INTRODUCTION

The Gap Closing package is designed to help Grade 9 teachers provide precisely targeted remediation for students who they identify as being significantly behind in mathematics. The goal is to close gaps in Number Sense, Algebra, and Measurement so that the students can be successful in learning grade-appropriate mathematics. However, the materials may also be useful to Grades 7 and 8 teachers in providing alternative approaches they had not considered to some of the content or to Grade 10 applied math or Grade 11 or 12 teachers, particularly teachers of Mathematics for Workplace courses.

For each topic, there is a diagnostic and a set of intervention materials that teachers can use to help students be more successful in their learning.

The diagnostics are designed to uncover the typical problems students have with a specific topic. Each diagnostic should be used as instructional decisions are being made for the struggling student.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with students.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

The style and content of the consolidating questions may be useful to teachers in their regular course preparation as well.

Getting Started

- Identify students who are underperforming in Number Sense, Algebra, and Measurement or in mathematics in general – students who are not where you think they should be.
- Select the Number, Algebra, and Measurement topics to focus on.
- Administer the diagnostic, allowing 20-40 minutes or as much additional time as seems reasonable for a particular student.
  - For students with accommodations, e.g., a scribe, supports should remain in place; they may require a longer time to complete the diagnostic.
- Refer to the Facilitator’s Guide as you administer and mark the diagnostic.
- Use the chart, Evaluating Diagnostic, Results to identify which of the intervention materials aligns with the student’s learning needs.
  - This is the stage where you personalize remediation based on the precisely identified area of concern. It may be appropriate to use more than one of the intervention materials with the student.
- Select either the Open Question or the Think Sheet approach:
  - The Open Question provides the student with a variety of possible responses and approaches.
  - The Think Sheet offers a more guided and structured approach.
  - Teachers should consider student preferences and readiness when deciding which of these two approaches to use.
- Use the suggestions for the three-part structure included for each topic to provide remediation.
  - Allow approximately 45 minutes to complete each topic, recognizing that time may vary for different students.

Considerations

Materials required for each intervention approach are indicated in the Facilitator’s Guide. There are templates included in the student book should the concrete materials not be available.

Sample answers to questions are included to give teachers language that they might use in further discussions with students. These samples sometimes may be more sophisticated than the responses that students would give. Teachers should probe further with individual students in their questioning, as needed.

Some students might need more practice with the same topic than other students. Teachers should use their professional judgment when working with individual students to decide how much practice they need. For additional practice, teachers can create similar questions to those provided using alternate numbers.

Access the Gap Closing materials at www.edugains.ca
Fractions

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FRACTIONS

Relevant Expectations for Grade 9

MPM1D

Number Sense and Algebra
- substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases)
- simplify numerical expressions involving integers and rational numbers, with and without the use of technology
- solve problems requiring the manipulation of expressions arising from applications of percent, ratio, rate, and proportion
- solve first-degree equations, including equations with fractional coefficients, using a variety of tools and strategies

Linear Relations
- identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear

Analytic Geometry
- determine, through investigation, various formulas for the slope of a line segment or a line and use the formula to determine the slope of a line segment or a line

MPM1P

Number Sense and Algebra
- illustrate equivalent ratios, using a variety of tools
- represent, using equivalent ratios and proportions, directly proportional relationships arising from realistic situations
- solve for the unknown value in a proportion, using a variety of methods
- make comparisons using unit rates
- solve problems involving ratios, rates, and directly proportional relationships in various contexts, using a variety of methods
- solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms
- simplify numerical expressions involving integers and rational numbers, with and without the use of technology
- substitute into algebraic equations and solve for one variable in the first degree
Linear Relations

- identify, through investigation, some properties of linear relations (i.e., numerically, the first difference is a constant, which represents a constant rate of change; graphically, a straight line represents the relation), and apply these properties to determine whether a relation is linear or non-linear
- determine, through investigation, that the rate of change of a linear relation can be found by choosing any two points on the line that represents the relation, finding the vertical change between the points (i.e., the rise) and the horizontal change between the points (i.e., the run) and writing the ratio \( \frac{\text{rise}}{\text{run}} \)

Possible reasons why a student might struggle when working with fractions

Students may struggle with fraction comparisons and operations as well as when to apply the various operations.

Some of the problems include:
- misunderstandings about how to create equivalent fractions, e.g., adding the same amount to both numerator and denominator
- misunderstandings about how to compare fractions when the denominators are not the same, e.g., assuming that if the numerator and denominator are closer, the fraction is greater although this is not always true; even though \( \frac{6}{10} \) is greater than \( \frac{7}{9} \), \( \frac{1}{3} \) is not greater than \( \frac{4}{7} \)
- adding (or subtracting) fractions by adding the numerators and adding (or subtracting) the denominators
- not recognizing that fractions can only be added or subtracted if they are parts of the same whole
- lack of understanding that \( \frac{5}{6} \times \frac{2}{3} \) is a portion of \( \frac{2}{3} \) if \( \frac{5}{6} \) is a proper fraction
- misinterpreting a remainder when dividing fractions, e.g., thinking that \( \frac{1}{2} + \frac{1}{3} = 1 \frac{1}{6} \) (since \( \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \)) instead of \( 1 \frac{1}{2} \)
- inverting the wrong fraction when dividing fractions using an invert-and-multiply strategy
- inability to extend knowledge of comparisons and operation rules with proper fractions to improper fractions and/or mixed numbers
- multiplying mixed numbers by separately multiplying whole number parts and fraction parts
Administer the diagnostic

Using the diagnostic results to personalize interventions

Intervention materials are included on each of these topics:
• comparing fractions
• adding fractions
• subtracting fractions
• multiplying fractions
• dividing fractions
• relating situations to fraction operations

You may use all or only part of these sets of materials, based on student performance with the diagnostic. If students need help in understanding the intent of a question in the diagnostic, you are encouraged to clarify that intent.

<table>
<thead>
<tr>
<th>Evaluating Diagnostic Results</th>
<th>Suggested Intervention Materials</th>
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<td>If students struggle with Questions 1-3</td>
<td>use <em>Comparing Fractions</em></td>
</tr>
<tr>
<td>If students struggle with Questions 4-6</td>
<td>use <em>Adding Fractions</em></td>
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<td>If students struggle with Questions 7-9</td>
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<tr>
<td>If students struggle with Questions 13-15</td>
<td>use <em>Dividing Fractions</em></td>
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<tr>
<td>If students struggle with Questions 6, 9, 12, 15, and 16</td>
<td>use <em>Relating Situations to Fraction Operations</em></td>
</tr>
</tbody>
</table>

**Solutions**

1. a) e.g., \(\frac{4}{6}, \frac{6}{9}\) and \(\frac{12}{18}\)
   b) e.g., \(\frac{4}{5}, \frac{16}{20}\) and \(\frac{32}{40}\)
   c) e.g., \(\frac{6}{25}, \frac{12}{50}, \frac{18}{75}\)

2. a) >
   b) >
   c) <
   d) >
   e) >
   f) <

3. \(\frac{2}{7}, \frac{1}{3}, \frac{4}{10}, \frac{6}{10}, 1\frac{1}{3}, 2\frac{2}{10}\)

4. a) e.g.,
5. a) \( \frac{6}{9} \)  
   b) \( \frac{13}{15} \)  
   c) \( \frac{29}{24} \)  
   d) \( \frac{16}{4} \)  
   e) \( \frac{5}{5} \)  
   f) \( \frac{8}{7} \)  

   **Note:** Equivalent forms are acceptable.

6. e.g., There were 1\( \frac{1}{2} \) Hawaiian pizzas and \( \frac{2}{3} \) of a vegetarian pizza left. How many pizzas were left altogether?

7. a) e.g., 
   b) e.g., 

8. a) \( \frac{4}{8} \)  
   b) \( \frac{7}{15} \)  
   c) \( \frac{7}{12} \)  
   d) \( \frac{14}{15} \)  
   e) \( 2\frac{3}{5} \)  
   f) \( 1\frac{11}{15} \)  

9. e.g., I had 3\( \frac{1}{2} \) cartons of eggs and used 1\( \frac{1}{3} \) of them for some baking. How many cartons of eggs are left?

10. a) e.g., 
   b) e.g., 

11. a) \( \frac{1}{2} \)  
     b) \( \frac{8}{15} \)  
     c) \( \frac{3}{2} \)  
     d) \( \frac{2}{3} \)  
     e) \( \frac{63}{12} \)  

12. e.g., If \( \frac{5}{6} \) of a project had been completed, and one student had done \( \frac{2}{3} \) of it, then \( \frac{2}{3} \times \frac{5}{6} \) tells what part of the whole project she had completed.

13. a) e.g., 
   b) e.g., 

14. a) 3  
     b) 2\( \frac{3}{7} \)  
     c) 6  
     d) \( \frac{16}{5} \)  
     e) \( \frac{9}{25} \)  
     f) \( \frac{21}{26} \)  

15. \( \frac{1}{10} \) of a room

16. a) \( 1 - \frac{2}{3} = ? \)  
     b) \( \frac{1}{3} \times \frac{2}{5} = ? \)  
     c) \( \frac{3}{8} + \frac{1}{2} = ? \)
Diagnostic

1. List three fractions equivalent to each fraction.
   a) \( \frac{1}{2} \)
   b) \( \frac{2}{5} \)
   c) \( \frac{4}{8} \)

2. Use a greater than (>) or less than (<) sign to make these statements true.
   a) \( \frac{3}{4} \) \( \frac{2}{3} \)
   b) \( \frac{5}{6} \) \( \frac{1}{3} \)
   c) \( \frac{7}{8} \) \( \frac{1}{2} \)
   d) \( \frac{2}{3} \) \( \frac{3}{5} \)
   e) \( \frac{5}{6} \) \( \frac{3}{10} \)

3. Order these values from least to greatest:
   \( \frac{4}{10} \) \( \frac{1}{3} \) \( \frac{3}{5} \) \( \frac{4}{5} \) \( \frac{1}{2} \)

4. Draw a picture to show why each statement is true:
   a) \( \frac{3}{4} \times \frac{2}{3} = \frac{6}{9} \)
   b) \( \frac{5}{6} \times \frac{1}{2} = \frac{5}{12} \)
   c) \( \frac{7}{8} \times \frac{1}{2} = \frac{7}{16} \)
   d) \( \frac{2}{3} \times \frac{3}{5} = \frac{6}{15} \)
   e) \( \frac{5}{6} \times \frac{3}{10} = \frac{15}{60} \)

5. Add each pair of numbers.
   a) \( \frac{1}{2} + \frac{2}{3} = \frac{7}{6} \)
   b) \( \frac{3}{4} + \frac{1}{3} = \frac{13}{12} \)
   c) \( \frac{5}{6} + \frac{2}{5} = \frac{37}{30} \)
   d) \( \frac{3}{4} + 2 = \frac{11}{4} \)
   e) \( \frac{5}{6} + 3 = \frac{23}{6} \)

6. Write a story problem that you could solve by adding \( \frac{2}{3} \) and \( \frac{1}{2} \):
   a) Mia read \( \frac{2}{3} \) of her book. How much of her book does she have left to read?
   b) Mia read \( \frac{1}{3} \) of her book. She read \( \frac{1}{2} \) of that amount on Monday. What fraction of the whole book did she read on Monday?
   c) Mia read \( \frac{3}{4} \) of a book. If she read \( \frac{1}{2} \) of the book each hour, how many hours was she reading?
USING INTERVENTION MATERIALS

The purpose of the suggested work is to help students build a foundation for working with rational expressions, proportions, and slope and solving equations where fractions are involved.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:
• Questions to ask before using the approach
• Using the approach
• Consolidating and reflecting on the approach
Comparing Fractions

Learning Goal

• selecting a strategy to compare fractions based on their numerators and denominators.

