GAP CLOSING

Decimals

Intermediate / Senior
Student Book

Topic 2
Decimals

This entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/2-Decimals_SB_IS.pdf. The Facilitator's Guide for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/2-Decimals_FG_IS.pdf.

Diagnostic

DO NOT USE A CALCULATOR FOR THIS DIAGNOSTIC.

1. Estimate each product, without calculating.

a) 2.4 × 1.6

b) 14.28 × 6.9

c) 2.345 × 1.2

2. Calculate each product.

a) 5 × 4.2

b) 6 × 7.25

c) 0.9 × 0.8

d) 1.9 × 0.8

e) 0.1 × 3.5

3. Suppose you know that 32 × 45 = 1440.

Explain why 3.2 × 4.5 has to be 14.40.

4. Estimate each quotient without calculating.

a) 6.5 ÷ 1.6

b) 26.88 ÷ 3.2

c) 7.316 ÷ 1.9

5. Calculate each quotient:

a) 6.4 ÷ 0.4

b) 6.4 ÷ 0.04

c) 12.2 ÷ 5

6. How can you predict why 3.4 ÷ 2 will be one tenth of 3.4 ÷ 0.2 without calculating either quotient?

7. In which order would you perform the calculations that are part of this question?
4.2 + (8.5 – 4.2) ÷ 0.6

8. **a)** Circle the correct equation.

1.5 + 4.5 × 2.5 = 15 or 1.5 + 4.5 × 2.5 = 12.75

b) Explain why it is correct.

9. Circle the greater expression. Explain your thinking.

6.4 × 1.5 + 0.4 ÷ [4.4 + 0.6] or 6.4 × 1.5 + (0.4 ÷ 4.4) + 0.6

Multiplying Decimals

Learning Goal

- reasoning about the relationship between the products of decimals and related whole numbers.

Open Question

- Choose two of the items.
- List five possible amounts of each you could buy that meet the rules below.
  - The numbers of kilograms for each item must be in the form □.□ kg or □□.□ kg.
  - Altogether you must spend between $50 and $100.

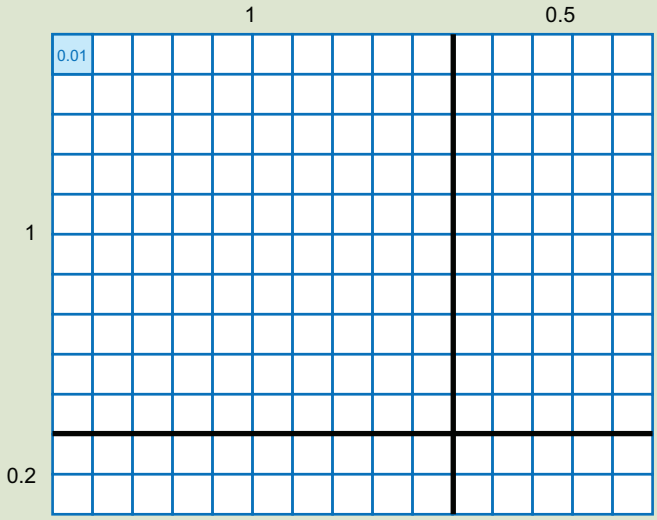
Tell the total price for each and tell why your answer makes sense.

Think Sheet

- We multiply a decimal by a whole number by adding the decimal the correct number of times. For example, 4 × 1.3 = 1.3 + 1.3 + 1.3 + 1.3 since 4 × a number means four groups of that number.

  Sometimes, we multiply a decimal by another decimal the same way. For example, we think of 1.5 × 4.2 as 1.5 groups of 4.2. That is 4.2 + half of 4.2. Since half of 4.2 is 2.1, we add 4.2 + 2.1 = 6.3.

- But it is easier to think about multiplying decimals in terms of area.
  For example, 1.5 × 1.2 is the area of a rectangle that is 1.2 units wide and 1.5 units long.



  Each section in the rectangle is 0.01 of a whole, since there are 100 squares in one whole.

  Notice that there are 12 rows of 15 sections. That makes 180 sections of 0.01.

  180 × 0.01 = 1.80 since 1.80 is 180 hundredths

  Multiplying 1.5 × 1.2 is the same as multiplying 15 × 12 and knowing that the units are hundredths (2 decimal places) or realizing the answer is close to 2 × 1 = 2.

- 0.12 is one tenth of 1.2. So if we multiply 1.5 × 0.12 instead, we could think of it as one tenth of 1.5 × 1.2.

  Since 1.5 × 1.2 = 1.80, 1.5 × 0.12 must have 0.180 as a product.

