

GAP CLOSING

Fractions

Intermediate / Senior
Student Book

Fractions

Diagnostic	3
Comparing Fractions	6
Adding Fractions	13
Subtracting Fractions	21
Multiplying Fractions	28
Dividing Fractions	34
Relating Situations to Fraction Operations	40
Template	
Fraction Tower	46

Diagnostic

1. List three fractions equivalent (equal) to each fraction.

a) $\frac{2}{3}$

b) $\frac{8}{10}$

c) $\frac{24}{100}$

2. Use a greater than (>) or less than (<) sign to make these statements true.

a) $\frac{1}{2} \square \frac{3}{8}$

b) $\frac{3}{8} \square \frac{3}{10}$

c) $\frac{5}{6} \square \frac{8}{9}$

d) $\frac{9}{5} \square \frac{5}{9}$

e) $2\frac{1}{3} \square 1\frac{5}{4}$

f) $1\frac{1}{8} \square 1\frac{3}{10}$

3. Order these values from least to greatest:

$\frac{4}{10}$ $\frac{1}{3}$ $\frac{2}{7}$ $\frac{6}{10}$ $1\frac{1}{3}$ $2\frac{7}{10}$

4. Draw a picture to show why each statement is true:

a) $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

b) $\frac{2}{5} + \frac{1}{4} = \frac{13}{20}$

5. Add each pair of numbers.

a) $\frac{4}{9} + \frac{2}{9}$

b) $\frac{2}{3} + \frac{1}{5}$

c) $\frac{3}{8} + \frac{5}{6}$

d) $\frac{9}{4} + \frac{7}{4}$

e) $\frac{8}{3} + 2\frac{1}{2}$

f) $3\frac{2}{3} + 4\frac{5}{8}$

6. Write a story problem that you could solve by adding $\frac{2}{3}$ and $1\frac{1}{2}$.

7. Draw a picture to show why each statement is true:

a) $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

b) $\frac{9}{10} - \frac{2}{5} = \frac{5}{10}$

8. Subtract:

a) $\frac{7}{8} - \frac{3}{8}$

b) $\frac{2}{3} - \frac{1}{5}$

c) $\frac{5}{6} - \frac{1}{4}$

d) $\frac{8}{5} - \frac{2}{3}$

e) $4 - 1\frac{2}{3}$

f) $4\frac{1}{3} - 2\frac{3}{5}$

9. Write a story problem that you could solve by subtracting $1\frac{1}{3}$ from $3\frac{1}{4}$.

10. Draw a picture to show why each statement is true:

a) $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

b) $\frac{2}{3} \times \frac{5}{8} = \frac{10}{24}$

11. Multiply each pair of numbers.

a) $\frac{3}{5} \times \frac{5}{6}$

b) $\frac{4}{5} \times \frac{2}{3}$

c) $\frac{9}{4} \times \frac{2}{3}$

d) $2\frac{1}{3} \times 2\frac{1}{4}$

12. Describe a situation where you might multiply $\frac{2}{3} \times \frac{5}{6}$.

13. Draw a picture to show why each statement is true:

a) $\frac{8}{10} \div \frac{2}{10} = 4$

b) $\frac{8}{10} \div \frac{3}{10} = 2\frac{2}{3}$

14. Divide:

a) $\frac{6}{9} \div \frac{2}{9}$

b) $\frac{5}{8} \div \frac{2}{8}$

c) $\frac{9}{4} \div \frac{3}{8}$

d) $\frac{8}{3} \div \frac{5}{6}$

e) $\frac{3}{10} \div \frac{5}{6}$

f) $3\frac{1}{2} \div 4\frac{1}{3}$

15. A painter uses $2\frac{1}{2}$ cans of paint to paint $\frac{1}{4}$ of a room. How much of a room could he paint with 1 can of paint?

16. Write an equation involving fractions and an operation sign that you would complete to solve the problem.

a) Mia read $\frac{5}{8}$ of her book. How much of her book does she have left to read?

b) Mia read $\frac{5}{8}$ of her book. She read $\frac{1}{3}$ of that amount on Monday. What fraction of the whole book did she read on Monday?

c) Mia read $\frac{5}{8}$ of a book. If she read $\frac{1}{5}$ of the book each hour, how many hours was she reading?

Comparing Fractions

Learning Goal

- selecting a strategy to compare fractions based on their numerators and denominators.

Open Question

- Choose two pairs of numbers from 3, 4, 6, 8, 9, 10, 12, 16, 20 to use as numerators and denominators of two fractions. For example, you could use $\frac{4}{6}$ and $\frac{10}{20}$.
 - Make sure that some of your fractions are **improper** and some are **proper**.
 - Make sure that some of your fractions use the same numerator and some do not.

Tell which of your fractions is greater and how you know.

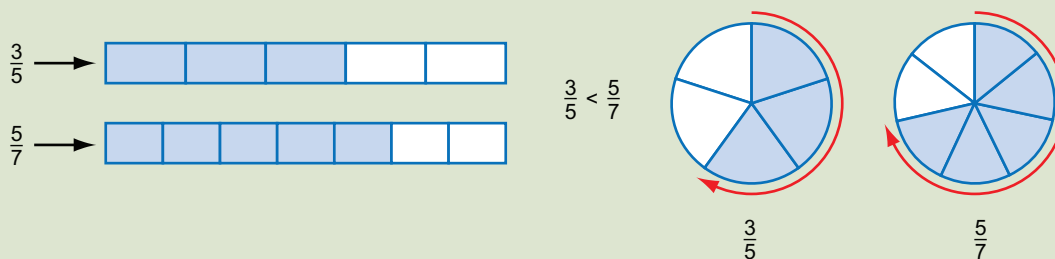
- Make more fractions following the rules above and compare at least six pairs of them. Tell how you know which fraction is greater each time.

Think Sheet

Pairs of fractions are either equal or one fraction is greater than the other fraction. We can decide which statement is true by using a model, by renaming one or both fractions, or by using benchmarks.

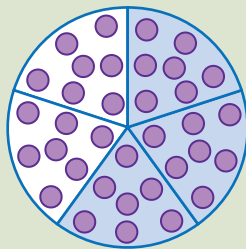
Using a Model

- To compare $\frac{3}{5}$ and $\frac{5}{7}$, we can use a picture that shows the two fractions lined up, so we can see which extends farther.

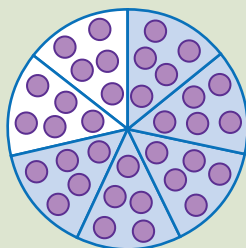


- We can use parts of sets. For example, use a number of counters, such as 35, that is easy to divide into both fifths and sevenths.

$\frac{1}{5}$ of 35 counters is 7 counters, so $\frac{3}{5}$ of 35 counters is 21 counters.



$\frac{1}{7}$ of 35 counters is 5 counters, so $\frac{5}{7}$ of 35 counters is 25 counters.



25 is more than 21, so $\frac{5}{7} > \frac{3}{5}$.

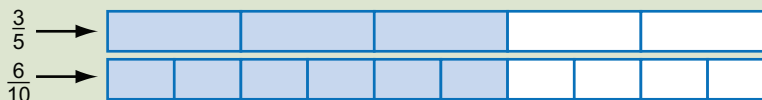
Renaming Fractions

To rename a fraction, we can think about how to express the fraction as an equivalent fraction or equivalent decimal.

- **Equivalent Fractions**

Two fractions are equivalent, or equal, if they take up the same part of a whole or wholes.

For example, $\frac{3}{5} = \frac{6}{10}$.



Each section of the $\frac{3}{5}$ model is split into two sections in the $\frac{6}{10}$ model. So, there are twice as many sections in the second model, and twice as many are shaded. The numerator and denominator have both doubled.

We can multiply the numerator and denominator by any amount (except 0) and the same thing happens as in the example above. There are more sections shaded and more sections unshaded, but the amount shaded does not change.

For example, if we multiply by 3: $\frac{3}{5} = \frac{3 \times 3}{3 \times 5} = \frac{9}{15}$.

or

If we multiply by 10: $\frac{3}{5} = \frac{10 \times 3}{10 \times 5} = \frac{30}{50}$.

