GAP CLOSING

Decimals

Intermediate / Senior
Facilitator's Guide

Topic 2
Decimals

The Facilitator's Guide for this entire module, including the Diagnostic plus all Intervention Materials, can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/2-Decimals_FG_IS.pdf. The Student Book for this module can be downloaded at http://www.edugains.ca/resources/LearningMaterials/GapClosing/Grade9/2-Decimals_SB_IS.pdf.

DECIMALS

Relevant Learning Expectations for Grade 9

MPM1D

Number Sense and Algebra

- substitute into and evaluate algebraic expressions involving exponents
- simplify numerical expressions involving integers and rational numbers, with and without the use of technology
- solve problems requiring the manipulation of expressions arising from application of percent, ratio, rate, and proportion
- solve first-degree equations, including equations with fractional coefficients
- solve problems that can be modelled with first-degree equations …

Linear Relations

- determine values of a linear relation by using a table of values, by using the equation of the relation, …
- determine, through investigation, various formulas for the slope of a line segment or a line
- determine the equation of a line from information about the line …

Measurement and Geometry

- pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures
- solve problems involving the area and perimeters of composite two-dimensional shapes
- solve problems involving the surface area and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures

MPM1P

Number Sense and Algebra

- solve for the unknown value in a proportion, using a variety of methods
- make comparisons using unit rates
- solve problems involving ratios, rates and directly proportional relationships in various contexts, using a variety of methods
- solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms
- simplify numerical expressions involving integers and rational numbers, with and without the use of technology
- substitute into and evaluate algebraic expressions involving exponents
- solve first-degree equations with non-fractional coefficients, using a variety of tools and strategies
- substitute into algebraic equations and solve for one variable in the first degree

Linear Relations

- determine, through investigation, that the rate of change of a linear relation can be found by choosing any two points on the line that represents the relation, finding the vertical change between the points and the horizontal change between the point and writing the ratio $\frac{\text{rise}}{\text{run}}$
- determine values of a linear relation by using a table of values, by using the equation of the relation …

Measurement and Geometry

- solve problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area
- solve problems involving the areas and perimeters of composite two-dimensional shapes
- solve problems involving the volumes of prisms, pyramids, cylinders, cones and spheres

Possible reasons why a student might struggle when multiplying and dividing with decimals

Students may struggle with decimal multiplication and division. Some of the problems include:

- not estimating to see if an answer makes sense;
- misplacing of a decimal point in the answer;
- not explaining why the processes for multiplying and dividing are what they are;
- not appling the order of operations rules properly.

DIAGNOSTIC

Administer the diagnostic

Using the diagnostic results to personalize the interventions

Materials
- 2-sided or 2 colours of counters
- blank number lines

Intervention materials are included on each of these topics:
- multiplying decimals
- dividing decimals
- order of operations

You may use all or only part of these sets of materials, based on student performance with the diagnostic. If students need help in understanding the intent of a question in the diagnostic, you are encouraged to clarify that intent.

Evaluating Diagnostic Results	Suggested Intervention Materials
If students struggle with Questions1–3	use *Multiplying Decimals*
If students struggle with Questions 4–6	use *Dividing Decimals*
If students struggle with Questions 7–9	use *Order of Operations*

Solutions

1. **a)** e.g., 4 **b)** e.g., 100
c) e.g., 3

2. **a)** 21.0 **b)** 43.50
c) 0.72 **d)** 1.52
e) 0.35

3. e.g., 3.2 × 4.5 is 32 tenths × 45 tenths, so it is 32 × 45 tenths × tenths, which would be hundredths.

4. **a)** e.g., 4 **b)** e.g., 9
c) e.g., 4

5. **a)** 16 **b)** 160
c) 2.44

6. e.g., 0.2 is $\frac{1}{10}$ as big as 2, so 10 times as many groups of 0.2 will fit into 3.4

7. I would first do the subtraction, then the division, and then the addition.

8. **a)** the second one.
b) BEDMAS says do multiplication calculations before doing addition calculations.

9. the second one, e.g., The first part is the same so it does not matter. The difference is whether you add 0.4 ÷ 5 which is 0.08 or whether you add 0.6 and some more. The second one is obviously more.

Diagnostic

DO NOT USE A CALCULATOR FOR THIS DIAGNOSTIC.