Open Question

<table>
<thead>
<tr>
<th>Questions to Ask Before Using the Open Question</th>
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<tbody>
<tr>
<td>◊ How can you decide whether one fraction is greater than another? (e.g., You can draw them and see which takes up more space.)</td>
</tr>
<tr>
<td>◊ What if the fractions were really close and you could not tell from the drawing? (e.g., You could use equivalent fractions with the same denominator.) Then what? (e.g., The one with the greater numerator is greater.)</td>
</tr>
<tr>
<td>◊ How could you tell that (\frac{4}{3} &gt; \frac{3}{2}) without using common denominators or a drawing? (e.g., (\frac{4}{3}) is more than 1 whole and (\frac{3}{2}) is not, so (\frac{4}{3}) is greater.)</td>
</tr>
</tbody>
</table>

Using the Open Question

Make sure students understand that they must use four numbers for each comparison, but they can use the same number twice. (This is required for fraction pairs with the same numerator.) Also ensure that they explain their thinking to justify their decisions.

By viewing or listening to student responses, note:
• how or whether they use benchmarks to compare fractions;
• whether they always use a common denominator to compare fractions;
• how they compare fractions with a common numerator;
• whether they are comfortable with improper fractions.

Consolidating and Reflecting on the Open Question

◊ What fractions did you use with the same numerator? (e.g., \(\frac{3}{8}\) and \(\frac{3}{12}\))
◊ How did you know which was greater? (e.g., I knew that \(\frac{3}{12}\) is only \(\frac{1}{4}\) and that is \(\frac{3}{8}\) and so \(\frac{3}{8}\) is more.)
◊ Could you have known without changing \(\frac{3}{12}\) to \(\frac{2}{8}\)? (e.g., Yes, I know that eighths are bigger than twelfths, so \(\frac{3}{8}\) has to be more than \(\frac{3}{12}\).)
◊ Which of your fractions did you compare by using common denominators? (e.g., When I compared \(\frac{3}{8}\) and \(\frac{5}{9}\), I got a common denominator of 72 and then compared them.)
◊ Did you compare any fractions by deciding how they compared to \(\frac{1}{2}\)? (Yes, e.g., I knew that \(\frac{3}{8}\) was less than \(\frac{1}{2}\) and that \(\frac{15}{16}\) was more than \(\frac{1}{2}\), so \(\frac{15}{16}\) was greater.)

Solutions

e.g.,

• \(\frac{4}{6} > \frac{10}{20}\) I know that \(\frac{10}{20}\) is \(\frac{1}{2}\) and that \(\frac{3}{6}\) is also \(\frac{1}{2}\). \(\frac{4}{6}\) is more sixths than \(\frac{3}{6}\), so it is greater.

• \(\frac{9}{12} > \frac{9}{12}\) I know that ninths are bigger than twelfths since if you cut a whole into nine parts, each part will be bigger than if you cut the whole into 12 parts. So four big parts is more than four little parts.

• \(\frac{12}{5} > \frac{16}{8}\) I know that \(\frac{12}{5}\) is 2 but \(\frac{6}{16}\) is not even 1.

• \(\frac{20}{8} > \frac{12}{6}\) I know that \(\frac{20}{8}\) is 5, but \(\frac{12}{6}\) is only 2.

• \(\frac{3}{20} < \frac{1}{8}\) I know that twentieths are smaller than eighths since, if you cut a whole into 20 parts, each part will be smaller than if you cut the whole into eight parts. So three big parts is more than three little parts.

• \(\frac{9}{10} > \frac{8}{10}\) I know that \(\frac{9}{10}\) is only \(\frac{1}{10}\) away from 1 but \(\frac{8}{10}\) is \(\frac{1}{10}\) away. I know that \(\frac{1}{10}\) away is closer since \(\frac{1}{10} < \frac{1}{2}\). I know that because, if you cut a whole into 10 pieces, each piece is smaller than if you cut it into nine pieces.
# Think Sheet

## Questions to Ask Before Assigning the Think Sheet

◊ **How can you decide whether one fraction is greater than another?** (e.g., You can draw them and see which takes up more space.)

◊ **Why is it important that you use the same size whole when you are drawing?** (e.g., You can make even a little fraction seem big if it is part of a big whole.)

◊ **Do you think that \( \frac{3}{5} \) or \( \frac{4}{5} \) is more?** (e.g., Do you need a drawing to tell? (e.g., No, since I know that \( \frac{4}{5} \) is more fifths.)

## Using the Think Sheet

Read through the introductory box with the students; respond to any questions they might have.

Make sure they realize that different comparisons suit different strategies.

Assign the tasks.

By viewing or listening to student responses, note:

- how or whether they use benchmarks to compare fractions;
- whether they always get a common denominator to compare fractions;
- how they compare fractions with a common numerator;
- whether they are comfortable with improper fractions;
- whether they gravitate toward using decimals to compare fractions.

## Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ **How can \( \frac{2}{5} = \frac{6}{15} \) even though 2 and 5 are only 3 apart, but 6 and 15 are 9 apart?** (e.g., It does not matter how far apart the numerator and denominator are; what matters is what fraction of the denominator each numerator is.

◊ **Which fraction would you rename to compare \( \frac{3}{4} \) and \( \frac{5}{6} \)?** (e.g., I would rename \( \frac{3}{8} \) to \( \frac{5}{16} \) and I know \( \frac{6}{16} > \frac{5}{17} \).)

◊ **Think of a different way to compare each of these pairs of fractions: \( \frac{2}{3} \) and \( \frac{3}{4} \); \( \frac{9}{10} \) and \( \frac{4}{5} \). Which way would you choose for each?** (e.g., I would know that fifths are more than sevenths and just know that \( \frac{2}{3} > \frac{3}{4} \). I would know that \( \frac{3}{8} \) is less than \( \frac{2}{3} \) and \( \frac{3}{10} \) is more to compare \( \frac{2}{3} \) and \( \frac{9}{10} \). I would change \( \frac{3}{8} \) to \( \frac{6}{15} \) to compare \( \frac{2}{3} \) and \( \frac{4}{5} \).)

## Solutions

1. a) \( \frac{2}{5} < \frac{4}{7} \)
   
   b) \( \frac{7}{8} > \frac{5}{6} \)
   
   c) \( \frac{5}{7} > \frac{3}{5} \)
   
   d) \( \frac{5}{6} < \frac{8}{9} \)

2. a) e.g.,

   ![Fraction Strip Diagram]

   b) e.g.,

   ![Fraction Circle Diagram]

## Materials

- fraction strips or other fraction materials
3. e.g., You can look at the models and see.

4. a) \( \frac{6}{9} \) since there are more ninths
   b) \( \frac{5}{9} \) since each ninth is bigger than each eleventh and there are the same number
      of ninths and elevenths.

5. a) e.g., \( \frac{4}{5} \) as \( \frac{8}{10} \) since 8 tenths is more than 6 tenths
    b) e.g., \( \frac{4}{8} \) as \( \frac{9}{10} \) since 8 tenths is more than 8 fifteenths (fifteenths are smaller)
    c) \( \frac{9}{8} \) as \( \frac{27}{24} \) and \( \frac{14}{12} \) as \( \frac{28}{24} \) (since 27 twenty-fourths is less than 28 of them)
    d) \( \frac{8}{3} \) as \( \frac{16}{6} \) (since 16 sixths is less than 17 sixths)

6. a) e.g., Division means sharing. If 9 people share 4 things, they each get \( \frac{1}{9} \) of each of
    the things; that is a total of \( \frac{4}{9} \) for each person
    b) \( \frac{3}{7} = 0.428571428571… \) and \( \frac{4}{9} = 0.4444…. \)
    c) Since \( \frac{3}{7} \) is not even 43 hundredths and \( \frac{4}{9} \) is more than 44 hundredths, \( \frac{4}{9} \) is greater.

7. a) e.g., I know that \( \frac{8}{5} \) is more than 1 and \( \frac{2}{5} \) is less than 1, so \( \frac{8}{5} \) is greater.
    b) e.g., \( \frac{7}{2} \) is a bit more than 2, but \( \frac{17}{4} \) is more than 4, so \( \frac{17}{4} \) is greater.
    c) e.g., \( 3\frac{1}{4} \) is more since it is more than 3, but \( 2\frac{1}{4} \) is not even 3.
    d) e.g., \( \frac{1}{10} \) is less since it is less than \( \frac{1}{2} \) and \( \frac{7}{9} \) is more than \( \frac{1}{2} \).
    e) e.g., \( \frac{7}{8} \) is less since it is \( \frac{1}{8} \) away from 1, but \( \frac{9}{10} \) is only \( \frac{1}{10} \) away from 1, which is closer
    f) e.g., \( \frac{8}{9} \) is less since it is less than 1, but \( 1\frac{2}{3} \) is more than 1.

8. a) e.g., If you were deciding which of two groups was closer to reaching their
    fund-raising goal, when one is \( \frac{3}{5} \) of the way and the other is \( \frac{1}{2} \) the way
    b) e.g., If you were deciding which of two turkeys needed more cooking time if the
    number of hours for one was \( 2\frac{1}{2} \) and for the other was \( 2\frac{5}{5} \).

9. e.g., \( \frac{2}{3} < \frac{5}{6} < \frac{6}{4} \)
    \( \frac{4}{3} < \frac{10}{6} < \frac{12}{4} \)
    \( \frac{1}{3} < \frac{3}{6} < \frac{3}{4} \)

10. a) \( \frac{7}{3}, \frac{8}{3}, \frac{11}{4} \)
    
    b) \( \frac{9}{4}, \frac{10}{4}, \frac{11}{4} \)
    
    c) e.g., No since there will be fractions with each denominator and there are all the
        possible counting numbers that could be denominators
Comparing Fractions

Learning Goal
• selecting a strategy to compare fractions based on their numerators and denominators.

Open Question
• Choose two pairs of numbers from 3, 4, 6, 8, 9, 10, 12, 16, 20 to use as numerators and denominators of two fractions. For example, you could use \( \frac{3}{5} \) and \( \frac{5}{10} \).
  - Make sure that some of your fractions are improper and some are proper.
  - Make sure that some of your fractions use the same numerator and some do not.

Tell which of your fractions is greater and how you know.

• Make more fractions following the rules above and compare at least six pairs of them. Tell how you know which fraction is greater each time.

Think Sheet

Pairs of fractions are either equal or one fraction is greater than the other fraction. We can decide which statement is true by using a model, by renaming one or both fractions, or by using benchmarks.

Using a Model
• To compare \( \frac{3}{5} \) and \( \frac{5}{7} \), we can use a picture that shows the two fractions lined up, so we can see which extends farther.

Using a Model
• We can use parts of sets. For example, use a number of counters, such as 35, that is easy to divide into both fifths and sevenths:
  - \( \frac{1}{5} \) of 35 counters is 7 counters, so \( \frac{2}{5} \) of 35 counters is 21 counters.
  - \( \frac{1}{5} \) of 35 counters is 5 counters, so \( \frac{5}{7} \) of 35 counters is 25 counters.

\[ \frac{7}{35} \text{ is more than } \frac{21}{35}, \text{ so } \frac{2}{5} \text{ is more than } \frac{5}{7}. \]

Renaming Fractions

To rename a fraction, we can think about how to express the fraction as an equivalent fraction or equivalent decimal.

• Equivalent Fractions
Two fractions are equivalent, or equal, if they take up the same part of a whole or wholes.
For example, \( \frac{3}{5} = \frac{6}{10} \).