- **Common Denominators and Common Numerators**

Renaming fractions to get common denominators or common numerators is helpful when comparing fractions. If we compare fractions with the same denominator, the one with the greater numerator is greater. If we compare fractions with the same numerator, the one with the lesser denominator is greater.

To determine if $\frac{5}{8}$ or $\frac{2}{3}$ is more, we might use common denominators:

$$\frac{5}{8} = \frac{10}{16} = \frac{15}{24}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21} = \frac{16}{24}$$

Since $\frac{5}{8} = \frac{15}{24}$ and $\frac{2}{3} = \frac{16}{24}$, 24 is a **common denominator**.

Since $15 < 16$, then $\frac{5}{8} < \frac{2}{3}$.

$$\frac{5}{8} = \frac{10}{16}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$$

Since $\frac{5}{8} = \frac{10}{16}$ and $\frac{2}{3} = \frac{10}{15}$, 10 is a **common numerator**.

Since fifteenths are bigger than sixteenths, $\frac{10}{15} > \frac{10}{16}$, and $\frac{2}{3} > \frac{5}{8}$.

- Decimals**

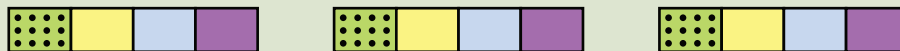
Another way to rename a fraction is to represent it as a decimal.

For example,

$$\frac{2}{3} = 2 \div 3. \text{ Using a calculator, we see that } 2 \div 3 = 0.6666\dots$$

The reason $\frac{a}{b} = a \div b$ is because we can imagine a objects, each being shared by b people. Each person gets one section of each object, so altogether each gets a sections of size $\frac{1}{b}$; that is $\frac{a}{b}$.

For example, if four people share three bars each one gets one fourth from each of the 3 whole bars and that is $\frac{3}{4}$.



We can use decimal renaming to compare fractions, such as $\frac{2}{3}$ to $\frac{4}{5}$.

$$\text{Since } \frac{2}{3} = 0.6666\dots \text{ and } \frac{4}{5} = 0.8, \text{ we see that } \frac{2}{3} < \frac{4}{5}.$$

Using Benchmarks

We can use benchmarks to compare fractions.

- Sometimes we can tell that one fraction is more than another by comparing them to 1. For example, $\frac{17}{12}$ is more than 1, and $\frac{3}{4}$ is less than 1, so $\frac{17}{12} > \frac{3}{4}$.
- Sometimes we can tell that one fraction is more than another since one is greater than one half and one is not. We can mentally rename one half to compare the other fractions to it.

For the fraction $\frac{5}{6}$, since one half is $\frac{3}{6}$, $\frac{5}{6}$ is more than one half.

For the fraction $\frac{3}{10}$, since one half is $\frac{5}{10}$, $\frac{3}{10}$ is less than one half.

$$\text{So, } \frac{5}{6} > \frac{3}{10}.$$

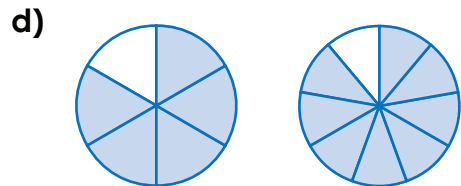
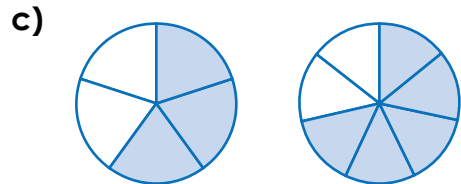
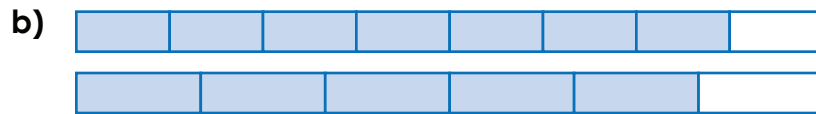
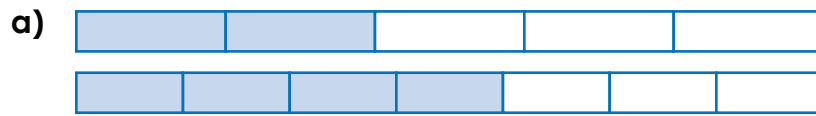
- Sometimes we can tell one fraction is more than another by comparing them to whole numbers other than 1.

For the fraction $\frac{11}{4}$, since 2 is $\frac{8}{4}$, $\frac{11}{4}$ is more than 2.

For the fraction $\frac{5}{3}$, since 2 is $\frac{6}{3}$, $\frac{5}{3}$ is less than 2.

$$\text{So, } \frac{11}{4} \text{ is more than } \frac{5}{3}.$$

1. What fraction comparison is being shown?



2. Draw a picture to show why this statement is true.

a) $\frac{3}{8} = \frac{6}{16}$

b) $\frac{5}{4} = \frac{10}{8}$

3. Tell or show why $\frac{3}{8}$ is not equivalent to $\frac{4}{9}$.

4. Circle the greater fraction. Explain why.

a) $\frac{5}{9}$ or $\frac{6}{9}$

b) $\frac{5}{9}$ or $\frac{5}{11}$

5. How might you rename one or both fractions as other fractions to make it easier to compare them? Tell how it helps.

a) $\frac{4}{5}$ and $\frac{6}{10}$

b) $\frac{4}{5}$ and $\frac{8}{15}$

c) $\frac{9}{8}$ and $\frac{14}{12}$

d) $\frac{8}{3}$ and $\frac{17}{6}$

6. a) Why does it make sense that $\frac{4}{9} = 4 \div 9$?

b) What are the decimal equivalents of $\frac{3}{7}$ and $\frac{4}{9}$?

c) How could you use decimal equivalents to compare $\frac{3}{7}$ and $\frac{4}{9}$?

7. How can each of these fraction pairs be compared without renaming them as other fractions or as decimals?

a) $\frac{8}{3}$ and $\frac{2}{5}$

b) $\frac{7}{3}$ and $\frac{17}{4}$

c) $2\frac{1}{3}$ and $3\frac{1}{5}$

d) $\frac{1}{10}$ and $\frac{7}{9}$

e) $\frac{7}{8}$ and $\frac{9}{10}$

f) $\frac{8}{9}$ and $1\frac{2}{3}$

8. Describe a real-life situation when you might make each comparison:

a) $\frac{3}{5}$ to $\frac{1}{2}$

b) $2\frac{2}{3}$ to $2\frac{1}{2}$

9. A fraction with a denominator of 5 is between one fraction with a denominator of 3 and one fraction with a denominator of 4. Fill in all 9 blanks to show three ways this could be true.

$$\frac{\square}{3} < \frac{\square}{5} < \frac{\square}{4}$$

$$\frac{\square}{3} < \frac{\square}{5} < \frac{\square}{4}$$

$$\frac{\square}{3} < \frac{\square}{5} < \frac{\square}{4}$$

10. a) List all the fractions with a denominator of 3 between 2 and 3.

b) List all the fractions with a denominator of 4 between 2 and 3.

c) Is it possible to list all the fractions between 2 and 3? Explain.

Adding Fractions

Learning Goal

- selecting an appropriate unit and an appropriate strategy to add two fractions.

Open Question

- Choose two different, non equivalent, fractions to add to meet these conditions.
 - Their sum is a little more than 1.
 - Their denominators are different.
 - At least one denominator is odd.

Tell how you predicted the sum would be a little more than 1.

Calculate the sum and explain your process.

Verify that the sum is just a little more than 1.

- Repeat the steps at least three more times with other fractions.

Think Sheet

Adding fractions means combining them.

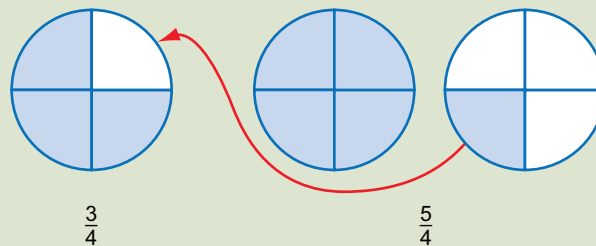
Same Denominators

- To combine fractions with the same denominator we can count.