1. Estimate each product, without calculating.

a) 2.4 × 1.6

b) 14.28 × 6.9

c) 2.345 × 1.2

2. Calculate each product.

a) 5 × 4.2

b) 6 × 7.25

c) 0.9 × 0.8

d) 1.9 × 0.8

e) 0.1 × 3.5

Diagnostic (Continued)

3. Suppose you know that 32 × 45 = 1440.
Explain why 3.2 × 4.5 has to be 14.40.

4. Estimate each quotient without calculating.

a) 6.5 ÷ 1.6

b) 26.88 ÷ 3.2

c) 7.316 ÷ 1.9

5. Calculate each quotient:

a) 6.4 ÷ 0.4

b) 6.4 ÷ 0.04

c) 12.2 ÷ 5

Diagnostic (Continued)

6. How can you predict why 3.4 ÷ 2 will be one tenth of 3.4 ÷ 0.2 without calculating either quotient?

7. In which order would you perform the calculations that are part of this question?
4.2 + (8.5 – 4.2) ÷ 0.6

8. **a)** Circle the correct equation.
1.5 + 4.5 × 2.5 = 15 or 1.5 + 4.5 × 2.5 = 12.75

b) Explain why it is correct.

9. Circle the greater expression. Explain your thinking.
6.4 × 1.5 + 0.4 ÷ [4.4 + 0.6] or 6.4 × 1.5 + (0.4 ÷ 4.4) + 0.6

USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for successfully working with equations, expressions, and calculating measurements involving decimals.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using you choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

Multiplying Decimals

Learning Goal

- reasoning about the relationship between the products of decimals and related whole numbers.

Materials
- calculators

Questions to Ask Before Using the Open Question

◊ *Suppose one item costs $34.29 and another costs $53.72. How would you estimate the total?* (I would use $30 + $50 to get $80.)
◊ *Would the actual amount be more or less? About how much?* (e.g., about $8 more since 34 is 4 more than 30 and 53.72 is almost 54 and that is 4 more than 50.)
◊ *Suppose you bought two of the $34.29 items. About how much would you have to pay?* (e.g., about $70) *How would you figure out the exact amount if you did not have a calculator?* (e.g., I would multiply 34 by 2 for the dollars and 29 by 2 for the cents.)
◊ *Suppose the $34.29 was for 1 kg of an item. How much would 0.1 kg cost? How do you know?* (e.g., $3.43, since it is one tenth as much.)

Using the Open Question

Make sure students provide five possible answers.

By viewing or listening to student responses, note if they:
- can estimate decimal products and sums;
- can calculate decimal products;
- can use one answer to help determine another one.

Consolidating and Reflecting on the Open Question

◊ *How did you come up with your first solution?* (e.g., I figured that salmon is about $16 for 1 kg and I know that 4 × 16 = 64, so, if I added about 25, that would be okay. That could be about two kg of chicken.)
◊ *Once you knew that 4 kg of salmon and 2 kg of chicken was $91.70, how would you know that 4.1 kg of salmon and 2 kg of chicken would also work?* (e.g., Since 0.1 kg of salmon is only an extra $1.58.)
◊ *How would knowing that the salmon and chicken cost almost the same help you get another answer once you had one?* (e.g., I could just use salmon in one answer and chicken in the other.)
◊ *How would comparing the costs of salmon and ground meat help you get another answer once you had one?* (e.g., If I used salmon in one answer, I could use double as much ground meat instead.)

Solutions

e.g.,
- 3.0 kg of salmon and 4.2 kg of ground meat → Total price is $84.38
- 3.1 kg of salmon and 4.0 kg of ground meant → Total price is $84.20
- 3.2 kg of salmon and 4.1 kg of ground meat → Total price is $86.66
- 5.0 kg of salmon and 1.2 kg of chicken → Total price is $96.05
- 4.9 kg of salmon and 1.1 kg of chicken → Total price is $93.04

Think Sheet

Materials
- grid paper
- cards
- calculators

Questions to Ask Before Assigning the Think Sheet

◊ *Why might you draw a rectangle to show what 5 × 8 means?* (e.g., The area of a rectangle that is 8 units by 5 units is what 5 × 8 is.)
◊ *What would you draw to show 5.5 × 8?* (e.g., a rectangle with a length of 8 and width of 5.5)
◊ *Why does it make sense that 5.5 × 8 is one tenth of 55 × 8?* (e.g., Since 5.5 is 55 tenths, so it is like 55 × 8 tenths, instead of 55 × 8 ones.)
◊ *How do you know that the area is actually 40 + 4?* (e.g., 5 × 8 is 40 and then 0.5 × 8 means half of 8 and that it is 4.)