- If we know how to multiply fractions, we could think of 1.25 × 1.4 as $\frac{125}{100} \times \frac{14}{10}$, which is also (125 × 14) thousandths.

1. Suppose you know that 14 × 23 = 322. Use that information to help you determine each of these products without using a calculator.

a) 14 × 2.3

b) 1.4 × 23

c) 1.4 × 2.3

d) 0.14 × 2.3

2. Use estimation to explain your answers to Questions 1c) and 1d).

3. Predict the number of decimal places that will be in each product without using a calculator.

a) 4.3 × 1.4

b) 2.75 × 1.4

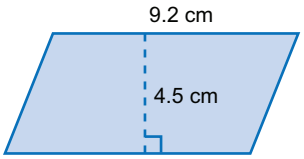
b) 8 × 2.3

d) 0.8 × 0.23

4. a) Sirloin steak costs $13.39 for 1 kilogram. How much do you pay for 2.1 kilograms?

b) How could you figure out the price for 0.21 kg without using your calculator?

5. What is the area of the parallelogram?



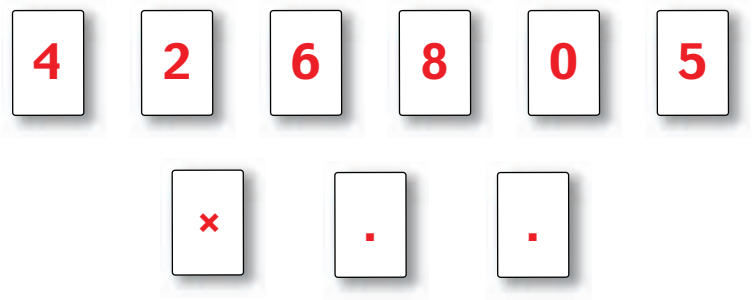
6. Suppose Shira is 1.12 times as tall as her sister, Lyla. If Lyla is 135 cm tall, how tall is Shira?

7. A car drives an average of 62.5 km per hour for 2.1 hours. How far did it go during that time?

8. Kellan used a calculator and said that $4.3 \times 1.8 = 77.4$.

Explain what is wrong with Kellan's thinking.

9. Copy each number or symbol on a separate card. (Try this question without a calculator.) Rearrange the cards to create a multiplication.



a) How could you arrange the cards to create a product with three decimal places?

b) How could you arrange the cards to create a product of about 200?

c) How could you arrange the cards to create a product of about 2000?

10. Rhys says that when you multiply two numbers with digits after the decimal point, the number of digits after the decimal point in the product is the total of the number of digits after the decimal point in the two numbers you multiply. Do you agree? Explain.

Dividing Decimals

Learning Goal

- reasoning about the relationship between the quotients of decimals and related whole numbers.

Open Question

A very large pot might hold 17.98 L of soup.

- Imagine a pot that holds just a little more. Decide how much that amount will be, but make sure it is of the form ☐☐.☐ L.

- Choose a portion size for a bowl of soup for one person. It might be anywhere between 0.2 L and 0.4 L, but make it of the form 0.☐☐ L.

- Decide how many portions (or parts of portions) the larger pot could hold.

- Justify how you know your answer makes sense.

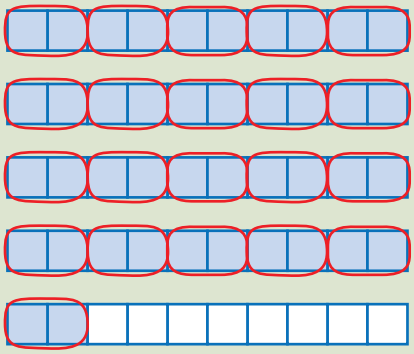
- Repeat using two more pot sizes of the form ☐☐.☐ L and portion sizes of the form 0.☐☐ L.

Think Sheet

- When we divide 4 tens by 2 tens (40 by 20), the answer is the same as the answer to 4 ÷ 2. That is because, if we divide numbers with the same units, we do not need to consider the units. The number of 2 ones in 4 ones is the same as the number of 2 tens in 4 tens or 2 hundreds in 4 hundreds.

  4.2 ÷ 0.2 can be thought of as 42 tenths ÷ 2 tenths; that is the same as 42 ÷ 2 = 21.

- We can check this another way. Since 0.2 is the same as $\frac{1}{5}$ and there are five $\frac{1}{5}$'s in a whole, there should be 5 × 4.2 sets of 0.2 in 4.2 and 5 × 4.2 = 21.0.