For example, $\frac{3}{4} + \frac{5}{4}$ is 3 fourths + 5 fourths. If we count the fourths, we get 8 fourths, so $\frac{3}{4} + \frac{5}{4} = \frac{8}{4}$. Another name for $\frac{8}{4}$ is 2.

Notice we add the numerators, not the denominators, because we are combining fourths. If we combined the two denominators, we would get eighths not fourths.

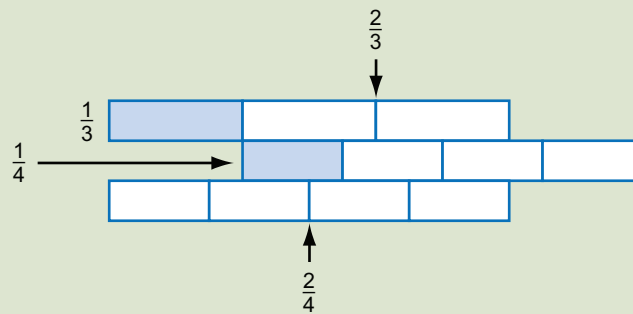
Using fraction pieces, we can see that $\frac{3}{4} + \frac{5}{4} = \frac{8}{4}$. If we combine the last $\frac{1}{4}$ part of the $\frac{5}{4}$ with the $\frac{3}{4}$, we see that 2 wholes are shaded. That shows that $\frac{8}{4}$ is 2.



Different Denominators

- To combine fractions with different denominators requires more thinking. For example, look at this picture of $\frac{1}{3} + \frac{1}{4}$.

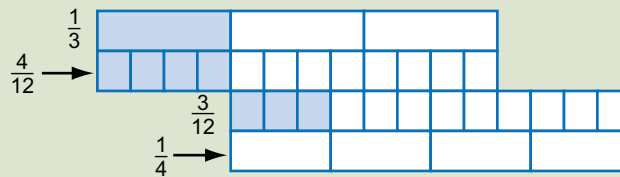
The total length is not as much as $\frac{2}{3}$ but it is more than $\frac{2}{4}$.



If the fractions had the same denominator, we could count sections.

We can create equivalent fractions for $\frac{1}{3}$ and $\frac{1}{4}$ that use the same denominator. That denominator has to be a multiple of both 3 and 4, so 12 is a possibility because $3 \times 4 = 12$.

$\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$, so we add 4 twelfths + 3 twelfths to get 7 twelfths. In this picture, there are 7 sections shaded and each is a twelfth section.

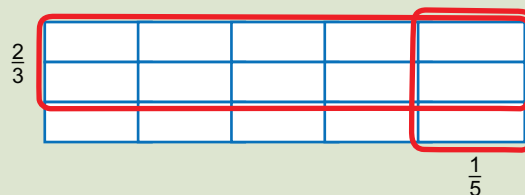


So, $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$.

Use a Grid

- Another way to combine fractions is to use a grid.

For example, to add $\frac{2}{3}$ and $\frac{1}{5}$, we could create a 3-by-5 grid to show thirds and fifths → $\frac{2}{3}$ is two rows and $\frac{1}{5}$ is one column.



Use the letter x to fill two rows to show $\frac{2}{3}$. Use the letter o to fill a column to show $\frac{1}{5}$.

Move counters to empty sections of the grid so each section holds only one counter.

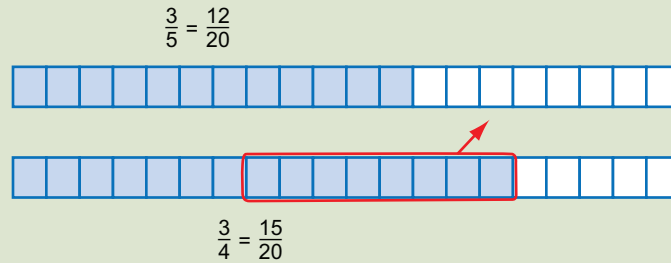
Since $\frac{13}{15}$ of the grid is covered, $\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$.

Notice that $\frac{2}{3} = \frac{10}{15}$ and $\frac{1}{5} = \frac{3}{15}$, so this makes sense.



- Sometimes the sum of two fractions is greater than 1.

To model this with **fraction strips**, we might need more than one whole strip. For example, to show $\frac{3}{5} + \frac{3}{4}$ we can use 20 as the common denominator.



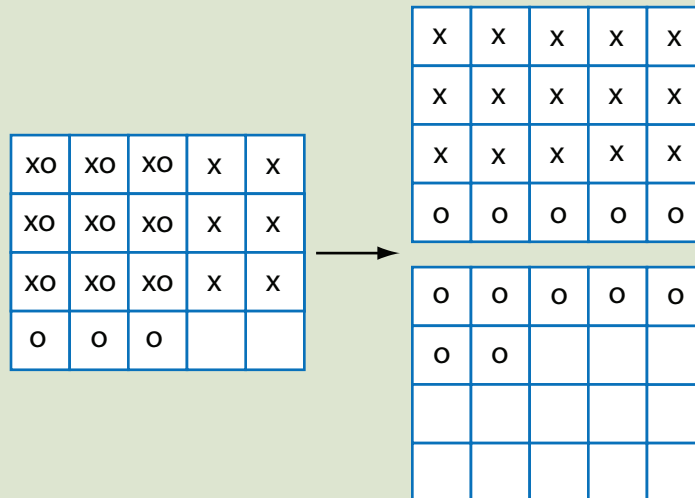
Move $\frac{8}{20}$ up to the first strip to fill the whole. Then, $\frac{7}{20}$ is left in the second strip.

$$\frac{3}{5} + \frac{3}{4} = \frac{27}{20} \text{ or } 1\frac{7}{20}$$

We can model the sum of two fractions that is greater than 1 with a **grid**. When you move the counters so there is only one counter in each section, the grid is filled, with 7 counters extra.

$$\frac{3}{5} = \frac{12}{20} \text{ (marked with o's)}$$

$$\frac{3}{4} = \frac{15}{20} \text{ (marked with x's)}$$



$$\frac{3}{5} + \frac{3}{4} = 1\frac{7}{20}$$

Improper Fractions

To add **improper fractions**, we can use models or we can use equivalent fractions with the same denominators and count.

For example, $\frac{9}{2} + \frac{5}{3} = \frac{27}{6} + \frac{10}{6} = \frac{37}{6} = 6\frac{1}{6}$.

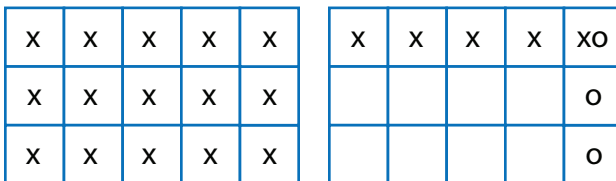
We can check by estimating. $4\frac{1}{2} + 1\frac{2}{3}$ is close to $4 + 2 = 6$.

Mixed Numbers

To add **mixed numbers**, such as $2\frac{1}{3} + 3\frac{3}{4}$, we could add the whole number parts and fraction parts separately.

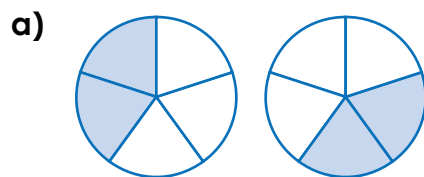
$$2\frac{1}{3} + 3\frac{3}{4} = 5\frac{13}{12} = 6\frac{1}{12}$$

1. a) How does this model show $\frac{4}{3} + \frac{1}{5}$?



- b) What is the sum?