Using the Think Sheet

Read through the introductory box with the students and clarify any questions they might have.

Encourage students to copy the rectangle to see the 100 squares for themselves. Encourage them to use the grid to model some of the other products in the questions.

Assign the tasks.

By viewing or listening to student responses, note if they:
- can estimate decimal products;
- can calculate decimal products;
- can use one answer to help determine another one.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How would you explain to someone why the product of 1.4 and 2.3 has the same digits as the digits for the product of 14 × 23?* (e.g., If there is a rectangle with length 2.3 and with 1.4, you would see 14 rows of 23 squares, which is 14 × 23, but the sections would be hundredths.)
◊ *How could you predict that 8 × 2.3 is between 16 and 20?* (e.g., 8 × 2 is 16 and 8 × 2.3 is a bit more. I know that 8 × 2.5 is 20 and that is too much.)
◊ *How could you predict that 0.8 × 2.3 is less than 2?* (e.g., It is not even one group of 2.3). *How do you know the answer is hundredths?* (e.g., One tenth of 2.3 is 0.23 and eight groups of hundredths is hundredths.)
◊ *How did you know that 80.5 × 4.62 would end up as a number involving thousandths?* (e.g., 80.5 is one tenth of 805. So 80.5 × 4.62 is one tenth of 805 × 4.62. But 805 × hundredths is hundredths. If you have tenths of hundredths, you have thousandths.)

Solutions

1. **a)** 32.2 **b)** 32.2 **c)** 3.22 **d)** 0.322
2. For Question 1c) e.g., the answer is about one and a half groups of 2, which is about 3.
For Question 1d) e.g., the answer is about one tenth of one and a half groups of 2; that is one tenth of 3, or about 0.3.
3. **a)** 2 **b)** 3 **c)** 1 **d)** 3
4. **a)** $28.12 **b)** I would divide the first price by ten.
5. 41.4 cm^2
6. 151.2 cm
7. 131.25 km
8. e.g., The answer is less than twice 4.3, so 77.4 is too high. He probably keyed the decimal point in the wrong spot.
9. **a)** e.g., 4.52 × 80.6 **b)** e.g., 50.8 × 4.26 **c)** e.g., 50.8 × 42.6
10. Yes. e.g., If there were two decimal places in each number, you would be multiplying hundredths × hundredths which are ten thousandths, or four decimal places. But, if it were hundredths × tenths, it would be thousandths or only three decimal places.

Open Question

Multiplying Decimals

Learning Goal

- reasoning about the relationship between the products of decimals and related whole numbers.

Open Question

- Choose two of the items.
- List five possible amounts of each you could buy that meet the rules below.
  - The numbers of kilograms for each item must be in the form ☐.☐ kg or ☐☐.☐ kg.
  - Altogether you must spend between $50 and $100.

Tell the total price for each and tell why your answer makes sense.

Think Sheet

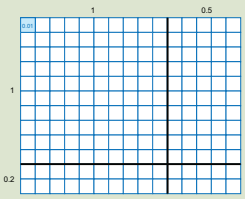
Multiplying Decimals (Continued)

Think Sheet

- We multiply a decimal by a whole number by adding the decimal the correct number of times. For example, 4 × 1.3 = 1.3 + 1.3 + 1.3 + 1.3 since 4 × a number means four groups of that number.

  Sometimes, we multiply a decimal by another decimal the same way. For example, we think of 1.5 × 4.2 as 1.5 groups of 4.2. That is 4.2 + half of 4.2. Since half of 4.2 is 2.1, we add 4.2 + 2.1 = 6.3.

- But it is easier to think about multiplying decimals in terms of area. For example, 1.5 × 1.2 is the area of a rectangle that is 1.2 units wide and 1.5 units long.



  Each section in the rectangle is 0.01 of a whole, since there are 100 squares in one whole.

  Notice that there are 12 rows of 15 sections. That makes 180 sections of 0.01.

  180 × 0.01 = 1.80 since 1.80 is 180 hundredths

  Multiplying 1.5 × 1.2 is the same as multiplying 15 × 12 and knowing that the units are hundredths (2 decimal places) or realizing the answer is close to 2 × 1 = 2.

- 0.12 is one tenth of 1.2. So if we multiply 1.5 × 0.12 instead, we could think of it as one tenth of 1.5 × 1.2.

  Since 1.5 × 1.2 = 1.80, 1.5 × 0.12 must have 0.180 as a product.

- If we know how to multiply fractions, we could think of 1.25 × 1.4 as $\frac{125}{100} \times \frac{14}{10}$, which is also (125 × 14) thousandths.