Each section of the \( \frac{3}{5} \) model is split into two sections in the \( \frac{6}{10} \) model. So, there are twice as many sections in the second model, and twice as many are shaded. The numerator and denominator have both doubled.
We can multiply the numerator and denominator by any amount (except 0) and the same thing happens as in the example above. There are more sections shaded and more sections unshaded, but the amount shaded does not change.
For example, if we multiply by 3: \( \frac{3}{5} = \frac{3 \times 3}{3 \times 5} = \frac{9}{15} \).

• Common Denominators and Common Numerators
Renaming fractions to get common denominators or common numerators is helpful when comparing fractions. If we compare fractions with the same denominator, the one with the greater numerator is greater. If we compare fractions with the same numerator, the one with the lesser denominator is greater.
To determine if \( \frac{2}{3} \) or \( \frac{3}{5} \) is more, we might use common denominators:
\[ \frac{2}{3} \times \frac{10}{10} = \frac{20}{30} \]
\[ \frac{3}{5} \times \frac{6}{6} = \frac{18}{30} \]
Since \( \frac{20}{30} \) and \( \frac{18}{30} \) are a common denominator.
Since \( 15 \times 2 = 30 \), then \( \frac{2}{3} \times \frac{2}{2} = \frac{4}{6} \).

Since \( \frac{4}{6} \) is less than \( \frac{5}{5} \), \( \frac{2}{3} < \frac{3}{5} \).

Since fractions are bigger than sixteens, \( \frac{3}{7} \) and \( \frac{3}{8} \),
Comparing Fractions (Continued)

5. How might you rename one or both fractions as other fractions to make it easier to compare them? Tell how it helps.
   a) \( \frac{3}{4} \) and \( \frac{5}{8} \)
   b) \( \frac{2}{3} \) and \( \frac{4}{6} \)
   c) \( \frac{1}{2} \) and \( \frac{3}{4} \)
   d) \( \frac{5}{6} \) and \( \frac{7}{8} \)

4. a) Why does it make sense that \( \frac{3}{4} \times 4 = \frac{3}{4} \)?
   
   b) What are the decimal equivalents of \( \frac{3}{4} \) and \( \frac{7}{8} \)?
   
   c) How could you use decimal equivalents to compare \( \frac{3}{4} \) and \( \frac{7}{8} \)?

Comparing Fractions (Continued)

7. How can each of these fraction pairs be compared without renaming them as other fractions or as decimals?
   a) \( \frac{3}{4} \) and \( \frac{7}{8} \)
   b) \( \frac{5}{6} \) and \( \frac{11}{12} \)
   c) \( \frac{2}{3} \) and \( \frac{5}{6} \)
   d) \( \frac{4}{5} \) and \( \frac{7}{10} \)
   e) \( \frac{3}{4} \) and \( \frac{5}{8} \)
   f) \( \frac{7}{12} \) and \( \frac{5}{8} \)

8. Describe a real-life situation when you might make each comparison:
   a) \( \frac{1}{2} \) to \( \frac{1}{3} \)
   b) \( \frac{2}{3} \) to \( \frac{2}{3} \)

9. A fraction with a denominator of 5 is between one fraction with a denominator of 3 and one fraction with a denominator of 4. Fill in all 9 blanks to show three ways this could be true.
   
   a) \( \frac{\_}{\_} \)
   b) \( \frac{\_}{\_} \)
   c) \( \frac{\_}{\_} \)

10. a) List all the fractions with a denominator of 3 between 2 and 3.
    
    b) List all the fractions with a denominator of 4 between 2 and 3.
    
    c) Is it possible to list all the fractions between 2 and 3? Explain.
Adding Fractions

Learning Goal
• selecting an appropriate unit and an appropriate strategy to add two fractions.

Open Question

Questions to Ask Before Using the Open Question

◊ Why is it easy to add \( \frac{2}{5} + \frac{2}{5} \)? (e.g., It is just 4 sections that are fifths, so it is \( \frac{4}{5} \).)
◊ How would you add \( \frac{3}{4} + \frac{2}{3} \)? (e.g., I would use a model, such as fraction strips.)
   How would you do that? (e.g., I would look for equivalent strips for \( \frac{3}{4} \) and \( \frac{2}{3} \) with the same denominator. Then I would count how many sections were shaded.)
◊ Which way would you add \( \frac{2}{3} + \frac{2}{3} \)? (e.g., The same way I did \( \frac{2}{5} + \frac{2}{5} \).)
◊ How do you know that \( \frac{1}{3} + \frac{1}{4} \) is more than \( \frac{1}{2} \)? (e.g., I know that \( \frac{1}{2} + \frac{1}{2} \) is \( \frac{1}{2} \) and \( \frac{1}{3} \) is more than \( \frac{1}{4} \), so the answer has to be more.)

Using the Open Question

Make sure students understand that they must create at least four fraction sums, each time predicting why the sum would only be a bit more than 1, how they calculated the sum, and a verification that the sum meets the required conditions. Some students might choose to use models.

By viewing or listening to student responses, note if they:
• can estimate fraction sums;
• realize that if both fractions are slightly greater than \( \frac{1}{2} \), the sum will be slightly greater than 1;
• realize that if one fraction is \( \frac{1}{2} \) and the other slightly greater than half, the sum will be greater than 1;
• realize that they can take any fraction, figure out how much less than 1 it is and add a bit more;
• realize that they can start with a fraction greater than, but not a lot greater than, 1 and add a very small amount.

Consolidating and Reflecting on the Open Question

◊ Suppose your first fraction had been \( \frac{1}{2} \). How would you have picked the second one? (e.g., I would pick something just a little more than \( \frac{1}{2} \).) What might it have been and how would you have added? (e.g., It had to have an odd denominator, so maybe \( \frac{3}{5} \). To add \( \frac{3}{5} \) to \( \frac{1}{2} \), I would write them both as tenths and count how many tenths.)
◊ Could one of your fractions have been greater than 1? (Yes) How would you pick your other one? (e.g., I would make it really small.)
◊ What strategy did you use most often to add your fractions? (e.g., I used equivalent fractions with the same denominator.)

Solutions

e.g.,

**Pair 1:** \( \frac{5}{7} + \frac{1}{2} = \)

I knew that \( \frac{5}{7} \) was just a little more than half since \( \frac{5}{7} \) is half. Sevenths are bigger than eighths, so \( \frac{5}{7} \) is more than \( \frac{4}{8} \). I knew that if I added exactly \( \frac{1}{2} \), I would be just a little more than 1.

The sum is \( \frac{15}{14} \) which equals \( 1 \frac{1}{14} \). I got the sum by using equivalent fractions: \( \frac{8}{14} \) and \( \frac{7}{14} \).

\( \frac{1}{14} \) is not much, so it is just a little more than 1.
Pair 2: $\frac{3}{5} + \frac{12}{25}$

I knew that $\frac{12}{25}$ was just a little under $\frac{1}{2}$ and thought that $\frac{3}{5}$ was enough more than $\frac{1}{2}$ that the sum would be more than 1.

$\frac{18}{25} + \frac{12}{25} = \frac{30}{25} = \frac{12}{25}$. I got the sum by writing $\frac{3}{5}$ as the equivalent fraction $\frac{15}{25}$.

$\frac{12}{25}$ is not much, so the sum is just a little more than 1.

Pair 3: $\frac{5}{6}$ and $\frac{1}{5}$.

I knew that $\frac{5}{6} + \frac{1}{6}$ would be exactly 1 and I knew that $\frac{1}{5}$ was just a bit more than $\frac{1}{6}$ so the sum would be just a bit more than 1.

$\frac{5}{6} + \frac{1}{5} = \frac{31}{30} = 1\frac{1}{30}$. I got the sum by writing $\frac{5}{6}$ as the equivalent fraction $\frac{25}{30}$ and $\frac{1}{5}$ as the equivalent fraction $\frac{6}{30}$. The sum is $\frac{31}{30}$.

Pair 4: $\frac{10}{9} + \frac{1}{100}$.

I knew that $\frac{10}{9}$ is already just a little more than 1, but that $\frac{1}{100}$ is so small that adding it keeps the answer close to 1.

The sum is $1\frac{109}{900}$. I wrote $\frac{10}{9}$ as $1 + \frac{1}{9}$. I added $\frac{1}{9}$ and $\frac{1}{100}$ using the common denominator of 900. The equivalent fractions were $\frac{100}{900} + \frac{9}{900} = \frac{109}{900}$. But there was still the 1 whole.

$\frac{109}{900}$ is close to $\frac{100}{900}$ which is $\frac{1}{9}$ and that is not much more than 0, so $\frac{109}{900}$ is not that much more than 1.
**Think Sheet**

### Questions to Ask Before Assigning the Think Sheet

◊ Why is it easy to add \( \frac{2}{5} + \frac{2}{5} \)? (e.g., It is just 4 sections that are fifths, so it is \( \frac{4}{5} \).)

◊ How would you add \( \frac{2}{3} \) and \( \frac{2}{3} \)? (e.g., I would use a model, such as fraction strips.)

   *How would you do that?* (e.g., I would look for equivalent strips for \( \frac{2}{3} \) and \( \frac{2}{3} \) with the same denominator. Then I would count how many sections were shaded.)

◊ Which way would you add \( \frac{2}{3} \) and \( \frac{2}{3} \)? (e.g., The same way I did \( \frac{2}{5} + \frac{2}{5} \).)

◊ How do you know that \( \frac{1}{3} + \frac{1}{4} \) is more than \( \frac{1}{2} \)? (e.g., I know that \( \frac{1}{4} + \frac{1}{4} \) is \( \frac{1}{2} \) and \( \frac{1}{3} \) is more than \( \frac{1}{4} \), so the answer has to be more.)

### Using the Think Sheet

Read through the introduction box with students and make sure students understand the material.

Assign the tasks.

By viewing or listening to student responses, note if they:

- relate appropriate fraction models to fraction addition;
- recognize how to add fractions with the same denominator as well as those with different denominators;
- can estimate fraction sums;
- add mixed numbers by adding the whole number parts and fraction parts separately;
- relate situations to fraction additions;
- recognize why they cannot just add numerators and denominators.

### Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What different strategies could you use to add \( \frac{1}{6} \) to \( \frac{7}{8} \)? (e.g., I could use fraction strips, or grids or equivalent fractions.)

◊ How do you know the result would be a little more than 1? (e.g., If you add \( \frac{1}{6} \) to \( \frac{7}{8} \), it would be 1, but \( \frac{1}{8} \) is a little more than \( \frac{1}{6} \).)

◊ What size grid would you use? (e.g., 6 by 8) *How would you use it?* (e.g., I would fill 7 of the 8 columns. Then I would fill 1 of the 6 rows. I would move any doubled-up counters. I think I will need a second grid, though.)

◊ If you used equivalent fractions, what denominator would you use? (e.g., I would use 24, since you can write eighths and sixths as twenty-fourths.)

