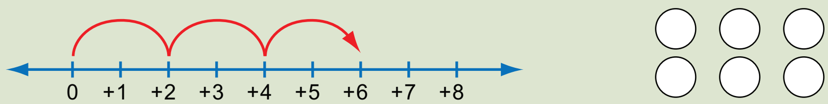
- Make up two other rules describing a product and quotient of two integers (some of the results must be negative).

Choose four possible pairs of integers to satisfy your rules.

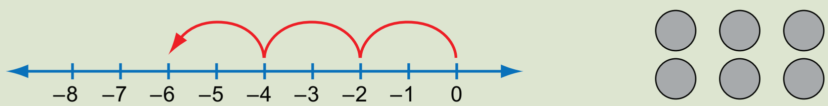
Think Sheet

Multiplying

- We know how to multiply two positive integers. For example, (+3) × (+2) means three groups of +2. That is the same as three jumps of 2 starting at 0 on the number line or three sets of two light counters.

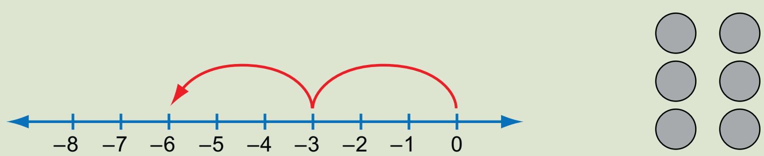


- In a similar way, (+3) × (–2) is three jumps of (–2) starting at 0 on the number line or it could be modelled as three sets of two dark counters.



(+3) × (–2) = –6

- Since it does not matter in which order you multiply numbers, (–3) × (+2) is the same as (+2) × (–3). That is 2 jumps of (–3) starting from 0 or two groups of three dark counters.



(–3) × (+2) = (–6)

Notice that the products for (+3) × (–2) or (+2) × (–3) are the same and the opposite of the product for (+3) × (+2).

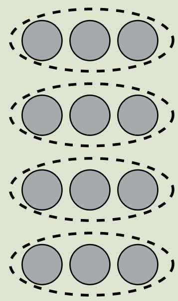
- There is no simple way to model (–3) × (–2), but it does make sense that the product should be the opposite of (+3) × (–2) and the result is +6. See Questions 5 and 6 for other ways to understand why (–a) × (–b) = +ab.

  Notice that (–3) × (–2) = (+3) × (+2) and (–3) × (+2) = (+3) × (–2).

Dividing

- We already know how to divide two positive integers. For example, (+12) ÷ (+4) = (+3), since division is the opposite of multiplication and (+3) × (+4) = (+12).
- In a similar way, (–12) ÷ (+4) = (–3) since (+4) × (–3) = –12.

  We model this calculation by thinking of dividing 12 dark counters into four equal groups and noticing that there are three dark counters (–3) in each group.



- (–12) ÷ (–4) = (+3) since (–4) × (+3) = (–12).

  We model this calculation by thinking: *How many groups of four dark counters are there in 12 dark counters?* Since the answer is three groups, (–12) ÷ (–4) = +3.
- It is difficult to model (+12) ÷ (–4), but it does make sense that the quotient is –3 since (–4) × (+3) = –12. See Question 11 for another way to understand why (+a) ÷ (–b) = –(a ÷ b).

  Notice that (12) ÷ (3) = (–12) ÷ (–3) and (–12) ÷ (+3) = (+12) ÷ (–3).

1. Solve each multiplication. Model parts a, b, and c.

a) (–6) × (+2)

b) (+3) × (–4)

c) (–3) × (+4)

d) (–7) × (+7)

e) (–8) × (+7)

f) (–10) × (+19)

2. You multiply a number by (–3) and the product is negative. List four possible numbers you could have multiplied by –3 and four you could not have multiplied.

3. You multiply two integers and the product is (–36).

a) List four possible pairs of integers.

b) Explain why –4 × (–9) is not a solution.

4. Is the statement true or false? Explain.

a) The product of two positive integers is always positive.

b) The product of two negative integers is always negative.

5. a) Complete this pattern. What do you notice?

3 × (–2) =

2 × (–2) =

1 × (–2) =

0 × (–2) =

(–1) × (–2) =

(–2) × (–2) =

b) What pattern could you create to show why (–3) × (–6) = (+18)?

6. Karan says that since 3 × 4 is the opposite of –3 × 4, then –3 × (–2) should be the opposite of 3 × (–2).

a) Do you agree with Karan?

b) How would that help Karan figure out (–3) × (–2)?