Multiplying Decimals (Continued)

1. Suppose you know that 14 × 23 = 322. Use that information to help you determine each of these products without using a calculator.

a) 14 × 2.3 **b)** 1.4 × 23

c) 1.4 × 2.3 **d)** 0.14 × 2.3

2. Use estimation to explain your answers to Questions 1c) and 1d).

3. Predict the number of decimal places that will be in each product without using a calculator.

a) 4.3 × 1.4 **b)** 2.75 × 1.4

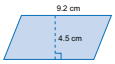
b) 8 × 2.3 **d)** 0.8 × 0.23

4. a) Sirloin steak costs $13.39 for 1 kilogram. How much do you pay for 2.1 kilograms?

b) How could you figure out the price for 0.21 kg without using your calculator?

Multiplying Decimals (Continued)

5. What is the area of the parallelogram?



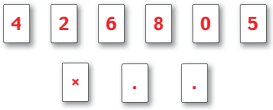
6. Suppose Shira is 1.12 times as tall as her sister, Lyla. If Lyla is 135 cm tall, how tall is Shira?

7. A car drives an average of 62.5 km per hour for 2.1 hours. How far did it go during that time?

8. Kellan used a calculator and said that 4.3 × 1.8 = 77.4.
Explain what is wrong with Kellan's thinking.

Multiplying Decimals (Continued)

9. Copy each number or symbol on a separate card. (Try this question without a calculator.) Rearrange the cards to create a multiplication.



a) How could you arrange the cards to create a product with three decimal places?

b) How could you arrange the cards to create a product of about 200?

c) How could you arrange the cards to create a product of about 2000?

10. Rhys says that when you multiply two numbers with digits after the decimal point, the number of digits after the decimal point in the product is the total of the number of digits after the decimal point in the two numbers you multiply. Do you agree? Explain.

Dividing Decimals

Learning Goal

- reasoning about the relationship between the quotients of decimals and related whole numbers.

Open Question

Materials
- calculators

Questions to Ask Before Using the Open Question

◊ *Suppose you walked 10.2 m and each step was 0.9 m. How would you figure out how many steps you took?* (e.g., I would draw a number line.) *What would it look like?* (e.g., I would use a number line going from 0 to 10.2 with jumps of 0.9 and would count the jumps)

◊ *What mathematical operation are you using?* (e.g., I was multiplying 0.9 × the number of steps.) *But you did not know the number of steps when you started. What could you have done with the 10.2 and the 0.9?* (I could have used division.)

◊ *Why did it make sense that the answer was more than 10.2?* (e.g., If the steps were 1 metre, it would take 10.2 steps and the steps are smaller, so it would take more of them.)

◊ *Why would 10.2 ÷ 0.9 have the same answer as 102 ÷ 9?* (e.g., It is really 102 tenths ÷ 9 tenths since you want to know how many 9s are in 102.)

Using the Open Question

Make sure students provide three possible answers.

By viewing or listening to student responses, note if they:
- can estimate decimal quotients;
- can recognize when to divide in a decimal situation;
- can use one answer to help determine another one.

Consolidating and Reflecting on the Open Question

◊ *How did you know that your first solution made sense?* (e.g., I used 18.1 L and 0.22 L for the portion size. Since 0.22 is close to ¼ of a litre and 18.1 is close to 18 L, I multiplied 4 × 18 = 72 and that was not too far from my calculator answer of about 82.)

◊ *If you had chosen 18.1 L and 0.25 L, would your answer have been higher or lower? Why?* (e.g., Lower, since the portion size is bigger, there will be not as many.)

◊ *Can you calculate 18.1 ÷ 0.25 by not considering decimals and just writing 181 ÷ 25?* (e.g., No, because that would be only about 7 or 8 and that is not enough.) *What is the mistake?* (It should have been 1810 ÷ 25 since you want to divide hundredths by hundredths.)

Solutions

e.g.

First Choice: Pot holds 18.0 L/Portion size is 0.25 L

The pot could hold 72 portions.

I know that since there are four groups of 0.25 L in 1 L, so I multiplied 4 × 18.

Second Choice: Pot holds 18.2 L/Portion size is 0.35 L

The pot could hold 52 portions.

I used a calculator to divide 18.2 by 0.35. The answer was 52. I think that makes sense since 0.35 is close to 1/3 of a litre, so 3 portions make 1 litre. If there are 52 portions, that's about 18 L.