### Solutions

1. a) e.g., A whole has 3 rows and 5 columns. Each row is \( \frac{1}{3} \) and each column is \( \frac{1}{5} \). \( \frac{4}{3} \) is four rows and \( \frac{1}{5} \) is one column.
   
   b) \( \frac{23}{15} \) or \( 1\frac{8}{15} \)

2. a) \( \frac{2}{5} + \frac{2}{5} \)
   
   b) \( \frac{3}{8} + \frac{2}{8} \)
   
   c) \( \frac{2}{3} + \frac{1}{3} \)
   
   d) \( \frac{3}{8} + \frac{1}{3} \)
   
   e) \( \frac{9}{14} + \frac{4}{5} \)

3. a) MORE than 1
   
   b) LESS than 1
   
   c) MORE than 1
   
   d) MORE than 1

### Materials

- fraction strips or other fraction materials
- grids and counters
4. a) \(\frac{9}{8}\)  
b) \(\frac{7}{5}\)  
c) \(\frac{31}{40}\)  
d) \(\frac{22}{15}\) or \(1\frac{7}{15}\)  
e) \(\frac{49}{15}\) or \(3\frac{4}{15}\)  
f) \(4\frac{4}{15}\)  

Model for part c:

\[
\begin{array}{cccccccc}
X & O & O & X & O & X & X & X & X \\
X & O & O & X & O & X & X & X & X \\
o & o & o & o & o & o & o & o & o \\
o & o & o & o & o & o & o & o & o \\
o & o & o & o & o & o & o & o & o \\
\end{array}
\]

Model for part e:

\[
\begin{array}{cccc}
X & X & X & X \\
X & X & X & X \\
o & o & o & o \\
o & o & o & o \\
o & o & o & o \\
\end{array}
\]

5. a) e.g., 3 and 9 since you could have changed the thirds to be ninths to combine them with the other ninths  
b) e.g., 9 and 9  

6. a) e.g., \(\frac{4}{8} + \frac{1}{3}\)  
b) e.g., \(\frac{6}{8} + \frac{1}{2}\)  

7. a) e.g., because \(1\frac{1}{2} + 1\frac{1}{2}\) is 3 and \(\frac{2}{3}\) and \(\frac{3}{4}\) are both more than \(\frac{1}{2}\)  
b) \(3\frac{5}{12}\) cups  

8. e.g., A baker used \(\frac{2}{3}\) cup of sugar for a cake and \(\frac{3}{4}\) cup of sugar for some cookies. How much sugar was used?  

9. a) \(\frac{9}{9} + \frac{5}{9} = \frac{14}{9}\) 
b) \(\frac{9}{3} + \frac{7}{5} = \frac{68}{15}\)  

10. a) yes  
b) e.g., You can always get equivalent fractions with the same denominator if you multiply the numerator and denominator of each fraction by the other denominator and that is what Kyle did. Once the denominators are the same, you just combine numerators.  

11. e.g., I would use an example, such as \(\frac{2}{3}\) and \(\frac{3}{4}\). If you add numerators and denominators, you get \(\frac{3}{7}\) which is less than 1. But I know that \(\frac{2}{3}\) and \(\frac{3}{4}\) are each almost 1, so the sum will be more than 1.
Adding Fractions

Learning Goal

• selecting an appropriate unit and an appropriate strategy to add two fractions.

Open Question

• Choose two different, non-equivalent fractions to add to meet these conditions.
  - Their sum is a little more than 1.
  - Their denominators are different.
  - At least one denominator is odd.

Tell how you predicted the sum would be a little more than 1.

Calculate the sum and explain your process.

Verify that the sum is just a little more than 1.

• Repeat the steps at least three more times with other fractions.

Think Sheet

Adding fractions means combining them.

Some Denominators

• To combine fractions with the same denominator we can count.
  For example, \( \frac{2}{3} + \frac{1}{3} \) is 3 fourths + 5 fourths. If we count the fourths, we get 8 fourths, so \( \frac{2}{3} + \frac{1}{3} = \frac{8}{12} \). Another name for \( \frac{2}{3} \) is \( \frac{8}{12} \).
  Notice we add the numerators, not the denominators, because we are combining fourths. If we combined the two denominators, we would get eighths, not fourths.

Using fraction pieces, we can see that \( \frac{2}{3} + \frac{1}{3} \) if we combine the last \( \frac{1}{3} \) part of the \( \frac{2}{3} \) with the \( \frac{1}{3} \) we see that 2 wholes are shaded. That shows that \( \frac{8}{12} = \frac{2}{3} \).

Different Denominators

• To combine fractions with different denominators requires more thinking. For example, look at this picture of \( \frac{1}{2} + \frac{1}{4} \).
  The total length is not as much as \( \frac{1}{2} \) but it is more than \( \frac{1}{4} \).

Use a Grid

• Another way to combine fractions is to use a grid.
  For example, to add \( \frac{1}{3} \) and \( \frac{1}{5} \), we could create a 3-by-5 grid to show thirds and fifths. \( \frac{1}{3} + \frac{1}{5} \) is two rows and \( \frac{1}{5} \) is one column.

Use the letter \( x \) to fill two rows to show \( \frac{1}{3} \). Use the letter \( o \) to fill a column to show \( \frac{1}{5} \).

Move counters to empty sections of the grid so each section holds only one counter.

Since \( \frac{3}{15} \) of the grid is covered, \( \frac{1}{3} + \frac{1}{5} \) is \( \frac{8}{15} \). Notice that \( \frac{8}{15} \) and \( \frac{8}{15} \) is \( \frac{8}{15} \), so this makes sense.
Adding Fractions (Continued)

• Sometimes the sum of two fractions is greater than 1.

To model this with fraction strips, we might need more than one whole strip. For example, to show \( \frac{3}{5} + \frac{3}{4} \) we can use 20 as the common denominator.

\[
\frac{3}{5} = \frac{12}{20}, \quad \frac{3}{4} = \frac{15}{20}
\]

Move \( \frac{8}{20} \) up to the first strip to fill the whole. Then, \( \frac{7}{20} \) is left in the second strip.

\[
\frac{3}{5} + \frac{3}{4} = \frac{27}{20} = 1 \frac{7}{20}
\]

We can model the sum of two fractions that is greater than 1 with a grid. When you move the counters so there is only one counter in each section, the grid is filled, with 7 counters extra.

\[
\frac{3}{5} = \frac{12}{20} \quad \text{marked with } o's
\]
\[
\frac{3}{4} = \frac{15}{20} \quad \text{marked with } x's
\]

Adding Fractions (Continued)

Improper Fractions

To add improper fractions, we can use models or we can use equivalent fractions with the same denominators and count.

For example, \( \frac{9}{2} + \frac{5}{3} \) = \( \frac{27}{6} + \frac{10}{6} = \frac{37}{6} = \frac{6}{1} \)

We can check by estimating. \( \frac{4}{2} + \frac{2}{3} \) is close to \( 4 + 2 = 6 \).

Mixed Numbers

To add mixed numbers, such as \( 2 \frac{1}{3} + 3 \frac{3}{4} \), we could add the whole number parts and fraction parts separately.

\[
2 \frac{1}{3} + 3 \frac{3}{4} = 5 \frac{13}{12} = 6 \frac{1}{12}
\]

1. a) How does this model show \( \frac{4}{5} + \frac{1}{8} \)?

b) What is the sum?

2. What addition is the model showing?

a) \[ \frac{2}{3} + \frac{1}{4} \]

b) \[ \frac{3}{4} + \frac{2}{5} \]

3. Estimate to decide if the sum will be more or less than 1. Circle MORE than 1 or LESS than 1.

a) \[ \frac{5}{6} + \frac{2}{3} \]

b) \[ \frac{2}{7} + \frac{3}{5} \]

c) \[ \frac{3}{8} + \frac{1}{4} \]

d) \[ \frac{1}{3} + \frac{3}{5} \]

e) \[ \frac{2}{3} + \frac{1}{2} \]

4. Add each pair of fractions or mixed numbers. Draw models for parts c) and e).

a) \[ \frac{3}{4} + \frac{1}{2} \]

b) \[ \frac{1}{3} + \frac{1}{5} \]

c) \[ \frac{3}{5} + \frac{2}{8} \]

d) \[ \frac{8}{10} + \frac{3}{4} \]

e) \[ \frac{5}{3} + \frac{1}{2} \]

5. The sum of two fractions is \( \frac{14}{9} \).

a) What might their denominators have been? Explain.

b) List another possible pair of denominators.

6. Choose values for the blanks to make each true.

a) \[ \frac{1}{3} + \frac{1}{2} = \frac{5}{6} \]

b) \[ \frac{1}{4} + \frac{1}{3} = \frac{7}{12} \]

7. Lisa used \( 1 \frac{2}{3} \) cups of flour to make cookies and another \( 1 \frac{3}{4} \) cups of flour to bake a cake.

a) How do you know that she used more than 3 cups of flour for her baking?

b) How much flour did she use?

8. Write a story problem that would require you to add \( \frac{2}{3} + \frac{1}{2} \) to solve it. Solve your problem.
9. a) Use the digits 3, 5, 7 and 9 in the boxes to create the least sum possible.

b) Use the digits again to create the greatest sum possible.

10. Kyle noticed that if you add two fractions, you get the sum’s denominator by multiplying the denominators and you get the sum’s numerator by multiplying each numerator by the other fraction’s denominator and adding. For example, for \( \frac{3}{5} + \frac{7}{8} \), the denominator is 5 \( \times \) 8 and the numerator is 3 \( \times \) 8 \( + \) 7 \( \times \) 5.

a) Do you agree?

b) Explain why or why not.

11. You have to explain why it does not work to add numerators and denominators to add two fractions. What explanation would you use?
Subtracting Fractions

Learning Goal

• selecting an appropriate unit and an appropriate strategy to subtract two fractions

Open Question

Questions to Ask Before Using the Open Question

◊ How would you subtract $\frac{2}{6}$ from $\frac{5}{6}$? (e.g., I would start with $\frac{2}{6}$ and count how many more sixths to get to $\frac{5}{6}$.)
◊ How would you subtract $\frac{3}{4}$ from $\frac{5}{4}$? (e.g., I would use a model, such as fraction strips.) How would you do that? (e.g., I would look for equivalent strips for $\frac{3}{4}$ and $\frac{5}{4}$ with the same denominator. Then I would count how many more sections were shaded in the $\frac{3}{4}$ than in the $\frac{5}{4}$.)
◊ Which way would you subtract $\frac{2}{3}$ from $\frac{5}{3}$? (e.g., More as I did $\frac{5}{6} - \frac{2}{6}$.)
◊ How do you know that $\frac{7}{8} - \frac{3}{9}$ is close to $\frac{1}{2}$? (e.g., I know that $\frac{7}{8} - \frac{3}{9}$ is $\frac{1}{2}$ and $\frac{3}{9}$ is close to $\frac{1}{3}$, so it should be close to $\frac{1}{2}$.)

Using the Open Question

Make sure students understand that they must create at least four fraction differences, each time predicting why the difference would be close to 1, explaining how they calculated the difference and verifying the fact that the difference meets the criteria required. Some students might choose to use models.

By viewing or listening to student responses, note if they:

• can estimate fraction differences;
• realize that if one fraction is slightly greater than 1, they should not subtract too much;
• realize that if one fraction is close to, but less than 1, the other fraction should be fairly small;
• realize that the original fraction cannot be too much less than 1;
• realize that they can use improper fractions whose mixed number equivalents have wholes about 1 apart.

Consolidating and Reflecting on the Open Question

◊ Suppose your first fraction had been $\frac{5}{4}$. How would you have picked the second one? (e.g., I would pick something just a little more or less than $\frac{1}{4}$, since $\frac{5}{4}$ is $\frac{1}{4}$ over 1.) What might it have been and how would you have subtracted? (e.g., It had to have an odd denominator, so maybe $\frac{2}{5}$. To subtract $\frac{2}{5}$ from $\frac{5}{4}$, I would use equivalent decimals.)
◊ Could one of your fractions have been greater than 3? (Yes) How would you pick your other one? (e.g., I would choose a fraction near 2.)
◊ What strategy did you use most often to subtract your fractions? (e.g., I used equivalent fractions with the same denominator.)

Solutions

e.g.,

Pair 1: $\frac{7}{5} - \frac{1}{2}$

I knew that $\frac{7}{5}$ was $\frac{2}{5}$ more than 1, so I wanted to subtract a little more than $\frac{2}{5}$. $\frac{1}{2}$ works since it is $2\frac{1}{2}$ fifths. $\frac{7}{5} - \frac{1}{2} = \frac{9}{10}$. I used equivalent fractions that were tenths ($\frac{14}{10}$ and $\frac{9}{10}$).