2. What addition is the model showing?



5. The sum of two fractions is $\frac{14}{9}$.
- a) What might their denominators have been? Explain.
- b) List another possible pair of denominators.
6. Choose values for the blanks to make each true.
- a) $\frac{\square}{8} + \frac{\square}{\square} = \frac{3}{4}$
- b) $\frac{\square}{6} + \frac{\square}{\square} = \frac{3}{2}$
7. Lisa used $1\frac{2}{3}$ cups of flour to make cookies and another $1\frac{3}{4}$ cups of flour to bake a cake.
- a) How do you know that she used more than 3 cups of flour for her baking?
- b) How much flour did she use?
8. Write a story problem that would require you to add $\frac{2}{3} + \frac{3}{4}$ to solve it. Solve your problem.

9. a) Use the digits 3, 5, 7 and 9 in the boxes to create the least sum possible.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

- b) Use the digits again to create the greatest sum possible.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

10. Kyle noticed that if you add two fractions, you get the sum's denominator by multiplying the denominators and you get the sum's numerator by multiplying each numerator by the other fraction's denominator and adding. For example, for $\frac{3}{5} + \frac{7}{8}$, the denominator is 5×8 and the numerator is $3 \times 8 + 7 \times 5$.

- a) Do you agree?

- b) Explain why or why not.

11. You have to explain why it does not work to add numerators and denominators to add two fractions. What explanation would you use?

Subtracting Fractions

Learning Goal

- selecting an appropriate unit and an appropriate strategy to subtract two fractions.

Open Question

- Choose two fractions to subtract to meet these conditions.
 - The difference is close to 1, but not exactly 1.
 - Their denominators are different.
 - One denominator is odd.

Tell how you predicted the difference would be close to 1.

Calculate the difference and explain your process.

Verify that the difference is close to 1.

- Repeat the steps at least three more times with other fractions.

Think Sheet

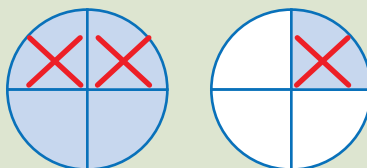
Subtracting can involve take away, comparing, or looking for a number to add.

- To subtract **fractions with the same denominator** we count.

For example, $\frac{5}{4} - \frac{3}{4}$ is 5 fourths – 3 fourths. Since the denominators are fourths, if we take 3 from 5, there are 2 left, so $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$ or $\frac{1}{2}$.

We subtracted the numerators because the fourths were there just to tell the size of the pieces. Nothing was done to the 4s.

We can see that $\frac{5}{4} - \frac{3}{4} = \frac{2}{4} \left(\frac{1}{2} \right)$ using fraction pieces.



- To subtract **fractions with different denominators** requires a number of steps. For example, look at the picture of $\frac{2}{5} - \frac{1}{4}$. The difference is — the part of the $\frac{2}{5}$ that extends beyond the $\frac{1}{4}$.

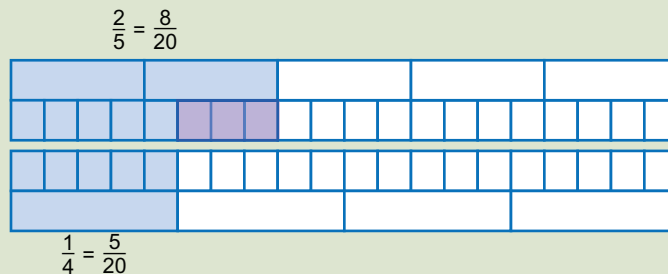


It is hard to tell how exactly long the dark piece is by looking at the picture.

So we can create equivalent fractions for $\frac{2}{5}$ and $\frac{1}{4}$ that use the same denominator. That denominator has to be a multiple of both 5 and 4, so 20 (5×4) is a possibility.

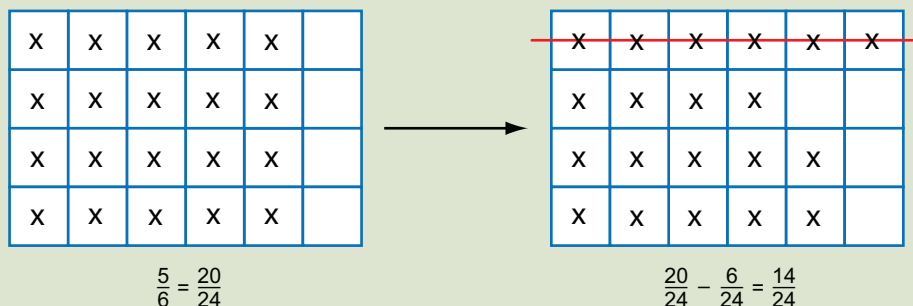
$\frac{2}{5} = \frac{8}{20}$ and $\frac{1}{4} = \frac{5}{20}$, so we subtract 5 twentieths from 8 twentieths, leaving 3 twentieths.

In the diagram there are 3 extra sections in shaded on top and each is a twentieth section.

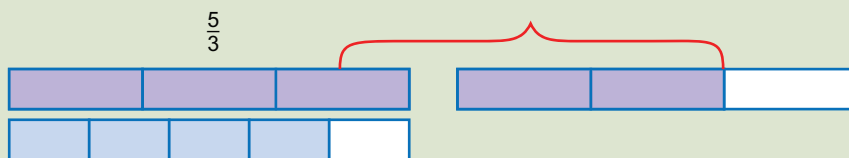


$$\frac{2}{5} - \frac{1}{4} = \frac{3}{20}$$

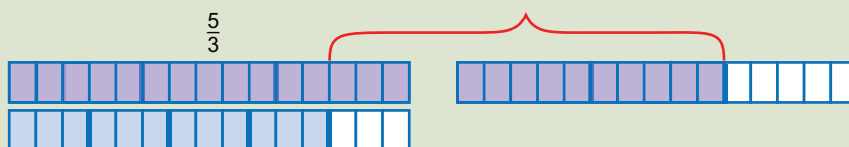
- To subtract fractions, we can use a grid. For example, to subtract $\frac{1}{4}$ from $\frac{5}{6}$, we could create a 4-by-6 grid to show both fourths and sixths. $\frac{5}{6}$ is five of six columns and $\frac{1}{4}$ is one of four rows. Using counters to model $\frac{5}{6}$, we rearrange them so that $\frac{1}{4}$ of the counters (one full row) can be removed. Since 14 sections remain full, the difference is $\frac{14}{24}$.



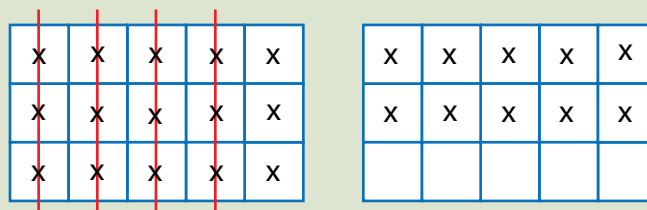
If one of the fractions is greater than 1, more than one strip or grid may need to be used. For example, to show $\frac{5}{3} - \frac{4}{5}$:



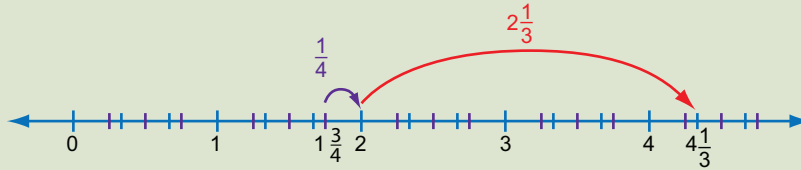
Renaming the fractions as fifteenths can help. $\frac{5}{3} = \frac{25}{15}$ $\frac{4}{5} = \frac{12}{15}$
 $\frac{25}{15} - \frac{12}{15} = \frac{13}{15}$



Using two 3-by-5 grids to solve the same question, cover $\frac{5}{3}$ by covering 5 rows. Remove counters from 4 columns to remove $\frac{4}{5}$ of a grid. $\frac{13}{15}$ of a grid are left covered.



- To subtract **mixed numbers**, such as $4\frac{1}{3} - 1\frac{3}{4}$, we could add up.
 For example, if we add $\frac{1}{4}$ to $1\frac{3}{4}$, we get to 2. If we add another $2\frac{1}{3}$ we get to $4\frac{1}{3}$,
 so $4\frac{1}{3} - 1\frac{3}{4} = \frac{1}{4} + 2\frac{1}{3} = 2\frac{7}{12}$.