Third Choice: Pot holds 18.5 L/Portion size is 0.33 L.

The pot could hold 56 portions and a tiny bit more.

I used a calculator to divide 18.5 by 0.33. I think that makes sense, since there is just about one more portion size worth of soup than in my second choice. But the portion is a bit smaller, so 56 compared to 52 make sense.

Think Sheet

Materials
- calculators
- hundredth grids

Questions to Ask Before Assigning the Think Sheet

Note: Show the hundredths grid.

◊ *How many sets of 2 hundredths are in 32 hundredths?* (16)
◊ *Why could you write what you did as 0.32 ÷ 0.02?* (e.g., You are finding out how many groups of one thing are in another and that is dividing.)
◊ *Show four hundredths grids. How many sets of 2 tenths are in 32 tenths?* (16)
◊ *Why could you write what you did as 3.2 ÷ 0.2?* (e.g., You are finding out how many groups of 2 tenths are in 3.2 tenths.)
◊ *Why is that the same as 32 ÷ 2?* (e.g., You are counting how many groups of 2 strips are in 32 strips.)

Using the Think Sheet

Read through the introductory box with the students or clarify any questions they might have.

Assign the tasks.

By viewing or listening to student responses, note if they:
- can estimate decimal quotients;
- can recognize when to divide in a decimal situation;
- can use one answer to help determine another one.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *How would you explain to someone why the quotient for 71.4 ÷ 1.4 is the same as for 7.14 ÷ 0.14?* (e.g., Both expressions are really asking how many 14 of something are in 714 of that thing. The first time it is tenths and the second time it is hundredths.)
◊ *How could you predict that 22.25 ÷ 2.5 is a bit less than 10?* (e.g., 10 × 2.5 = 25 and 22.25 is less, but not a lot less.)
◊ *How could you predict that 128.52 ÷ 1.8 is about 65?* (e.g., it is about 2 into 130.)
◊ *When was the quotient of tenths divided by tenths a whole number?* (e.g., only if the whole number of tenths you are dividing by is a factor of the whole number of tenths you are dividing into.)
◊ *If someone asked you to quickly come up with a different division question with the same answer as 57.3 ÷ 3.4, what would it be?* (e.g., 573 ÷ 34)

Solutions

1. **a)** 51 **b)** 51
c) 510 **d)** 5.1

2. For Question 1c) e.g., 0.14 is hundredths, so I wanted to write 71.4 as hundredths. I wrote 71.40, so I was dividing 7140 by 14. That is 10 times as much as 714 ÷ 14. For Question 1d) e.g., 7.14 is hundredths, so I rewrote 1.4 as 1.40; that is 140 hundredths. I divided 714 by 140. That is one tenth as much as 714 ÷ 14.

3. **a)** e.g., about 10 **b)** e.g., about 3
c) e.g., about 5 **d)** e.g., about 9.5

4. $12.85

5. 2.75 cm

6. 71.4 km/h

7. **a)** Yes, e.g., 0.4 ÷ 0.2 = 2 **b)** Yes, e.g., 0.5 ÷ 0.2 = 2.5
c) Yes, e.g., 0.5 ÷ 0.4 = 1.25

8. e.g., If you are dividing about 48 by about 3, the quotient should be about 16, so the answer should have been 15.65.

9. **a)** 36 since I divided by the greatest amount that I could so the fewest number of pieces would fit in; 32.4 ÷ 0.9 = 36.

b) 324 since I divided by the least amount; 32.4 ÷ 0.1 = 324

10. The first one; the first answer is 13.6 which is an exact answer. The second answer is close to 11.33, but it is a repeating decimal answer – 11.333…..

11. e.g., I disagree. You could divide 4.24 by 0.2 by figuring out how many sets of 0.2 are in 4.24. You know that there are five sets of 0.2 in 1, so you just multiply 5 × 4.24 to get an answer and then you have done the division.

Open Question

Dividing Decimals

Learning Goal

- reasoning about the relationship between the quotients of decimals and related whole numbers.

Open Question

A very large pot might hold 17.98 L of soup.

- Imagine a pot that holds just a little more. Decide how much that amount will be, but make sure it is of the form ☐☐.☐ L.
- Choose a portion size for a bowl of soup for one person. It might be anywhere between 0.2 L and 0.4 L, but make it of the form 0.☐☐ L.
- Decide how many portions (or parts of portions) the larger pot could hold.
- Justify how you know your answer makes sense.
- Repeat using two more pot sizes of the form ☐☐.☐ L and portion sizes of the form 0.☐☐ L.