$\frac{9}{10}$ is almost $\frac{10}{10}$ so it is just a little less than 1.
**Pair 2:** \(\frac{24}{25} - \frac{1}{100}\)

I knew that \(\frac{24}{25}\) is already close to 1 and \(\frac{1}{100}\) is not very much, so if you subtract it, the answer will still be close to 1.

\[
\frac{24}{25} - \frac{1}{100} = \frac{95}{100}.
\]

I used a hundredths grid and shaded in nine columns and 6 more squares (96 squares) for \(\frac{24}{25}\) and then I removed the shading on one square, so there were 95 squares left shaded.

**Pair 3:** \(\frac{3}{2} - \frac{3}{5}\)

I knew that \(\frac{3}{2}\) is \(\frac{1}{2}\) more than 1, so I wanted to take away more than \(\frac{1}{2}\), but not a lot more. That is why I chose \(\frac{3}{5}\).

\[
\frac{3}{2} - \frac{3}{5} = \frac{9}{10}.
\]

I figured it out using fraction strips. It takes 2 strips to represent \(\frac{3}{2}\). I saw that there were \(\frac{2}{5}\) left on the first strip and then another \(\frac{1}{2}\) on the second strip. I know that \(\frac{2}{5} + \frac{1}{2} = \frac{4}{10} + \frac{5}{10} = \frac{9}{10}\)

\[
\frac{9}{10}\text{ is almost a whole, so it is close to 1.}
\]

**Pair 4:** \(2\frac{1}{3} - 1\frac{1}{4}\)

I know that \(2\frac{1}{3}\) is just a bit more than 2 and \(1\frac{1}{4}\) is a bit more than 1, so the difference should be close to 1.

\[
2\frac{1}{3} - 1\frac{1}{4} = 1\frac{1}{12}.
\]

I subtracted the whole number parts and then the fraction parts. I used a grid for the fraction parts.

\[
1\frac{1}{12}\text{ is close to 1 since }\frac{1}{12}\text{ is quite small.}
\]
Questions to Ask Before Assigning the Think Sheet

◊ How would you subtract \( \frac{2}{6} \) from \( \frac{5}{6} \)? (e.g., I would start with \( \frac{2}{6} \) and count how many more sixths to get to \( \frac{5}{6} \).)

◊ How would you subtract \( \frac{2}{3} \) from \( \frac{3}{4} \)? (e.g., I would use a model, such as fraction strips.) How would you do that? (e.g., I would look for equivalent strips for \( \frac{3}{4} \) and \( \frac{2}{3} \) with the same denominator. Then I would count how many sections were shaded in the \( \frac{3}{4} \) than the \( \frac{2}{3} \).)

◊ Which way would you subtract \( \frac{2}{3} \) from \( \frac{5}{3} \)? (e.g., More as I did \( \frac{5}{6} - \frac{2}{6} \).)

◊ How do you know that \( \frac{7}{8} - \frac{3}{9} \) is close to \( \frac{1}{2} \)? (e.g., I know that \( \frac{7}{8} - \frac{3}{8} \) is \( \frac{1}{8} \) and \( \frac{3}{9} \) is close to \( \frac{1}{8} \), so it should be close to \( \frac{1}{2} \).)

Using the Think Sheet

Read through the introductory box with students and make sure students understand the material explained in the instructional box.

Assign the tasks.

By viewing or listening to student responses, note if they:
• relate appropriate fraction models to fraction subtraction;
• recognize how to subtract fractions with the same denominator as well as those with different denominators;
• can estimate fraction differences;
• subtract mixed numbers by using the whole number parts and fraction parts separately;
• relate situations to fraction subtractions;
• recognize why they cannot just subtract numerators and denominators.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What different strategies could you use to subtract \( \frac{1}{2} \) from \( \frac{7}{8} \)? (e.g., I could use fraction strips, grids, or equivalent fractions.)

◊ How do you know the result would be a little less than \( \frac{3}{4} \)? (e.g., If you subtract \( \frac{1}{8} \), it would be \( \frac{6}{8} \), which is \( \frac{3}{4} \), but \( \frac{1}{8} \) is a little more than \( \frac{1}{4} \) to take away.)

◊ What size grid would you use? (e.g., 6 by 8) How would you use it? (e.g., I would fill 7 of the 8 columns. Then I would move one counter to fill a row and remove the counters in one row.)

◊ If you used equivalent fractions, what denominator would you use? (e.g., I would use 24, since you can write eighths and sixths as twenty-fourths)

Solutions

1. a) e.g., each row is \( \frac{1}{3} \) of a grid and 4 rows are covered and one column is removed; each column is \( \frac{1}{2} \) of a grid.
   b) \( \frac{7}{15} \)

2. a) \( \frac{5}{6} - \frac{1}{6} \)
   b) \( \frac{7}{12} - \frac{5}{12} \)
   c) \( \frac{4}{5} - \frac{2}{3} \)
   d) \( \frac{5}{6} - \frac{2}{5} \)
   e) \( \frac{8}{3} - \frac{3}{5} \)

3. a) closer to \( \frac{1}{2} \)
   b) closer to 1
   c) closer to \( \frac{5}{6} \)
   d) closer to \( \frac{1}{2} \)
4. a) $\frac{6}{5}$
   b) $\frac{5}{24}$
   c) $\frac{6}{35}$
   d) $\frac{17}{12} = 1\frac{5}{12}$
   e) $1\frac{5}{6}$
   f) $2\frac{3}{10}$

Example 4 c)

Example 4 f)

5. a) e.g., 4 and 3 since fourths and thirds can both be written as twelfths
   b) e.g., 12 and 2

6. a) e.g., $\frac{4}{3} - \frac{1}{2}$
   b) e.g., $\frac{17}{12} - \frac{1}{12}$

7. a) e.g., about 1 cup
   b) $1\frac{1}{3}$ cups

8. e.g., I had $3\frac{1}{3}$ cups of flour and used $1\frac{1}{2}$ cups for a cake. How much flour do I have left?

9. a) e.g., $3\frac{7}{9} - 2\frac{5}{9}$
   b) e.g., $9\frac{3}{5} - 3\frac{5}{8}$

10. a) yes
    b) e.g., When you do that, you are just getting equivalent fractions for the original fractions since you are multiplying the numerator and denominator by the same thing — the other denominator.

11. e.g., I would use an example: I know that $\frac{1}{3} - \frac{3}{5} = \frac{1}{15}$, but if I subtracted numerators and denominators, I would get $\frac{6}{1}$, which, I know, is wrong.
Open Question

Subtracting Fractions

Learning Goal

- selecting an appropriate unit and an appropriate strategy to subtract two fractions.

Open Question

- Choose two fractions to subtract to meet these conditions.
  - The difference is close to 1, but not exactly 1.
  - Their denominators are different.
  - One denominator is odd.

Tell how you predicted the difference would be close to 1.

Calculate the difference and explain your process.

Verify that the difference is close to 1.

- Repeat the steps at least three more times with other fractions.

Think Sheet

Subtracting Fractions (Continued)

Think Sheet

Subtracting can involve take away, comparing, or looking for a number to add.

- To subtract fractions with the same denominator we count.
  - For example, \( \frac{5}{4} - \frac{4}{4} \) is 5 fourths – 3 fourths. Since the denominators are fourths, if we take 3 from 5, there are 2 left, so \( \frac{5}{4} - \frac{4}{4} = \frac{1}{4} \).
  - We subtracted the numerators because the fourths were there just to tell the size of the pieces. Nothing was done to the 4s.
  - We can see that \( \frac{5}{4} - \frac{4}{4} = \frac{1}{4} \) using fraction pieces.

- To subtract fractions with different denominators requires a number of steps. For example, look at the picture of \( \frac{5}{3} - \frac{4}{5} \). The difference is — the part of the \( \frac{5}{3} \) that extends beyond the \( \frac{4}{5} \).
  - It is hard to tell how exactly long the dark piece is by looking at the picture.
  - So we can create equivalent fractions for \( \frac{5}{3} \) and \( \frac{4}{5} \) that share the same denominator. That denominator has to be a multiple of both 3 and 4, so 20 (5 x 4) is a possibility.
  - We subtract 5 twentieths from 8 twentieths, leaving 3 twentieths.
  - In the diagram there are 3 extra sections in shaded on top and each is a twentieth section.
  - \( \frac{5}{3} - \frac{4}{5} = \frac{13}{20} \).

If one of the fractions is greater than 1, more than one strip or grid may need to be used. For example, to show \( \frac{3}{2} - \frac{3}{5} \):

Renaming the fractions as fifteenths can help.

Using two 3-by-5 grids to solve the same question, cover \( \frac{3}{2} \) by covering 3 rows. Remove counters from 4 columns to remove \( \frac{3}{5} \) of a grid. 3 rows of a grid are left covered.
Subtracting Fractions

To subtract mixed numbers, such as $4\frac{1}{3} - 3\frac{1}{4}$, we could add up. For example, if we add $\frac{1}{3}$ to $\frac{1}{4}$, we get to $2$. If we add another $\frac{2}{3}$ we get to $\frac{5}{2}$.

We could subtract the whole number parts and the fraction parts. Since $\frac{1}{3} > \frac{1}{4}$, it might make sense to rename $4\frac{1}{3}$ as $3 + \frac{4}{3}$ first.

$4\frac{1}{3} - 3\frac{1}{4} = 3 - \frac{1}{4} + \frac{4}{3} - \frac{1}{3} = \frac{16}{12} - \frac{9}{12} = \frac{7}{12}$.

1. a) How does this model show $\frac{3}{4} - \frac{1}{2}$?

b) How much is $\frac{3}{4} - \frac{1}{5}$?

2. What subtraction is the model showing?

a) $\frac{3}{4} - \frac{1}{2}$

b) $\frac{2}{3} - \frac{1}{4}$

c) $\frac{1}{3} - \frac{1}{3}$

d) $\frac{7}{8} - \frac{3}{4}$

3. Estimate to decide if the difference will be closer to $\frac{1}{2}$ or 1. Circle your choice.

a) $\frac{1}{4} - \frac{1}{4}$ closer to $\frac{1}{2}$ closer to 1

b) $\frac{3}{4} - \frac{1}{2}$ closer to $\frac{1}{2}$ closer to 1

c) $\frac{1}{3} - \frac{1}{5}$ closer to $\frac{1}{2}$ closer to 1

d) $\frac{7}{8} - \frac{3}{4}$ closer to $\frac{1}{2}$ closer to 1

4. Subtract each pair of fractions or mixed numbers. Draw models for parts c) and f).

a) $9 - 5 - 3 - 5$

b) $7 - 8 - 2 - 3$

c) $3 - 5 - 3 - 7$

d) $11 - 3 - 9 - 4$

e) $4\frac{1}{5} - 1\frac{7}{8}$

f) $5 - 3\frac{5}{6}$

5. The difference of two fractions is $\frac{11}{12}$. a) What do you think their denominators might have been? Explain.

b) List another possible pair of denominators.

6. Choose values for the blanks to make each equation true.

a) $\frac{5}{2} \times \frac{1}{3} = \frac{1}{2}$

b) $\frac{5}{6} \times \frac{1}{4} = \frac{1}{12}$

7. Sakura started with 3 cups of flour. She used $\frac{1}{2}$ cups for one recipe.

a) About how much flour did she have left?

b) How much flour did she have left?