  We can record the computation like this:

  $0.2\overline{)4.2} = 0.2\overline{)4.2} = 2\overline{)42}$

- Similarly, 0.42 ÷ 0.02 is 42 hundredths ÷ 2 hundredths; that is also 42 ÷ 2 = 21.

- Sometimes, we might want to divide numbers where the number of decimal places is different; that means that the units are different.

  For example, 12.4 ÷ 0.02 is 124 tenths ÷ 2 hundredths.

  124 tenths = 1240 hundredths, so

  12.4 ÷ 0.02 = 1240 hundredths ÷ 2 hundredths
  = 1240 ÷ 2 = 620.

  You can check by realizing that there are 50 sets of 0.02 in one whole and that means there are 12.4 × 50 (620) sets of 0.02 in 12.4

  $0.02\overline{)12.4} = 0.02\overline{)12.40} = 2\overline{)1240}$

- Sometimes the quotient is a decimal, not a whole number. For example, consider 5.3 ÷ 0.25:

  5.3 ÷ 0.25 is 530 hundredths ÷ 25 hundredths = 530 ÷ 25.

  To divide 530 by 25, we can break the 530 into parts to divide by 25.
  For example since 530 = 500 + 25 + 5 , we can express 530 ÷ 25 as:
  $25\overline{)530} = 25\overline{)500 + 25 + 5}$
  $= 25\overline{)500 + 25 + 50\ tenths}$

  So 20 + 1 + 2 tenths = 21.2
  When we divide, we rename the "leftover" piece in terms of tenths or hundredths or thousandths.

1. Suppose you know that 714 ÷ 14 = 51. Use that information to help you determine each of these quotients.

a) 71.4 ÷ 1.4

b) 7.14 ÷ 0.14

c) 71.4 ÷ 0.14

d) 7.14 ÷ 1.4

2. Explain why your answers to Questions 1c) and 1d) make sense.

3. Estimate each quotient.

a) 22.25 ÷ 2.5

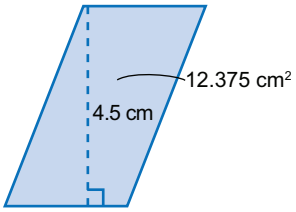
b) 6.65 ÷ 1.75

c) 14.065 ÷ 2.9

d) 18.72 ÷ 2.4

4. You bought 1.6 kg of meat. You paid $20.56. How much did 1 kg cost?

5. What is the base length of the parallelogram?



6. A car travelled 128.52 km in 1.8 hours. What was the average hourly speed?

7. Is each statement possible when you divide tenths by tenths? Explain each response.

a) The quotient could be a whole number.

b) The quotient could be a tenth.

c) The quotient could be a hundredth.

8. Kevin used a calculator and said that 48.515 ÷ 3.1 = 156.5

Explain what is wrong with Kevin's thinking.

9. You are dividing 32.4 by 0.□. You fill in the blank with a single digit.

a) What is the least possible quotient? How do you know?

b) What is the greatest possible quotient? How do you know?

10. For which division can you give an exact decimal answer? Explain.

3.4 ÷ 0.25 or 3.4 ÷ 0.3

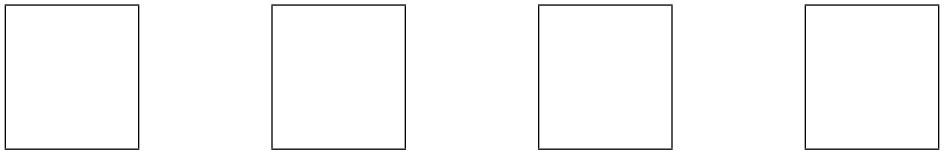
11. Rhys says that you can only divide decimals if they have the same number of decimal places. Explain whether you agree or disagree and why.

Order of Operations

Learning Goal

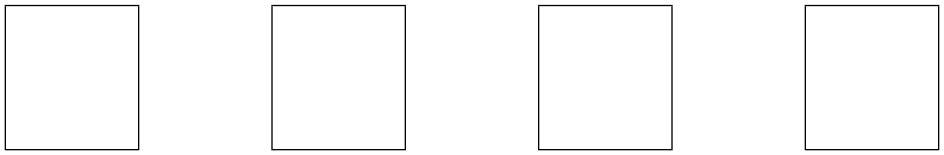
- recognizing that the same order of operations rules that apply to whole number calculations must apply to decimal calculations .