Think Sheet

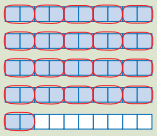
Dividing Decimals (Continued)

Think Sheet

- When we divide 4 tens by 2 tens (40 by 20), the answer is the same as the answer to 4 ÷ 2. That is because, if we divide numbers with the same units, we do not need to consider the units. The number of 2 ones in 4 ones is the same as the number of 2 tens in 4 tens or 2 hundreds in 4 hundreds.

  4.2 ÷ 0.2 can be thought of as 42 tenths ÷ 2 tenths; that is the same as 42 ÷ 2 = 21.

- We can check this another way. Since 0.2 is the same as $\frac{1}{5}$ and there are five $\frac{1}{5}$'s in a whole, there should be 5 × 4.2 sets of 0.2 in 4.2 and 5 × 4.2 = 21.0.



  We can record the computation like this:

  $0.2\overline{)4.2} = 0.2\overline{)4.2} = 2\overline{)42}$

- Similarly, 0.42 ÷ 0.02 is 42 hundredths ÷ 2 hundredths; that is also 42 ÷ 2 = 21.
- Sometimes, we might want to divide numbers where the number of decimal places is different; that means that the units are different.

  For example, 12.4 ÷ 0.02 is 124 tenths ÷ 2 hundredths.

  124 tenths = 1240 hundredths, so

  12.4 ÷ 0.02 = 1240 hundredths ÷ 2 hundredths
  = 1240 ÷ 2 = 620.

  You can check by realizing that there are 50 sets of 0.02 in one whole and that means there are 12.4 × 50 (620) sets of 0.02 in 12.4

  $0.02\overline{)12.4} = 0.02\overline{)12.40} = 2\overline{)1240}$

Dividing Decimals (Continued)

- Sometimes the quotient is a decimal, not a whole number. For example, consider 5.3 ÷ 0.25:

  5.3 ÷ 0.25 is 530 hundredths ÷ 25 hundredths = 530 ÷ 25.

  To divide 530 by 25, we can break the 530 into parts to divide by 25.
  For example since 530 = 500 + 25 + 5 , we can express 530 ÷ 25 as:

  $25\overline{)530} = 25\overline{)500 + 25 + 5}$
  $= 25\overline{)500 + 25 + 50\ tenths}$ (20 + 1 + 2 tenths)

  So 20 + 1 + 2 tenths = 21.2
  When we divide, we rename the "leftover" piece in terms of tenths or hundredths or thousandths.

1. Suppose you know that 714 ÷ 14 = 51. Use that information to help you determine each of these quotients.

a) 71.4 ÷ 1.4 **b)** 7.14 ÷ 0.14

c) 71.4 ÷ 0.14 **d)** 7.14 ÷ 1.4

2. Explain why your answers to Questions 1c) and 1d) make sense.

3. Estimate each quotient.

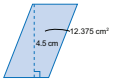
a) 22.25 ÷ 2.5 **b)** 6.65 ÷ 1.75

c) 14.065 ÷ 2.9 **d)** 18.72 ÷ 2.4

Dividing Decimals (Continued)

4. You bought 1.6 kg of meat. You paid $20.56. How much did 1 kg cost?

5. What is the base length of the parallelogram?



6. A car travelled 128.52 km in 1.8 hours. What was the average hourly speed?

7. Is each statement possible when you divide tenths by tenths? Explain each response.

a) The quotient could be a whole number.

b) The quotient could be a tenth.

c) The quotient could be a hundredth.

8. Kevin used a calculator and said that 48.515 ÷ 3.1 = 156.5
Explain what is wrong with Kevin's thinking.

Dividing Decimals (Continued)

9. You are dividing 32.4 by 0.□. You fill in the blank with a single digit.

a) What is the least possible quotient? How do you know?

b) What is the greatest possible quotient? How do you know?

10. For which division can you give an exact decimal answer? Explain.
3.4 ÷ 0.25 or 3.4 ÷ 0.3

11. Rhys says that you can only divide decimals if they have the same number of decimal places. Explain whether you agree or disagree and why.

Order of Operations

Learning Goal

- recognizing that the same order of operations rules that apply to whole number calculations must apply to decimal calculations .

Open Question

Materials
- cards
- calculators

Questions to Ask Before Using the Open Question

◊ *Suppose I asked you to add 4 to 8 and then multiply by 3. What would the result be?* (36)
◊ *Why would we not write (4 + 8 × 3)?* (e.g., That says to multiply 8 by 3 first and then add it to 4, so it is different.)
◊ *What do you remember about the rules for order of operations with whole numbers?* (e.g., I should do brackets first, then multiply and divide, and then add and subtract.)