8. Write a story problem that would require you to subtract $\frac{1}{2}$ from $\frac{3}{4}$ to solve it. Solve the problem.
# Multiplying Fractions

## Learning Goal
- representing multiplication of fractions as repeated addition or determining area.

## Open Question

### Questions to Ask Before Using the Open Question

- What is \( \frac{1}{3} \) of \( \frac{3}{5} \)? (\( \frac{1}{3} \times \frac{3}{5} \) means 4 of them, so \( \frac{1}{3} \times \frac{3}{5} \) should be \( \frac{1}{3} \) of them.)
- How would you multiply \( \frac{2}{3} \) and \( \frac{4}{5} \)? (e.g., I would think of \( \frac{4}{5} \) as \( \frac{1}{5} + \frac{3}{5} \) and multiply \( \frac{1}{5} \) to get \( \frac{1}{5} \) and add it to \( \frac{3}{5} \). I know that \( \frac{1}{5} \times \frac{3}{5} \) means \( \frac{1}{5} \) of \( \frac{3}{5} \), so it is \( \frac{3}{5} \). The answer, then is \( \frac{3}{5} + \frac{1}{5} = \frac{4}{5} \).)
- Would you multiply \( \frac{2}{3} \times \frac{4}{5} \) the same way? (e.g., No, I would draw a rectangle and divide it into equal sections and figure out what part of the whole a rectangle \( \frac{2}{3} \times \frac{4}{5} \) is.)
- How do you know that \( \frac{4}{3} \times \frac{3}{5} \) is more than \( \frac{3}{5} \), but not a lot more? (e.g., I know that \( 1 \times \frac{3}{5} \) is \( \frac{3}{5} \). Since \( \frac{4}{3} \) is more than 1, then \( \frac{4}{3} \times \frac{3}{5} \) is more than \( \frac{3}{5} \). But \( \frac{4}{3} \) is not a lot more than 1, so \( \frac{4}{3} \times \frac{3}{5} \) is not a lot more than \( \frac{3}{5} \).)

### Using the Open Question

Make sure students understand that they must create at least four fraction products, each time predicting why the product would be a bit more than 1, how they calculated the product and a verification that the product is the right size. Some students might choose to use models.

By viewing or listening to student responses, note if they:
- can estimate fraction products;
- realize that if one fraction is slightly greater than 1, they should multiply by something close to 1;
- realize that if one fraction is close to, but less than 1, the other fraction should be a bit more than 1;
- realize that they can use any mixed number if they multiply by a unit fraction with a denominator equal to the whole number part of the mixed number (e.g., \( 4 \frac{1}{3} \times \frac{1}{2} \)).

### Consolidating and Reflecting on the Open Question

- Suppose your first fraction had been \( \frac{5}{4} \). How would you have picked the second one? (e.g., I would pick something really close to 1, maybe a bit less.) What might it have been and how would you have multiplied? (e.g., I might have picked \( \frac{9}{10} \) and then just multiplied numerators and denominators.)
- Could one of your fractions have been greater than 3? (Yes) How would you pick your other one? (e.g., I would choose a fraction that is about \( \frac{3}{4} \)).
- How could you multiply \( 3 \frac{1}{2} \times \frac{4}{5} \)? (e.g., I know that you can add \( \frac{2}{5} \times 3 \) to \( \frac{2}{5} \times \frac{1}{5} \), so I would do that.)

### Solutions

- \( \frac{8}{9} \times \frac{2}{3} \)
  - I started with \( \frac{8}{9} \), I realized that if I multiplied by \( \frac{1}{3} \), it would be \( \frac{2}{5} \) and that was too low, so I picked a fraction more than \( \frac{1}{3} \) but not more than 1.
  - \( \frac{8}{9} \times \frac{2}{3} = \frac{16}{15} \).
  - \( \frac{16}{15} = 1 \frac{1}{15} \) which is just a little more than 1.
**Pair 2**: $\frac{2}{5} \times 2\frac{3}{5}$  
I started with $\frac{2}{5}$. I knew that double was only $\frac{4}{5}$, so I needed more than $\frac{2}{5} \times 2$. I thought about $2\frac{1}{2}$ times but it would be exactly 1, so I went higher than $\frac{2}{5}$ for the fraction part.

$\frac{2}{5} \times 2\frac{1}{2} = \frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1\frac{1}{2}$.

$\frac{1}{25}$ is not very much, so $1\frac{1}{25}$ is just a little more than 1.

**Pair 3**: $3\frac{1}{3} \times \frac{1}{3}$  
I started with $3\frac{1}{3}$ which is more than 3. I need $\frac{1}{3}$ of it to be close to 1.

$3\frac{1}{3} \times \frac{1}{3} = 1\frac{1}{9}$. I took $\frac{1}{3}$ of 3, which I know is 1, and $\frac{1}{3}$ of $\frac{1}{3}$ is $\frac{1}{9}$ since there are three 3s in 9.  
$1\frac{1}{9}$ just a little more than 1, since $\frac{1}{9}$ is not much.

**Pair 4**: $\frac{8}{9} \times 10\frac{10}{9}$  
I knew that $\frac{8}{9} \times \frac{9}{9}$ is 1 (since $\frac{8}{9} \times \frac{9}{9} = \frac{72}{81} = 1$). So I needed the second fraction to be a bit more than $\frac{9}{9}$. That means I could use $\frac{10}{9}$.

$\frac{8}{9} \times 10\frac{10}{9} = \frac{10}{9}$. I know that $\frac{1}{9}$ of $\frac{8}{9}$ is $\frac{1}{9}$ since there are 8 parts in $\frac{8}{9}$ and 1 out of 8 of them is $\frac{1}{9}$. That means that $\frac{10}{9}$ is 10 times as much; that is $\frac{10}{9}$.

I know that $\frac{10}{9} = 1\frac{1}{9}$ and $\frac{1}{9}$ is not that much more than 1.
Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ How much is \( \frac{3}{4} \) of \( \frac{3}{5} \)? *(\( \frac{1}{2} \)) Why does that make sense? *(e.g., \( \frac{1}{3} \) of \( \frac{3}{5} \) means 1 of the 3 sections in \( \frac{3}{5} \) and \( \frac{3}{5} \) means 2 of the sections. The sections are fifths.)

◊ Why might you write \( \frac{3}{4} \times \frac{3}{5} \) for \( \frac{3}{4} \) of \( \frac{3}{5} \)? *(e.g., 4 × \( \frac{3}{5} \) means 4 of them, so \( \frac{3}{4} \times \frac{3}{5} \) should be \( \frac{3}{5} \) of them.)

◊ About how much do you think \( \frac{3}{4} \times \frac{3}{5} \) might be? *(e.g., I think just a little less than \( \frac{3}{5} \) since \( \frac{3}{4} \times 1 \) is \( \frac{3}{4} \) and \( \frac{3}{4} \) is less than 1.)

◊ How do you know that \( \frac{3}{4} \times \frac{3}{5} \) is more than \( \frac{3}{5} \), but not a lot more? *(e.g., I know that \( \frac{3}{4} \times 1 \) is more than 1, then \( \frac{3}{4} \times \frac{3}{5} \) is more \( \frac{3}{5} \)s. But \( \frac{3}{4} \times \frac{3}{5} \) is not a lot more than \( \frac{3}{5} \).)

Using the Think Sheet

Read through the introductory box with students and make sure students understand the material.

Assign the tasks.

By viewing or listening to student responses, note if they:
• relate appropriate fraction models to fraction multiplication;
• recognize how to multiply both improper and proper fractions;
• can estimate fraction products;
• relate situations to fraction multiplications;
• recognize why they can multiply numerators and denominators.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What model could you draw to show what \( \frac{3}{4} \times \frac{3}{5} \) means? *(e.g., I could use a \( \frac{3}{4} \) fraction strip and just use 3 out of the 4 shaded sections; I could also draw a rectangle that has length \( \frac{3}{4} \) and width \( \frac{3}{5} \). I would do that inside a 1 × 1 rectangle.)

◊ How do you know the result would be less than \( \frac{3}{5} \)? *(e.g., I am only taking a part of the \( \frac{3}{5} \), so it has to be less.)

◊ How would you multiply \( 2 \frac{1}{4} \times 2 \frac{1}{4} \)? *(e.g., I would change to \( \frac{9}{4} \times \frac{9}{4} \) and just multiply the numerators and denominators.) Why can you not just multiply the whole number parts and the fraction parts separately? *(e.g., If you draw the rectangle that is \( 2 \frac{1}{4} \) by \( 2 \frac{1}{4} \), you would only be using two of the four sections in the rectangle.)

Solutions

1. e.g., I would multiply \( 5 \times 4 \) and remember it is ninths, so it is \( \frac{20}{9} \). If you added \( \frac{4}{9} \) five times, that is also what you would get.
2. a) e.g., The rectangle has a length of \( \frac{5}{6} \) and a width of \( \frac{3}{4} \), so the area is the product.
   b) \( \frac{15}{24} \)
3. a) \( \frac{5}{4} \times \frac{2}{3} \)
   b) \( \frac{2}{3} \times \frac{2}{5} \)
   c) \( \frac{4}{3} \times \frac{3}{5} \)
   d) \( 1 \frac{1}{2} \times 2 \frac{1}{2} \)
4. a) closer to \( \frac{1}{2} \)
   b) closer to 1
   c) closer to \( \frac{1}{2} \)
   d) closer to 1

Materials
• grid paper
5. a) \( \frac{12}{45} \)  
   b) \( \frac{6}{35} \)  
   c) \( \frac{20}{9} \)  
   d) \( \frac{42}{24} \)  
   e) \( \frac{24}{10} \)  
   f) \( 9\frac{1}{15} \)

Model for part b:

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Model for part e:

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\]

6. a) e.g., \( \frac{4}{3} \times \frac{5}{2} \) since, if you multiply numerators and denominators, you get \( \frac{20}{6} \).
   b) e.g., \( \frac{2}{3} \times \frac{10}{2} \)

7. e.g., \( \frac{5}{3} \times \frac{5}{4} \)

8. a) e.g., about 3 cups
   b) \( \frac{28}{9} \) cups or \( 3\frac{1}{3} \) cups

9. e.g., Jane read \( \frac{3}{4} \) of her book. She read \( \frac{3}{4} \) of that amount on Monday. What fraction of the whole book did she read on Monday?

10. a) e.g., \( 5\frac{3}{4} \times 6\frac{1}{2} = 37\frac{3}{8} \)
    b) e.g., \( 1\frac{5}{8} \times 2\frac{1}{4} = 5\frac{1}{24} \)

11. Yes, e.g., if you multiply \( \frac{2}{3} \times \frac{3}{2} \), you get 1.

12. e.g., I would say that if you are multiplying by \( \frac{2}{3} \), you are taking \( \frac{2}{3} \) of something and that is less than the whole thing; it is only part of it.
Open Question

Multiplying Fractions

Learning Goal
• representing multiplication of fractions as repeated addition or determining area.

Open Question
• Choose two fractions or mixed numbers to multiply to meet these conditions.
  – The product is a little more than 1.
  – If a mixed number is used, the whole number part is not 1.

Tell how you predicted the product would be a little more than 1.

Calculate the product and explain your process.

Verify that the product is close to 1.

• Repeat the steps at least three more times with other fractions.

Think Sheet

Multiplying Fractions (Continued)

Think Sheet
• We have learned that $4 \times 3$ means 4 groups of 3. So it makes sense that $4 \times \frac{2}{3}$ means 4 groups of two-thirds:

$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} \text{ or } 2 \frac{2}{3}$$

We multiply $4 \times 2$ to get the numerator 8. The denominator has to be 3 since we are combining thirds.

• To multiply two proper fractions, we can think of the area of a rectangle with the fractions as the lengths and widths just as with whole numbers.