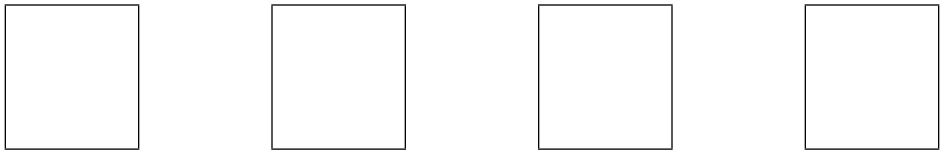
Open Question

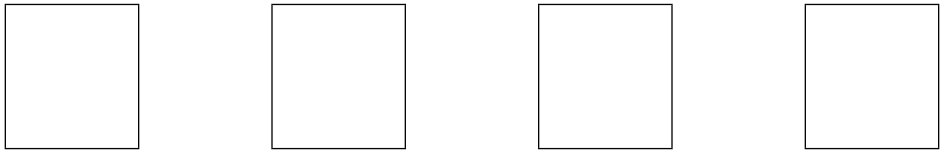


- Choose values of the form □.□ or □.□□ between 1 and 10, one for each of the boxes above.
- Choose operations to connect the boxes. Add brackets if you wish. Your choice of operations should get you as close to 1.5 as possible when you use the order of operations rules.

- Repeat three more times using different sets of decimal values and operations.







Think Sheet

When an expression involves more than one operation, different people might interpret it different ways. There need to be rules so that everyone gets the same answer. For example, consider 3.2 – 1.5 × 2:

If we subtract 1.5 from 3.2 first and then multiply the result by 2, the answer would be 3.4, but if we multiply 1.5 × 2 first and then subtract 3 from 3.2, the answer would be 0.2.

The rules for **order of operations** are:

Step 1: If there is a calculation within brackets (or parentheses), do what is inside the brackets first. For example, for 1.5 × [3 – 2.1], do the subtraction calculation inside the brackets first. If there are brackets inside brackets, work on the inside brackets first.

Note: Sometimes brackets are round ( ) and sometimes they are square [ ]; the shape does not matter.

Step 2: Perform all division and multiplication calculations next, in order from left to right.

Note: It does not matter whether the division or multiplication comes first. For example, for 2.25 × 1.5 + 4.2 ÷ 2, first do 2.25 × 1.5 and 4.2 ÷ 2 and then add.

Step 3: Perform all addition and subtraction calculations next, in order from left to right.

Note: It does not matter whether the addition or a subtraction comes first. For example, for 4.5 – 1.2 + 6.3, first subtract and then add.

If we are evaluating 5.5 ÷ 1.1 + [2.5 × 8 – 4], we:

- multiply 2.5 by 8, so the whole expression becomes (5.5) ÷ (1.1) + [20 – 4]
- subtract 4 from 20, so the whole expression becomes (5.5) ÷ (1.1) + 16
- divide 5.5 by 1.1, so the whole expression becomes 5 + 16
- add 5 and 16, so the whole expression becomes 21

Some people call the rules for Order of Operations **BEDMAS**:

B stands for brackets.

E stands for exponents. (If there are squares or cubes, etc., do them before multiplying and dividing.)

DM stands for dividing and multiplying.

AS stands for adding and subtracting.

1. Tell which calculation you would perform first.

a) 4.2 + (3.6 ÷ 0.9) × 5

b) 4.2 + 3.6 ÷ 0.9 × 5

c) (4.2 + 3.6) ÷ 0.9 × 5

d) 4.2 + (3.6 ÷ 0.9 × 5)

2. Calculate each expression using the order of operations.

a) 9.5 + 8.1 – 1.75 × 4 ÷ 3.5

b) 1.5 × 2.5 + 3.75 ÷ 1.25 – 0.8

c) 13.25 – 1.2 × 3 + 7.8 ÷ 0.6

d) 6.4 ÷ (0.4 + 1.2) × (4 – 3.1)

e) [11.3 + 0.67 – (2.8 + 1.7)] × 5 ÷ 2.5

f) 11.53 + 4.2 – (0.4 + 2.6) × 6.6 ÷ 2.2

3. Why are the answers to (4.5 – 2.5) × 4.2 ÷ 2.1 and 4.5 – 2.5 × (4.2 ÷ 2.1) different even though the calculations are the same?

4. Show that if you start at 0 and perform these three operations in different orders, you get different results:

Divide by 1.5	Multiply by 3	Add 2.5

5. Place brackets in the expression below to get 3.5 as a result.

0.3 + 1.7 × 2 – 2.5 ÷ 5

6. a) Create an expression involving at least three operations that would give the same result if you calculated in order from left to right as if you used the proper order of operation rules. Make sure to use decimals.

b) Explain why the rules did not matter.

7. a) Create an expression involving decimal operations that would require knowing the order of operations rules to get a result of 4.5.

b) What about the order of operations rules would you have needed to know?