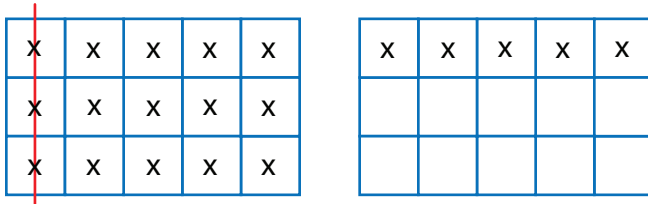
We could subtract the whole number parts and the fraction parts. Since $\frac{1}{3} > \frac{3}{4}$, it might make sense to rename $4\frac{1}{3}$ as $3 + \frac{4}{3}$ first.

$$4\frac{1}{3} = 3\frac{4}{3} \quad \frac{4}{3} - \frac{3}{4} = \frac{16}{12} - \frac{9}{12}$$

$$-1\frac{3}{4} = 1\frac{3}{4}$$

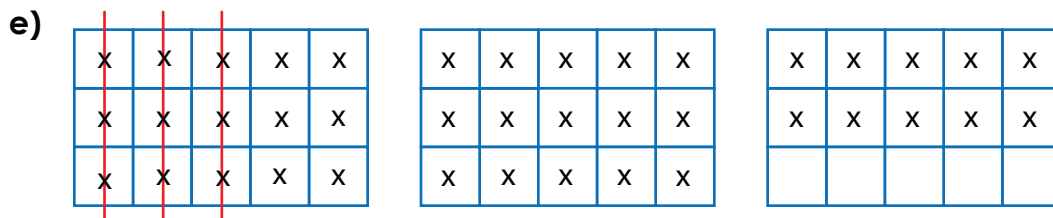
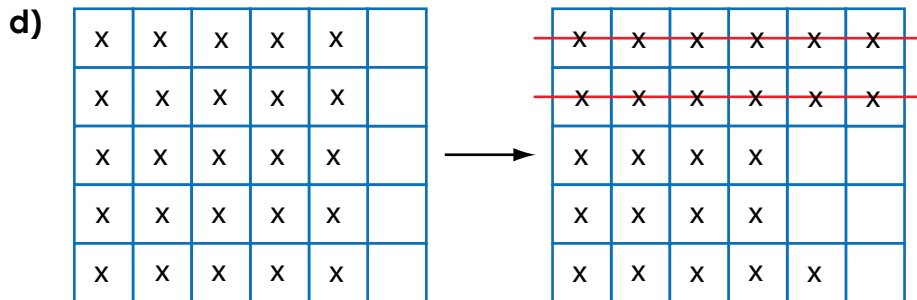
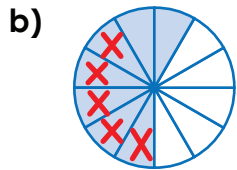
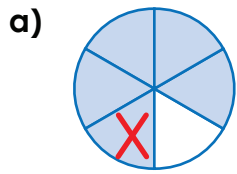
$$2\frac{7}{12}$$

1. a) How does this model show $\frac{4}{3} - \frac{1}{5}$?



- b) How much is $\frac{4}{3} - \frac{1}{5}$?

2. What subtraction is the model showing?



3. Estimate to decide if the difference will be closer to $\frac{1}{2}$ or 1. Circle your choice.

a) $\frac{5}{6} - \frac{1}{4}$ closer to $\frac{1}{2}$ closer to 1

b) $\frac{7}{3} - \frac{7}{5}$ closer to $\frac{1}{2}$ closer to 1

c) $\frac{3}{5} - \frac{1}{10}$ closer to $\frac{1}{2}$ closer to 1

d) $\frac{8}{5} - \frac{7}{6}$ closer to $\frac{1}{2}$ closer to 1

4. Subtract each pair of fractions or mixed numbers. Draw models for parts c) and f).

a) $\frac{9}{5} - \frac{3}{5}$

b) $\frac{7}{8} - \frac{2}{3}$

c) $\frac{3}{5} - \frac{3}{7}$

d) $\frac{11}{3} - \frac{9}{4}$

e) $4\frac{1}{5} - 1\frac{7}{8}$

f) $5 - 3\frac{5}{6}$

5. The difference of two fractions is $\frac{11}{12}$.

a) What do you think their denominators might have been? Explain.

b) List another possible pair of denominators.

6. Choose values for the blanks to make each equation true.

a) $\frac{\square}{3} - \frac{\square}{\square} = \frac{5}{6}$

b) $\frac{\square}{12} - \frac{\square}{\square} = 1\frac{1}{3}$

7. Sakura started with 3 cups of flour. She used $1\frac{2}{3}$ cups for one recipe.

a) About how much flour did she have left?

b) How much flour did she have left?

8. Write a story problem that would require you to subtract $1\frac{1}{2}$ from $3\frac{1}{3}$ to solve it. Solve the problem.

9. a) Use the digits 2, 3, 5, 6, 7 and 9 in the boxes to create a difference that is less than 1.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} - \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

- b) Use the digits again to create a difference greater than 5.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} - \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

10. Kyla noticed that if you subtract two fractions, you get the denominator of the answer by multiplying the denominators and you get the numerator of the answer by multiplying each numerator by the other fraction's denominator and subtracting. For example, for $\frac{8}{5} - \frac{4}{3}$, the denominator is 5×3 and the numerator is $8 \times 3 - 4 \times 5$.

a) Do you agree?

b) Explain why or why not.

11. You have to explain why it does not work to subtract numerators and denominators to subtract two fractions. What explanation would you use?

Multiplying Fractions

Learning Goal

- representing multiplication of fractions as repeated addition or determining area.

Open Question

- Choose two fractions or mixed numbers to multiply to meet these conditions.
 - The product is a little more than 1.
 - If a mixed number is used, the whole number part is not 1.

Tell how you predicted the product would be a little more than 1.

Calculate the product and explain your process.

Verify that the product is close to 1.

- Repeat the steps at least three more times with other fractions.

Think Sheet

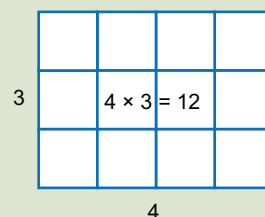
- We have learned that 4×3 means 4 groups of 3. So it makes sense that $4 \times \frac{2}{3}$ means 4 groups of two-thirds:

$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} \quad \text{or} \quad 2 \text{ thirds} + 2 \text{ thirds} + 2 \text{ thirds} + 2 \text{ thirds} = 8 \text{ thirds}$$

We multiply 4×2 to get the numerator 8. The denominator has to be 3 since we are combining thirds.

- To multiply two **proper fractions**, we can think of the area of a rectangle with the fractions as the lengths and widths just as with whole numbers.

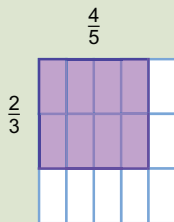
For example, 4×3 is the number of square units in a rectangle with length 4 and width 3. That is because there are 4 equal groups of 3.



To multiply $\frac{2}{3} \times \frac{4}{5}$, we think of the area of a rectangle that is $\frac{2}{3}$ wide and $\frac{4}{5}$ long.

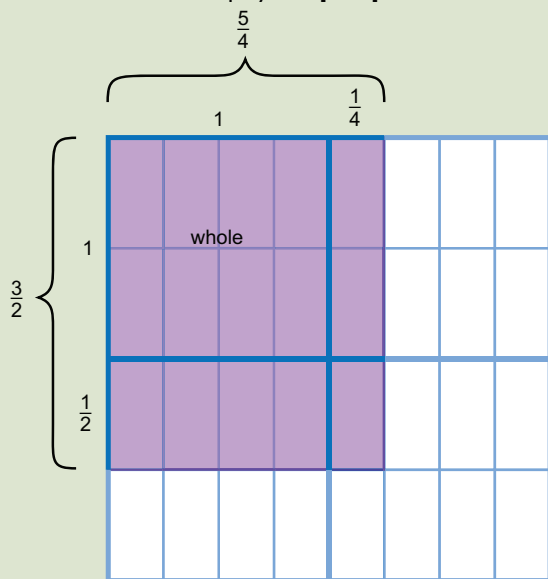
2×4 sections out of the total number of 3×5 sections are inside the rectangle.