For example, $4 \times 3$ is the number of square units in a rectangle with length 4 and width 3. That is because there are 4 equal groups of 3.

To multiply $\frac{2}{3} \times \frac{4}{5}$ we think of the area of a rectangle that is $\frac{2}{3}$ wide and $\frac{4}{5}$ long.

$2 \times 4$ sections out of the total number of $3 \times 5$ sections are inside the rectangle. That means the area is $\frac{8}{15}$ and so $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

The product of the numerator tells the number of sections in the shaded rectangle and the product of the denominator tells how many sections make a whole.

• We can multiply improper fractions the same way as proper fractions.

For example, $\frac{3}{4} \times \frac{2}{3}$ is the area of a rectangle that is $\frac{3}{4}$ units wide and $\frac{1}{2}$ units long. There are 15 sections in the shaded rectangle and each is $\frac{1}{4}$ of a whole, so the area is $\frac{3}{4} \times \frac{1}{2} = \frac{6}{12} = \frac{1}{4}$.

The shaded part of the diagram looks like it might be close to 2 wholes so the answer of $\frac{6}{12}$ makes sense. Each shaded square is $\frac{1}{4}$ of a whole.

Think Sheet

Multiplying Fractions (Continued)

1. How would you calculate $\frac{3}{4} \times \frac{2}{3}$? Why does your strategy make sense?

2. a) How does the model show $\frac{2}{3} \times \frac{4}{5}$?

   b) What is the product?
**Multiplying Fractions**

3. What multiplication is being modelled?
   - a) 
   - b) 
   - c) 
   - d) 

4. Estimate to decide if the product will be closer to $\frac{1}{2}$ or 1. Circle your choice.
   - a) $\frac{3}{4} \times \frac{2}{3}$ closer to $\frac{1}{2}$ closer to 1
   - b) $\frac{5}{6} \times \frac{3}{4}$ closer to $\frac{1}{2}$ closer to 1
   - c) $\frac{11}{12} \times \frac{5}{6}$ closer to $\frac{1}{2}$ closer to 1
   - d) $\frac{7}{8} \times \frac{1}{2}$ closer to $\frac{1}{2}$ closer to 1

5. Multiply each pair of fractions or mixed numbers. Draw models for parts b) and e).
   - a) $\frac{3}{4} \times \frac{2}{3}$
   - b) $\frac{5}{6} \times \frac{3}{4}$
   - c) $\frac{11}{12} \times \frac{5}{6}$
   - d) $\frac{2}{3} \times \frac{1}{2}$

6. The product of two fractions is $\frac{20}{6}$.
   - a) What fractions might they have been? Explain.
   - b) List another possible pair of fractions.

7. Choose values for the blanks to make this equation true.
   - $\frac{\Box}{\Box} \times \frac{\Box}{\Box} = \frac{\Box}{\Box}$

8. A recipe to serve 9 people requires $\frac{4}{3}$ cups of flour.
   - a) About how much flour is needed to make the recipe for 6 people?
   - b) How much flour is actually needed?

9. Write a story problem that would require you to multiply $\frac{2}{3} \times \frac{3}{4}$ to solve it. Then solve the problem.

10. a) Use the digits 1, 2, 3, 4, 5, and 6 in the boxes to create a fairly high product.
    - b) Use the digits again to create a fairly low product.

11. Is it possible to multiply two fractions and get a whole number product? Explain.

12. You have to explain why multiplying a number by $\frac{2}{3}$ results in less than you started with. What explanation would you use?
Dividing Fractions

Learning Goal
• representing division of fractions as counting groups, sharing or determining a unit rate.

Open Question

<table>
<thead>
<tr>
<th>Questions to Ask Before Using the Open Question</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Note:</strong> Provide fraction strips as you work through these questions.</td>
</tr>
<tr>
<td>◊ <em>How many times does</em> ( \frac{1}{3} ) <em>fit into</em> ( \frac{1}{2} )? <em>(1 ( \frac{1}{2} ) times)</em> Why do you think you might write that as ( \frac{1}{2} + \frac{1}{3} = \frac{5}{6} )? <em>(e.g., because you divide when you are trying to figure out how many of one thing fits into another,)</em></td>
</tr>
<tr>
<td>◊ <em>How would you figure out</em> ( \frac{2}{3} + \frac{1}{2} )? <em>(e.g., I know that there are 2 groups of ( \frac{2}{3} ) in ( \frac{2}{3} ), so the answer is 2.)</em></td>
</tr>
<tr>
<td>◊ <em>Why would</em> ( \frac{4}{5} + \frac{2}{3} ) <em>have the same answer?</em> <em>(e.g., It is still how many groups of 2 of something are in 4 of something.)</em></td>
</tr>
<tr>
<td>◊ <em>What about</em> ( \frac{5}{6} + \frac{1}{2} )? <em>(e.g., It would be 2 ( \frac{1}{2} ) since you can get to ( \frac{5}{6} ) with two groups of ( \frac{2}{3} ) but you need another half of ( \frac{5}{6} ).)</em> When you write ( \frac{3}{2} ) as an improper fraction, what do you notice? <em>(e.g., It is ( \frac{5}{2} ).)</em> Where do you see the ( \frac{5}{2} ) in the original problem? <em>(the two numerators)</em></td>
</tr>
</tbody>
</table>

Using the Open Question

Make sure students understand that they must create at least four fraction quotients, each time predicting why the quotient would be about \( \frac{3}{2} \), explaining how they calculated the quotient and verifying that the quotient is the right size. Some students might choose to use models.

By viewing or listening to student responses, note if they:
• can estimate fraction quotients;
• realize that the divisor should fit into the dividend between 1 and 2 times;
• relate division to multiplication;
• use either a common-denominator or invert-and-multiply strategy to divide.

Consolidating and Reflecting on the Open Question

◊ *Suppose your first fraction had been* \( \frac{5}{4} \). *How would you have picked the second one?* *(e.g., I would pick something between \( \frac{5}{8} \), which fits into \( \frac{3}{4} \) twice and \( \frac{5}{6} \).)* What fraction might you have chosen? *(e.g., \( \frac{1}{2} \).)* |
| ◊ *Could your first fraction be any size?* *(Yes)* Why? *(e.g., You just get another one that is smaller and fits into it about one and a half times.)* |
| ◊ *Once you chose your fractions, what strategies did you use to divide?* *(e.g., Sometimes I used the fraction tower and looked for a fraction that fit into another one about one and a half times. If it was exactly one and a half, then I just used a close fraction for one of them. Sometimes I wrote the division as a missing multiplication. I had to use an equivalent fraction for \( \frac{3}{4} \), though, to figure out what to multiply by. Other times I wrote the fractions as decimals and divided that way.)* |
Solutions

e.g.,

**Pair 1:** \( \frac{3}{2} + \frac{9}{10} \)
I knew that \( \frac{3}{2} \div 1 = \frac{3}{2} \), so I just chose a fraction close to 1, but not 1.

\[ \frac{3}{2} + \frac{9}{10} = \frac{15}{10} \]

I wrote 3.2 as a decimal (1.5) and divided by 0.9. I realized 1.5 ÷ 0.9 = \( \frac{15}{9} \) and that is \( \frac{15}{9} \) which is \( 1\frac{6}{9} \).

1\( \frac{6}{9} \) is pretty close to \( \frac{3}{2} \), which is \( 1\frac{1}{2} \).

**Pair 2:** \( \frac{3}{5} ÷ \frac{3}{8} \)
I knew that \( \frac{3}{5} ÷ \frac{2}{5} \) is \( \frac{3}{2} \) since there are \( 1\frac{1}{2} \) groups of \( \frac{2}{5} \) in \( \frac{3}{5} \). But the answer was not supposed to be exactly \( \frac{3}{2} \), so instead of \( \frac{2}{5} \), I used a close fraction. I used a fraction tower to help me see what was close and I picked \( \frac{3}{8} \).

\[ \frac{3}{5} ÷ \frac{3}{8} = \frac{8}{5} = 1\frac{3}{5} \]

To figure the answer out, I thought about multiplication. \( \frac{3}{8} \times \frac{2}{5} = \frac{3}{10} \). I changed \( \frac{3}{10} \) to \( \frac{24}{40} \), so that I could multiply 8 by something to get the new denominator. The answer was \( \frac{6}{5} \).

\( \frac{3}{5} \) is close to \( \frac{1}{2} \), so \( 1\frac{3}{5} \) is close to \( 1\frac{1}{2} \), which is \( \frac{3}{2} \).

**Pair 3:** \( \frac{4}{5} ÷ \frac{1}{2} \)
I started with \( \frac{3}{4} ÷ \frac{1}{2} \), since I know that \( \frac{1}{2} \) fits into \( \frac{3}{4} \) one and a half times. But I did not want an exact answer, so I changed \( \frac{3}{4} \) to a close-by fraction. I chose \( \frac{4}{5} \).

\[ \frac{4}{5} ÷ \frac{1}{2} = 0.8 ÷ 0.5 = \frac{8}{5} = 1\frac{3}{5} \]

I wrote each fraction as a decimal and divided. I realized that 8 tenths ÷ 5 tenths = 8 ÷ 5.

\( \frac{4}{5} \) is fairly close to \( \frac{1}{2} \), so \( 1\frac{3}{5} \) is fairly close to \( 1\frac{1}{2} \), which is \( \frac{3}{2} \).

**Pair 4:** \( \frac{6}{9} ÷ \frac{2}{5} \)
One way you can divide two fractions is to invert and multiply. So I needed two fractions that multiplied to something close to \( \frac{5}{9} \) and then I would just flip the second one. Since \( \frac{5}{9} = \frac{30}{18} \), I picked \( \frac{30}{18} \) to be close.

Since \( \frac{6}{9} \times \frac{5}{2} \) multiply to be \( \frac{30}{18} \), I used \( \frac{6}{9} \) and \( \frac{2}{5} \).

\[ \frac{6}{9} ÷ \frac{2}{5} = \frac{30}{18} = 1\frac{2}{3}, \text{which is close to } 1\frac{1}{2} \]

I checked by looking at fraction strips. I see that \( \frac{2}{5} \) fits into \( \frac{2}{3} \) (the same as \( \frac{6}{9} \)) more than 1 time, but less than two times, so that makes sense.

I also multiplied: \( \frac{2}{5} \times \frac{30}{18} = \frac{60}{90} = \frac{6}{9} \)
Think Sheet

Questions to Ask Before Assigning the Think Sheet

Note: Provide fraction strips as you work through these questions.
◊ How many times does \( \frac{1}{3} \) fit into \( \frac{1}{2} \)? (1 \( \frac{1}{3} \) times) Why do you think you might write that as \( \frac{1}{2} ÷ \frac{1}{3} = \frac{1}{2} \)? (e.g., You divide when you are trying to figure out how many of one thing fits into another.)
◊ How would you figure out \( \frac{4}{3} ÷ \frac{2}{3} \)? (e.g., I know that there are 2 groups of \( \frac{2}{3} \) in \( \frac{4}{3} \), so the answer is 2.)
◊ Why would \( \frac{4}{3} ÷ \frac{2}{3} \) have the same answer? (e.g., It is still how many groups of 2 of something are in 4 of something.)
◊ What about \( \frac{4}{5} ÷ \frac{2}{5} \)? (e.g., It would be 2 \( \frac{1}{2} \) since you can get to \( \frac{4}{5} \) with two groups of \( \frac{2}{5} \) but you need another half of \( \frac{2}{5} \).) When you write 2 \( \frac{1}{2} \) as an improper fraction, what do you notice? (e.g., It is \( \frac{5}{2} \).) Where do you see the \( \frac{2}{5} \) in the original problem? (the two numerators)

Using the Think Sheet

Read through the introductory box with students and make sure students understand the material.