7. Model and solve at least three of these.

a) (–49) ÷ 7

b) 49 ÷ (–7)

c) 36 ÷ (6)

d) (–81) ÷ 9

e) (–22) ÷ (–2)

f) (–40) ÷ (–8)

8. Why does it make sense that 30 ÷ (–6) is negative?

9. Two other integers have the same quotient as 40 ÷ (–5). List three possible pairs of integers.

10. You divide two integers and the quotient is –12. List four possible pairs of integers.

11. a) Complete the pattern. What do you notice?

(–12) ÷ (–4) =

(–8) ÷ (–4) =

(–4) ÷ (–4) =

0 ÷ (–4) =

4 ÷ (–4) =

8 ÷ (–4) =

b) What pattern could you create to show why (+9) ÷ (–3) = –3?

12. Complete the statement to make it true.

a) If you divide a positive integer by a negative one, the result ________

b) If you divide a positive integer by a positive one, the result ________

c) If you divide a negative integer by a negative one, the result ________

d) If you divide a negative integer by a positive one, the result ________.

Order of Operations

Learning Goal

- recognizing that the same order of operations rules that apply to whole number calculations must apply to integer calculations

Open Question

- Choose values anywhere from –10 to + 10 for numbers to insert in the boxes. Some of them should be negative.

- Then choose at least three different operations to connect the boxes and add brackets if you wish. Your choice should result in a –5 when you use the order of operations rules.

☐ ☐ ☐ ☐ ☐

- Repeat three more times using at least some different integers.

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Think Sheet

- If an expression involves more than one operation, we could interpret it different ways. There need to be rules so that everyone gets the same answer. For example, think about: (–2) – (–8) × (+2).

  If we subtract –8 from –2 first and then multiply the result by +2, the answer would be +12. But if we multiply (–8) × (+2) first to get –16 and then subtract –16 from –2, the answer would be +14.

The rules for **Order of Operations** are:

Step 1: Brackets first.

For example, for (–2) × [+3 – (–4)], subtract –4 from 3 first.

<u>Notes:</u> i) The brackets are not the parentheses we use for positive and negative integers, but ones that include calculations.

ii) Sometimes brackets are round ( ) and sometimes they are square [ ]; the shape does not matter. If there are brackets inside brackets, work on the inside brackets first.

Step 2: Division and multiplication calculations next, in order from left to right. It does not matter whether division or multiplication comes first.

For example, for (–12) × (–3) + (–12) ÷ (–2), first do (–12) × (–3) and then (–12) ÷ (–2) and then add.

Step 3: Addition and subtraction calculations next, in order from left to right.

It does not matter whether addition or a subtraction comes first.

For example, for (–12) – (+3) + (–4), first subtract and then add.

If we were evaluating (–15) ÷ (+3) – [(–2) × (–8) – (–4)], we would think:

(–15) ÷ (+3) – [16 – (–4)]
–5 – 20
–25.

Some people call the **Order of Operation** rules **BEDMAS**:

B stands for brackets.

E stands for exponents. (If there are squares or cubes, etc., do them before multiplying and dividing.)

DM stands for dividing and multiplying.

AS stands for adding and subtracting.

1. Calculate using the correct order of operations.

a) $(+12) + (-4) - (+3) \times (-2) \div (-6)$

b) $(-3) \times (-4) + (-12) \div (-3) - (-5)$

c) $9 - (-8) \times 3 + 24 \div (-3)$

d) $50 \div (-2 + (-3)) \times (4 - (-2))$

e) $[-6 + (-3) - (-4 + 8)] \times 4 \div (-2)$

f) $-6 + (-3) - (-4 + 8) \times 4 \div (-2)$

2. Why are the answers to Questions 1e) and 1f) different, even though the numbers and operations involved are the same?

3. Show that, if you start at 0 and perform the following three operations in different orders, you get different results:

Divide by –4	Multiply by 2	Add –6

4. **a)** Place brackets in the expression below to get a result of 31.

   $-12 \div (-2) \times 8 - 4 \times 3 + 5$

   b) Place brackets in the same expression to get a result of 77.

   $-12 \div (-2) \times 8 - 4 \times 3 + 5$

5. **a)** Create an expression that would give the same result, if you calculated in order from left to right, as if you used the proper order of operation rules. Make sure to use integers and include both a division and a subtraction.

   b) Explain why the rules did not matter in this case.

6. Create two of your own expressions, involving integer operations that would require knowing the order of operations rules, to get a result of –2.