That means the area is $\frac{2 \times 4}{3 \times 5}$ and so $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$.



The product of the numerators tells the number of sections in the shaded rectangle and the product of the denominators tells how many sections make a whole.

- We can multiply **improper fractions** the same way as proper fractions.

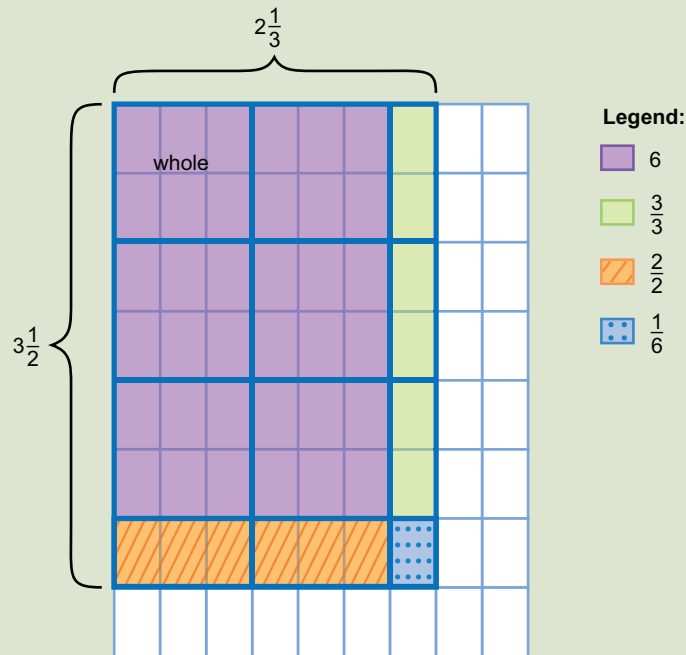


For example, $\frac{5}{4} \times \frac{3}{2}$ is the area of a rectangle that is $1\frac{1}{4}$ units wide and $1\frac{1}{2}$ units long. There are 15 sections in the shaded rectangle and each is $\frac{1}{8}$ of

1 whole, so the area is $\frac{5}{4} \times \frac{3}{2} = \frac{5 \times 3}{4 \times 2}$.

The shaded part of the diagram looks like it might be close to 2 wholes so the answer of $\frac{15}{8}$ makes sense. Each shaded square is $\frac{1}{8}$ of a whole.

- To multiply two mixed numbers, we can either rewrite each mixed number as an improper fraction and multiply, or we can build a rectangle. For example, for $3\frac{1}{2} \times 2\frac{1}{3}$.

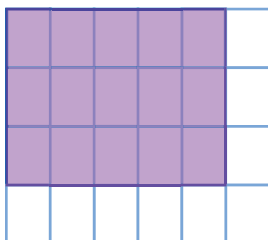


$$3\frac{1}{2} \times 2\frac{1}{3} = 6 + \frac{3}{3} + \frac{2}{2} + \frac{1}{6} = 8\frac{1}{6}$$

$$3\frac{1}{2} \times 2\frac{1}{3} \text{ is the same as } \frac{7}{2} \times \frac{7}{3} = \frac{49}{6} = 8\frac{1}{6}.$$

1. How would you calculate $5 \times \frac{4}{9}$? Why does your strategy make sense?

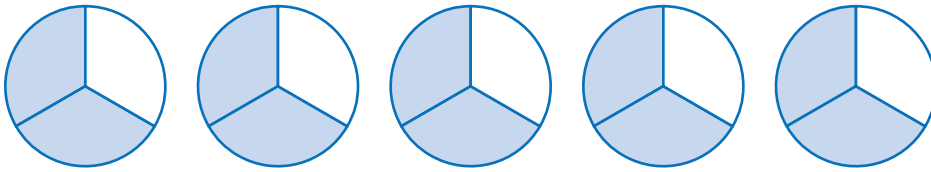
2. a) How does this model show $\frac{3}{4} \times \frac{5}{6}$?



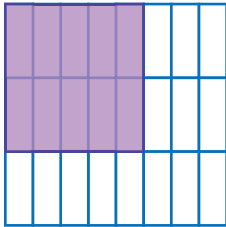
b) What is the product?

3. What multiplication is being modelled?

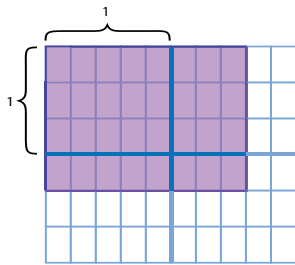
a)



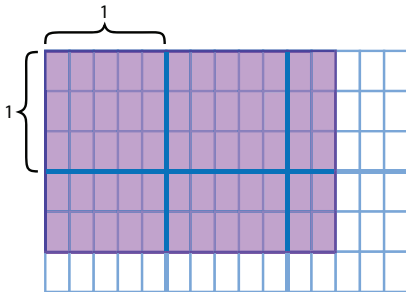
b)



c)



d)



4. Estimate to decide if the product will be closer to $\frac{1}{2}$ or 1. Circle your choice.

a) $\frac{3}{8} \times \frac{7}{8}$

closer to $\frac{1}{2}$

closer to 1

b) $\frac{5}{4} \times \frac{5}{6}$

closer to $\frac{1}{2}$

closer to 1

c) $\frac{9}{10} \times \frac{3}{8}$

closer to $\frac{1}{2}$

closer to 1

d) $2\frac{2}{3} \times \frac{1}{3}$

closer to $\frac{1}{2}$

closer to 1

5. Multiply each pair of fractions or mixed numbers. Draw models for parts b) and e). You may use grid paper.

a) $\frac{3}{5} \times \frac{4}{9}$

b) $\frac{2}{7} \times \frac{3}{5}$

c) $\frac{4}{3} \times \frac{5}{3}$

d) $\frac{6}{5} \times \frac{7}{4}$

e) $1\frac{1}{2} \times 1\frac{3}{5}$

f) $2\frac{2}{3} \times 3\frac{2}{5}$

6. The product of two fractions is $\frac{20}{6}$.

a) What fractions might they have been? Explain.

b) List another possible pair of fractions.

7. Choose values for the blanks to make this equation true.

$$\frac{\square}{3} \times \frac{\square}{\square} = 2\frac{1}{12}$$

8. A recipe to serve 9 people requires $4\frac{2}{3}$ cups of flour.

a) About how much flour is needed to make the recipe for 6 people?

b) How much flour is actually needed?

9. Write a story problem that would require you to multiply $\frac{2}{3} \times \frac{3}{4}$ to solve it. Then solve the problem.

10. a) Use the digits 1, 2, 3, 4, 5, and 6 in the boxes to create a fairly high product.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

- b) Use the digits again to create a fairly low product.

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$$

11. Is it possible to multiply two fractions and get a whole number product? Explain.

12. You have to explain why multiplying a number by $\frac{2}{3}$ results in less than you started with. What explanation would you use?

Dividing Fractions

Learning Goal

- representing division of fractions as counting groups, sharing or determining a unit rate.

Open Question

- Choose two fractions to divide to meet these conditions.
 - The quotient (the answer you get when you divide) is about, but not exactly, $\frac{3}{2}$.
 - The denominators are different.

Tell how you predicted the quotient would be about $\frac{3}{2}$.

Calculate the quotient and explain your process.

Verify that the quotient is about $\frac{3}{2}$.

- Repeat the steps at least three more times with other fractions.

Think Sheet

Just as with whole numbers, dividing fractions can describe the result of sharing, can tell how many of one size group fits in another, or can describe rates.