Using the Open Question

Encourage students to use different sets of operations in their four expressions. You might suggest students use cards for their numbers to move them around. They might even use cards for brackets and operation signs.

By viewing or listening to student responses, note if they:
- correctly add, subtract, multiply and divide decimals;
- correctly apply the rules for order of operations;
- recognize when the rules are not needed;
- demonstrate flexibility in creating their expressions.

Consolidating and Reflecting on the Open Question

◊ *How did you make sure the results would be close to 1.5?* (e.g., I figured out different ways to write 1.5, such as 3 – 1.5 and then I used combinations to get 3 that involved three numbers.)
◊ *Did you actually need to know the order of operations rules for all of them?* (e.g., No, since, for my third one, I used only multiplications and division and you can just do them in order from left to right.)
◊ *Would it have been easy to come up with an alternate answer once you had one?* (e.g., Yes, since you just change one or more of the values a little bit. For example, instead of 1.5 × (4.2 ÷ 2.1) – 1.5, you could use 1.49 × (4.19 ÷ 2.1) – 1.5 and it would be close.)

Solutions

e.g.,
1.5 × (4.2 ÷ 2.1) – 1.5 = 1.5
(9.63 – 8.1) × 5.0 ÷ 5.1 = 1.5
9.75 ÷ 3.25 × 4.2 ÷ 8.4 = 1.5
6.7 × 7.3 ÷ 7.5 ÷ 4.35 is about 1.499

Think Sheet

Materials
- integer chips (2 colours of counters)
- blank number lines

Questions to Ask Before Assigning the Think Sheet

◊ *Suppose I asked you to add 4 to 8 and then multiply by 3. What would the result be?* (36)
◊ *Why would we not write (4 + 8 × 3)?* (e.g., That says to multiply 8 by 3 first and then add it to 4, so it is different.)
◊ *What do you remember about the rules for order of operations with whole numbers?* (e.g., I should do the brackets first, then multiply and divide, and then add and subtract.)

Using the Think Sheet

Read through the introductory box with the students and clarify any questions they might have.

Assign the tasks.

By viewing or listening to student responses, note if they:
- correctly add, subtract, multiply and divide decimals;
- correctly apply the rules for order of operations;
- recognize when the rules are not needed;
- problem-solve to adjust expressions to result in a certain value;
- demonstrate flexibility in creating expressions that result in a certain value.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ *Did you actually need to know the order of operations rules for all of the parts of Question 2?* (e.g., Yes, since I had to know to do brackets first or to multiply or divide first.)
◊ *How did you solve Question 5?* (e.g., I realized that the last thing to do was to subtract 2.5 ÷ 5, which is 0.5, so I tried to get the rest to equal 4. I realized that if you multiplied 2 × 2, it would work, so I put 0.3 and 1.7 in brackets.)
◊ *How did you create the expression for Question 6?* (e.g., I made sure the addition was last and the other operations were all × and ÷.)
◊ *If you know the rules for order of operations for whole numbers, is there anything new you need to know for order of operations involving decimals?* (No)

Solutions

1. **a)** 3.6 ÷ 0.9 **b)** 3.6 ÷ 0.9
c) 4.2 + 3.6 **d)** 3.6 ÷ 0.9

2. **a)** 15.6 **b)** 5.95
c) 22.65 **d)** 3.6
e) 14.94 **f)** 6.73

3. e.g., In the first one you multiply 4.5 - 2.5 by 2 but in the second one you multiply 4.2 ÷ 2.1 by 2.5 and then subtract it from 4.5. Those are different things to do.

4. e.g., If you start with 0: multiply by 3, divide by 1.5, and then add 2.5, you end up with 2.5. If you add 2.5, multiply by 3 and then divide by 1.5, you end up with 5.

5. e.g., (0.3 + 1.7) × 2 – 2.5 ÷ 5

6. **a)** e.g., 4.3 × 3.1 ÷ 0.1 + 2.3
b) If you go from left to right you would do all the multiplications and divisions in order first anyway.

7. **a)** e.g., (2.6 + 0.4) × 3 ÷ (1.3 + 0.7)
b) I needed to know that I should do what is in brackets first.