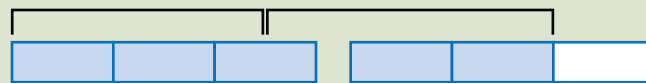
Sharing

- When we divide by a whole number, we can think about sharing. For example, $\frac{4}{5} \div 2$ means that 2 people share $\frac{4}{5}$ of the whole.



We divide the 4 sections into 2 and remember that we are thinking about fifths. Each person gets $\frac{2}{5}$.

- Sometimes the result is not a whole number of sections. For example, $\frac{5}{3} \div 2$ means that 2 people share $\frac{5}{3}$.



Each gets $2\frac{1}{3}$.

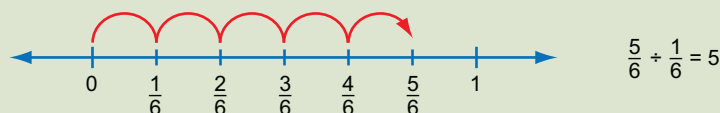
We can either multiply numerator and denominator by 2 to get the equivalent fraction $\frac{5}{6}$ or we can use an equivalent fraction model for $\frac{5}{3}$.



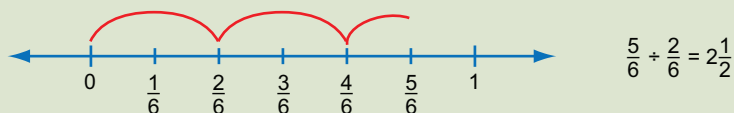
The thirds were split in half since there needed to be an even number of sections for 2 people to share them. Each person gets $\frac{5}{6}$. If it had been $\frac{5}{3} \div 4$, the thirds could be split into fourths so that it would be possible to divide the total amount into four equal sections.

Counting Groups

Sometimes it makes sense to **count** how many groups. For example, there are 5 groups of $\frac{1}{6}$ in $\frac{5}{6}$.

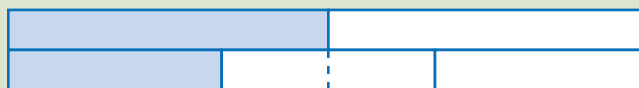


- To **model** $\frac{5}{6} \div \frac{2}{6}$, we need to see how many groups of $\frac{2}{6}$ are in $\frac{5}{6}$. We figure out how many 2s are in 5 or $5 \div 2$.

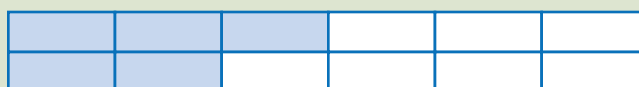


- If two fractions have the **same denominator**, all of the sections are the same size. To get the quotient, we figure out how many times one numerator fits into the other. For example, for $\frac{7}{8} \div \frac{3}{8}$, we calculate $7 \div 3$ since that tells how many groups of 3 of something are in 7 of that thing. For $\frac{3}{8} \div \frac{6}{8}$, we calculate $3 \div 6$, which is $\frac{1}{2}$, since that tells how much of the $\frac{6}{8}$ fits in $\frac{3}{8}$.
- If two fractions have **different denominators**, we can use a model to see how many times one fraction fits in another, or we can use equivalent fractions with the same denominator and divide numerators.

For example, for $\frac{1}{2} \div \frac{1}{3}$, the picture shows that the $\frac{1}{3}$ section fits into the $\frac{1}{2}$ section $1\frac{1}{2}$ times.



We could write $\frac{1}{2} \div \frac{1}{3}$ as $\frac{3}{6} \div \frac{2}{6} = \frac{3}{2}$ or $1\frac{1}{2}$. It is the same as representing the model using equivalent fractions.



Using a Unit Rate

- Another way to think about division is thinking about unit rates. For example, if we can drive 80 km in two hours, we think of $80 \div 2$ as the distance we can travel in one hour.

If a girl can complete $\frac{1}{3}$ of a project in two days, we divide $\frac{1}{3} \div 2$ to figure out how much she can complete in one day. Similarly, if she can complete $\frac{1}{3}$ of a project in $\frac{1}{2}$ day, we divide $\frac{1}{3} \div \frac{1}{2}$ to figure out how much she can complete in 1 day. Since you know that we could also multiply $\frac{1}{3} \times 2$; it makes sense that $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times 2$.

- Suppose we know that $\frac{2}{3}$ of a can of paint can cover $\frac{1}{4}$ of a space. To figure out how much of the space one full can of paint can cover, we calculate $\frac{1}{4} \div \frac{2}{3}$.
 If $\frac{2}{3}$ of a can covers $\frac{1}{4}$ of a space, then $\frac{1}{3}$ of a can covers $\frac{1}{8}$ of the space ($\frac{1}{4} \div 2$) and then $\frac{3}{3}$, which is 1, can covers $\frac{3}{8}$ of the space ($3 \times \frac{1}{8}$). That means $\frac{1}{4} \div \frac{2}{3} = \frac{3}{8}$. Since we know that $\frac{1}{4} \div \frac{2}{3} = \frac{3}{12} \div \frac{8}{12} = \frac{3}{8}$, this makes sense.

What we did was divide the $\frac{1}{4}$ by 2 and multiply by 3.

$$\text{So } \frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \div 2 \times 3 = \frac{1}{4} \times \frac{3}{2}.$$

We reversed the divisor and multiplied by the **reciprocal**, the fraction we get by switching the numerator and denominator.

- Another way to think about this is:

$1 \div \frac{1}{3} = 3$ since three are 3 groups of $\frac{1}{3}$ in 1 whole.

$1 \div \frac{2}{3} = \frac{3}{2}$ since there are only half as many groups of $\frac{2}{3}$ in 1 whole as there would be groups of $\frac{1}{3}$.

$\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2}$ since there are only one fourth as many groups of $\frac{2}{3}$ in $\frac{1}{4}$ as groups of $\frac{2}{3}$ in a whole.

- Improper fractions** can be divided in the same way as proper fractions.
- Mixed numbers** are usually written as improper fractions to divide them.

1. How would you calculate each? Explain why your strategy makes sense.

a) $\frac{6}{8} \div 4$

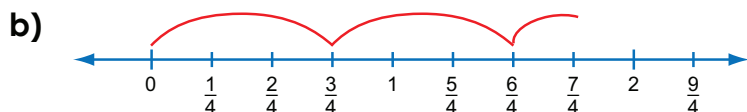
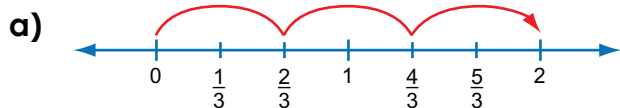
b) $\frac{4}{9} \div 3$

2. a) How does this model show $\frac{5}{8} \div \frac{1}{3}$?



b) Estimate the quotient.

3. What division is being modelled?



4. Estimate each quotient as closer to $\frac{1}{2}$ or 1 or 2. Circle your choice.

a) $\frac{3}{8} \div \frac{7}{8}$ closer to $\frac{1}{2}$ closer to 1 closer to 2

b) $\frac{5}{4} \div \frac{2}{3}$ closer to $\frac{1}{2}$ closer to 1 closer to 2

c) $\frac{11}{3} \div \frac{13}{4}$ closer to $\frac{1}{2}$ closer to 1 closer to 2

d) $2\frac{2}{3} \div 4\frac{1}{2}$ closer to $\frac{1}{2}$ closer to 1 closer to 2

5. Divide each pair of fractions or mixed numbers. Draw models for parts a) and c). You might use grid paper.

a) $\frac{3}{5} \div 4$

b) $\frac{5}{6} \div \frac{1}{3}$

c) $8 \div \frac{2}{3}$

d) $\frac{7}{8} \div \frac{9}{10}$

e) $\frac{14}{3} \div \frac{2}{5}$

f) $2\frac{2}{3} \div 3\frac{2}{5}$

6. You have $3\frac{1}{3}$ cups of flour to divide into equal batches for four recipes.

a) About how much flour is available for each batch?

b) How much flour is that per batch?

7. You can tile $\frac{2}{5}$ of a floor area in $\frac{3}{4}$ of a day.
- How much of the floor can you tile in one full day?
 - What computation can you do to describe this situation?
8. Write a story problem that would require you to divide $\frac{3}{5}$ by $\frac{1}{8}$ to solve it. Solve the problem.

9. a) Choose values for the blanks to make this true.

$$\frac{\square}{3} \div \frac{\square}{\square} = \frac{3}{4}$$

- b) List another possible set of values.

$$\frac{\square}{3} \div \frac{\square}{\square} = \frac{3}{4}$$

10. a) Use the digits 2, 3, 5, 6, and 9 in the boxes to create a small quotient.

$$\frac{\square\square}{\square} \div \frac{\square\square}{\square}$$

- b) Use the digits again to create a large quotient.

$$\frac{\square\square}{\square} \div \frac{\square\square}{\square}$$

11. Kevin noticed that $\frac{3}{8} \div \frac{1}{3}$ is $\frac{9}{8}$, but $\frac{1}{3} \div \frac{3}{8} = \frac{8}{9}$.

a) Explain why it makes sense that the first quotient is greater than 1, but the second one is less than 1.

b) Explain why it also makes sense that the quotients are reciprocals.

12. You have to explain why dividing by $\frac{5}{6}$ is the same as multiplying by $\frac{6}{5}$. What explanation would you use?

Relating Situations to Fraction Operations

Learning Goal

- connecting fraction calculations with real-life situations.

Open Question

- Create two problems that could be solved using each of the equations below. Make the problems as different as you can. You do not have to solve them.

$$\frac{3}{5} + \frac{1}{3} = \square$$

$$\frac{3}{5} - \frac{1}{3} = \square$$

$$\frac{3}{5} \times \frac{1}{3} = \square$$

$$\frac{3}{5} \div \frac{1}{3} = \square$$

- Explain your thinking for each.

Think Sheet

It is important to be able to tell what fraction operations make sense to use for solving fraction problems.

Addition

Addition situations always involve **combining**. For example:

I had $\frac{1}{3}$ of my essay done. I did another $\frac{1}{2}$ of the essay. How much of it is finished now? Answer: $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ of the essay

Subtraction

Subtraction situations could involve **take away**, **comparing**, or **deciding what to add**.

- An example of **take away** is: I had $\frac{3}{4}$ cup of juice. I poured out $\frac{1}{3}$ cup. How much of a cup of juice is left? Answer: $\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$ cup
- An example of **comparing** is: I finished $\frac{5}{8}$ of my project. Angela finished $\frac{1}{4}$ of her project. How much more did I finish? Answer: $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$
- An example of **deciding what to add** is: I have finished $\frac{5}{8}$ of my project. How much more is left? Answer: $1 - \frac{5}{8} = \frac{3}{8}$

Multiplication

Multiplication situations could involve **counting parts of sets**, **determining areas of rectangles**, or **applying rates**.

- An example of **counting parts of sets** is: Jeff filled 8 cups, each $\frac{1}{3}$ of the way. If he had filled the cups to the top instead, how many cups would he fill?
Answer: $8 \times \frac{1}{3} = \frac{8}{3}$.

Another example is: A class raised $\frac{2}{3}$ of the money they needed to go on a trip. The boys raised $\frac{2}{5}$ of the class's contribution. What fraction of the whole amount of money needed did the boys raise?

- An example of **determining areas of rectangles** is: One rectangle is $\frac{1}{3}$ as long and $\frac{2}{3}$ as wide as a 4×8 rectangle. What is its area? Answer: $\frac{1}{3} \times \frac{2}{3} \times 4 \times 8 = \frac{64}{9}$
- An example of **applying rates** is: Jane can paint a wall in $\frac{2}{3}$ of an hour. How long would it take her to paint $2\frac{1}{2}$ walls? Answer: $2\frac{1}{2} \times \frac{2}{3} = \frac{5}{3}$ hours

Division

Division situations could involve **sharing**, **determining how many groups**, or **determining unit rates**

- An example of **sharing** is: $\frac{2}{3}$ of a room must be painted. Four friends are going to share the job. What fraction of the room will each paint if they work at the same rate? Answer: $\frac{2}{3} \div 4 = \frac{2}{12}$ of the room
- An example of **determining how many groups** is: You want to measure $\frac{5}{3}$ of a cup of flour, but only have a $\frac{1}{2}$ cup measure. How many times must you fill the $\frac{1}{2}$ cup measure? Answer: $\frac{5}{3} \div \frac{1}{2} = 3\frac{1}{3}$ times
- An example of **determining unit rate** is: It takes you $\frac{1}{2}$ of an hour to clean $\frac{2}{5}$ of the house, how much of the house could you clean in 1 hour? Answer: $\frac{2}{5} \div \frac{1}{2} = \frac{4}{5}$ of the house in 1 hour.

More Than One Option

Sometimes the same situation can be approached using more than one operation. For example, if you can solve a problem by dividing, you can also solve it by multiplying.

The last problem above could be solved by thinking: $\frac{1}{2} \times \frac{\square}{\square} = \frac{2}{5}$.

- Every division can also be solved as a multiplication.
- Every subtraction can also be solved as an addition.

1. Tell what operation or operations you could use to solve each problem. Write the equation to represent the problem.
 - a) Cynthia has $1\frac{1}{2}$ containers of juice. Each smaller container holds $\frac{2}{5}$ as much as a large container. How many small containers can she fill?

 - b) About $\frac{3}{4}$ of the athletes in a school play basketball. About $\frac{1}{4}$ of those players are in Grade 9. What fraction of the students in the school are Grade 9 basketball players?

 - c) Stacey read $\frac{1}{5}$ of her book yesterday and $\frac{1}{3}$ of it today. How much of the book has she read?

 - d) It takes Lea's mom about $1\frac{2}{3}$ hours to drive to work in the morning and $1\frac{2}{3}$ hours to drive home every afternoon. If it is Wednesday at noon and she went to work Monday, Tuesday and Wednesday (and is still there), about how many hours has she spent driving to and from work?

 - e) The gas tank in Kyle's car was $\frac{7}{8}$ full when they started a trip. Later in the day, the tank registered $\frac{1}{4}$ full. How much of the tank of gas had been used?

- f) You can travel $\frac{2}{3}$ of the way to your grandmother's home in $1\frac{1}{2}$ hours. How much of the way can you travel in 1 hour?

2. a) What makes this a multiplication problem?

Fred has $2\frac{1}{2}$ times as much money as his sister. Aaron has $\frac{3}{4}$ as much money as Fred. How many times as much money as Fred's sister does Aaron have?

- b) What makes this a division problem?

A turkey is in the oven for $5\frac{1}{2}$ hours. You decide to check it every $\frac{1}{3}$ of an hour. How many times will you check the turkey?

- c) What makes this a subtraction problem?

Ed has $3\frac{1}{2}$ rolls of tape. He uses about $1\frac{3}{4}$ rolls to wrap gifts. How many rolls of tape does he have left?

3. Write an equation you could use to solve each problem in Question 2.

a)

b)

c)

4. Why might you solve this problem using either multiplication or division? You have worked out $3\frac{1}{3}$ hours this week. Each workout was $\frac{2}{3}$ of an hour. How many times did you workout?

5. How would you keep part of the information in Question 2b, but change some of it so that:
 - a) it becomes a multiplication question

 - b) it becomes a subtraction question

6. Finish this problem so that it is a division problem:
Alexander has $3\frac{1}{2}$ dozen eggs.

7. What hints can you suggest to decide if a problem requires subtraction to solve it?

Fraction Tower

1																											
$\frac{1}{2}$										$\frac{1}{2}$																	
$\frac{1}{3}$						$\frac{1}{3}$						$\frac{1}{3}$															
$\frac{1}{4}$				$\frac{1}{4}$				$\frac{1}{4}$				$\frac{1}{4}$															
$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$			$\frac{1}{5}$															
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$															
$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$													
$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$											
$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$		$\frac{1}{10}$									
$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$		$\frac{1}{12}$									
$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$		$\frac{1}{15}$							
$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$		$\frac{1}{18}$							
$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$		$\frac{1}{20}$							