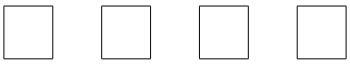
Open Question

Order of Operations

Learning Goal

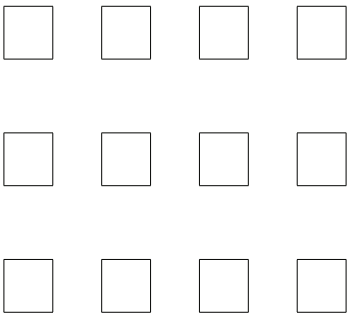
- recognizing that the same order of operations rules that apply to whole number calculations must apply to decimal calculations .

Open Question



- Choose values of the form ☐.☐ or ☐.☐☐ between 1 and 10, one for each of the boxes above.
- Choose operations to connect the boxes. Add brackets if you wish. Your choice of operations should get you as close to 1.5 as possible when you use the order of operations rules.

- Repeat three more times using different sets of decimal values and operations.



Think Sheet

Order of Operations (Continued)

Think Sheet

When an expression involves more than one operation, different people might interpret it different ways. There need to be rules so that everyone gets the same answer. For example, consider 3.2 – 1.5 × 2:

If we subtract 1.5 from 3.2 first and then multiply the result by 2, the answer would be 3.4, but if we multiply 1.5 × 2 first and then subtract 3 from 3.2, the answer would be 0.2.

The rules for **order of operations** are:

Step 1: If there is a calculation within brackets (or parentheses), do what is inside the brackets first. For example, for 1.5 × [3 – 2.1], do the subtraction calculation inside the brackets first. If there are brackets inside brackets, work on the inside brackets first.

Note: Sometimes brackets are round ( ) and sometimes they are square [ ]; the shape does not matter.

Step 2: Perform all division and multiplication calculations next, in order from left to right.

Note: It does not matter whether the division or multiplication comes first. For example, for 2.25 × 1.5 + 4.2 ÷ 2, first do 2.25 × 1.5 and 4.2 ÷ 2 and then add.

Step 3: Perform all addition and subtraction calculations next, in order from left to right.

Note: It does not matter whether the addition or a subtraction comes first. For example, for 4.5 – 1.2 + 6.3, first subtract and then add.

If we are evaluating 5.5 ÷ 1.1 + [2.5 × 8 – 4], we:

- multiply 2.5 by 8, so the whole expression becomes (5.5) ÷ (1.1) + [20 – 4]
- subtract 4 from 20, so the whole expression becomes (5.5) ÷ (1.1) + 16
- divide 5.5 by 1.1, so the whole expression becomes 5 + 16
- add 5 and 16, so the whole expression becomes 21

Some people call the rules for Order of Operations **BEDMAS**:

B stands for brackets.

E stands for exponents. (If there are squares or cubes, etc., do them before multiplying and dividing.)

DM stands for dividing and multiplying.

AS stands for adding and subtracting.

Order of Operations (Continued)

1. Tell which calculation you would perform first.

a) 4.2 + (3.6 ÷ 0.9) × 5

b) 4.2 + 3.6 ÷ 0.9 × 5

c) (4.2 + 3.6) ÷ 0.9 × 5

d) 4.2 + (3.6 ÷ 0.9 × 5)

2. Calculate each expression using the order of operations.

a) 9.5 + 8.1 – 1.75 × 4 ÷ 3.5

b) 1.5 × 2.5 + 3.75 ÷ 1.25 – 0.8

c) 13.25 – 1.2 × 3 + 7.8 ÷ 0.6

d) 6.4 ÷ (0.4 + 1.2) × (4 – 3.1)

e) [11.3 + 0.67 – (2.8 + 1.7)] × 5 ÷ 2.5

f) 11.53 + 4.2 – (0.4 + 2.6) × 6.6 ÷ 2.2

Order of Operations (Continued)

3. Why are the answers to (4.5 – 2.5) × 4.2 ÷ 2.1 and 4.5 – 2.5 × (4.2 ÷ 2.1) different even though the calculations are the same?

4. Show that if you start at 0 and perform these three operations in different orders, you get different results:

Divide by 1.5	Multiply by 3	Add 2.5

5. Place brackets in the expression below to get 3.5 as a result.

0.3 + 1.7 × 2 – 2.5 ÷ 5

6. a) Create an expression involving at least three operations that would give the same result if you calculated in order from left to right as if you used the proper order of operation rules. Make sure to use decimals.

b) Explain why the rules did not matter.

7. a) Create an expression involving decimal operations that would require knowing the order of operations rules to get a result of 4.5.

b) What about the order of operations rules would you have needed to know?