Assign the tasks.

By viewing or listening to student responses, note if they:
• can estimate fraction quotients;
• relate division to multiplication;
• use either a common-denominator or invert-and-multiply strategy to divide.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ What model could you draw to show what \( \frac{2}{3} ÷ \frac{3}{4} \) means? (e.g., I could use fraction strips to show how many \( \frac{2}{3} \) fits into \( \frac{3}{4} \).)
◊ How do you know the result would be less than 1? (e.g., \( \frac{2}{3} \) is not as much as \( \frac{3}{4} \).)
◊ How might you calculate \( \frac{2}{3} ÷ \frac{3}{4} \) if you did not have fraction strips? (e.g., I would use equivalent fractions with the same denominator. It would be \( \frac{6}{9} ÷ \frac{9}{12} \). I know that I could only fit \( \frac{6}{9} \) of \( \frac{9}{12} \) into \( \frac{6}{9} \).)
◊ Why and what would you divide to solve this problem: Kelly went \( \frac{2}{3} \) of the way to her grandmother’s home in \( \frac{2}{5} \) of an hour. How far could she get in a whole hour? (e.g., It is a rate problem. You would divide \( \frac{2}{3} \) by 2 to figure out how far in \( \frac{1}{5} \) of an hour and multiply by 5 to figure out a whole hour. Since you are doing \( \frac{3}{5} \times \frac{5}{2} \), you are actually dividing \( \frac{2}{3} \) by \( \frac{2}{5} \).)

Solutions

1. a) e.g., I renamed \( \frac{6}{3} \) as \( \frac{12}{6} \) so I could share into 4 equal groups of \( \frac{3}{12} \) each.
   b) e.g., I renamed \( \frac{3}{9} \) as \( \frac{27}{27} \) so I could share into 3 equal groups of \( \frac{9}{27} \)
2. a) e.g., It shows how many times \( \frac{1}{3} \) fits into \( \frac{5}{6} \).
   b) e.g., almost 2
3. a) \( 2 ÷ \frac{3}{4} \)
   b) \( \frac{7}{4} ÷ \frac{3}{4} \)
4. a) closer to \( \frac{1}{2} \)
   b) closer to 2
   c) closer to 1
   d) closer to \( \frac{3}{2} \)

Materials
• grid paper
• fraction strips
5. a) $\frac{3}{20}$  
   b) $2\frac{1}{2}$  
   c) 12  
   d) $\frac{70}{12}$  
   e) $\frac{70}{6}$  
   f) $\frac{40}{51}$  

   NOTE: Answers may be written as equivalent fractions.

Model for part a:

Model for part c:

6. a) about $\frac{3}{4}$ cup  
   b) $\frac{5}{6}$ (or $\frac{10}{12}$) cups  

7. a) $\frac{8}{15}$ of the floor  
   b) division  

8. e.g., A small paint can holds $\frac{1}{8}$ as much as a large one. If I pour $\frac{3}{5}$ of a large can of paint into small cans, how many full and part small cans can I fill?

9. a) e.g., $\frac{4}{3} + \frac{16}{15} = \frac{3}{4}$  
   b) $\frac{2}{3} + \frac{8}{9} = \frac{3}{4}$  

10. a) e.g., $3\frac{5}{6} + \frac{9}{2} = \frac{45}{24}$  
    b) e.g., $6\frac{5}{9} + \frac{2}{9} = 29\frac{5}{9}$  

11. a) e.g., Since $\frac{3}{8}$ is more than $\frac{1}{3}$, $\frac{1}{3}$ fits into $\frac{3}{8}$ more than once, but $\frac{3}{8}$ does not fit into $\frac{1}{3}$ even once.  
    b) e.g. If you show $\frac{3}{8}$ as $\frac{9}{24}$ and $\frac{1}{3}$ as $\frac{8}{24}$, you know that $\frac{9}{24} + \frac{8}{24} = \frac{9}{5}$, but $\frac{8}{24} + \frac{9}{24} = \frac{9}{5}$ since you are either fitting 8 things into 9 or else only using 8 parts of the 9.

12. e.g., I would use an example such as $2 + \frac{5}{6}$. That asks how many times $\frac{5}{6}$ fits into 2. I know that $\frac{1}{6}$ fits into 1 six times, so $\frac{1}{5}$ fits into 2, $2 \times 6$ times. But $\frac{5}{6}$ is five times as big, so only $\frac{1}{5}$ as many will fit in. That means you have to divide $2 \times 6$ by 5. That is the same as $2 \times \frac{6}{5}$.  

Open Question

Dividing Fractions

Learning Goal
- representing division of fractions as counting groups, sharing or determining a unit rate.

Open Question
- Choose two fractions to divide to meet these conditions:
  - The quotient (the answer you get when you divide) is about, but not exactly, \( \frac{2}{3} \).
  - The denominators are different.

Tell how you predicted the quotient would be about \( \frac{2}{3} \).

Verify that the quotient is about \( \frac{2}{3} \), but not exactly, \( \frac{2}{3} \). Check your estimate.

Explore other combinations.

Think Sheet

Just as with whole numbers, dividing fractions can describe the result of sharing, can tell how many of one size group fits in another, or can describe rates.

Sharing
- When we divide by a whole number, we can think about sharing. For example, \( \frac{5}{6} \div 2 \) means that 2 people share \( \frac{5}{6} \) of the whole.

We divide the 4 sections into 2 and remember that we are thinking about fifths. Each person gets \( \frac{5}{12} \).
- Sometimes the result is not a whole number of sections. For example, \( \frac{5}{6} \div 2 \) means that 2 people share \( \frac{5}{6} \) of the whole.

Each gets \( \frac{5}{12} \).

We can either multiply numerator and denominator by 2 to get the equivalent fraction \( \frac{5}{12} \) or we can use an equivalent fraction model for \( \frac{5}{6} \).

The thirds were split in half since there needed to be an even number of sections for 2 people to share them. Each person gets \( \frac{3}{12} \). If it had been \( \frac{1}{2} \), the thirds could be split into fourths so that it would be possible to divide the total amount into four equal sections.

Counting Groups
- Sometimes it makes sense to count how many groups. For example, there are 5 groups of \( \frac{5}{6} = \frac{5}{6} \times 1 \) each.

\[ \frac{5}{6} \div 2 = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12} \]

\[ 2 \text{ people} \times \frac{5}{12} \text{ section} = \frac{5}{6} \text{ of the whole} \]

\[ \text{Each person gets } \frac{5}{12} \text{ of } \frac{5}{6} \text{ of the whole} \]

Using a Unit Rate
- Another way to think about division is thinking about unit rates. For example, if we can travel 80 km in 2 hours, we think of 80 km/2 h as the distance we can travel in one hour.

If a girl can complete \( \frac{1}{2} \) of a project in 2 days, we divide \( \frac{1}{2} \) to figure out how much she can complete in one day. Similarly, if she can complete \( \frac{1}{3} \) of a project in \( \frac{1}{3} \) day, we divide \( \frac{1}{3} \) to figure out how much she can complete in 1 day. Since you know that we could also multiply \( \frac{1}{2} \) by 2, it makes sense that \( \frac{1}{2} \) of \( \frac{1}{2} = \frac{1}{2} \).
Suppose we know that \( \frac{3}{4} \) of a can of paint can cover \( \frac{1}{2} \) of a space. To figure out how much of the space one full can of paint can cover, we calculate \( \frac{3}{4} \times \frac{1}{2} \).

If \( \frac{3}{4} \) of a can covers \( \frac{1}{2} \) of a space, then \( \frac{1}{2} \) of a can covers \( \frac{3}{4} \) of the space \( \frac{1}{2} \times \frac{3}{4} \) and then \( \frac{3}{4} \), which is \( \frac{3}{4} \), can cover \( \frac{3}{8} \) of the space \( \frac{3}{4} \times \frac{3}{4} \), that means \( \frac{3}{8} \times \frac{3}{8} \).

Since we know that \( \frac{3}{8} \times \frac{3}{8} = \frac{9}{64} \), this makes sense.

What we did was divide the \( \frac{1}{2} \) by 2 and multiply by 3.

So \( \frac{3}{8} \times \frac{3}{8} = \frac{9}{64} \).

We reversed the divisor and multiplied by the reciprocal, the fraction we get by switching the numerator and denominator.

Another way to think about this is:

1. \( \frac{1}{2} \times \frac{3}{3} = \frac{3}{3} \) since there are three \( \frac{1}{3} \) in 1 whole.
2. \( \frac{2}{3} \times \frac{3}{3} = \frac{6}{3} = \frac{2}{1} \) since there are only half as many groups of \( \frac{1}{3} \) in 1 whole as there would be groups of \( \frac{1}{2} \).
3. \( \frac{4}{3} \times \frac{3}{3} = \frac{12}{3} = \frac{4}{1} \) since there are only one fourth as many groups of \( \frac{3}{8} \) as groups of \( \frac{3}{4} \) in a whole.

Improper fractions can be divided in the same way as proper fractions.

Mixed numbers are usually written as improper fractions to divide them.

1. How would you calculate each? Explain why your strategy makes sense.
   a) \( \frac{3}{4} \times \frac{1}{2} \)
   b) \( \frac{3}{4} \times \frac{1}{2} \)

2. a) How does this model show \( \frac{3}{4} \times \frac{1}{2} \)?
   b) Estimate the quotient.

3. a) How would you calculate each? Explain why your strategy makes sense.
   a) \( \frac{3}{4} \times \frac{1}{2} \)
   b) \( \frac{3}{4} \times \frac{1}{2} \)

4. a) What division is being modelled?
   b) Estimate the quotient as closer to \( \frac{1}{2} \) or \( \frac{3}{4} \). Circle your choice.
   a) \( \frac{3}{4} \times \frac{1}{2} \)
   b) \( \frac{3}{4} \times \frac{1}{2} \)
   c) \( \frac{3}{4} \times \frac{1}{2} \)
   d) \( \frac{3}{4} \times \frac{1}{2} \)

5. a) Divide each pair of fractions or mixed numbers. Draw models for parts a) and c).
   b) Explain why it also makes sense that the quotients are reciprocals.
   c) Describe the situation.

6. a) How much flour is that per batch?
   b) How much flour is available for each batch?

7. a) How much of the floor can you tile in one full day?
   b) What computation can you do to describe this situation?

8. Write a story problem that would require you to divide \( \frac{3}{4} \) by \( \frac{1}{2} \) to solve it. Solve the problem.

9. a) Choose values for the blanks to make this true.
   b) List another possible set of values.

10. a) Use the digits 2, 3, 5, and 9 in the boxes to create a small quotient.
    b) Use the digits again to create a large quotient.

11. a) Explain why it makes sense that the first quotient is greater than 1, but the second one is less than 1.
    b) Explain why it also makes sense that the quotients are reciprocals.

12. You have to explain why dividing by \( \frac{3}{4} \) is the same as multiplying by \( \frac{4}{3} \). What explanation would you use?
Relating Situations to Fraction Operations

Learning Goal

• connecting fraction calculations with real-life situations.

Open Question

<table>
<thead>
<tr>
<th>Questions to Ask Before Using the Open Question</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>◊ What might make it a subtraction one? (e.g., take-away) Could there be other problems that are not take-away that are subtraction problems? (e.g., Yes, it could be comparing.)</td>
</tr>
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<td>◊ What might make a problem a multiplication one? (e.g., You multiply to get areas or to count equal groups.)</td>
</tr>
<tr>
<td>◊ What might make a problem a division one? (e.g., You divide when you are sharing.) When else might you divide? (e.g., When you are figuring out how many of something fits into something else.)</td>
</tr>
</tbody>
</table>

Using the Open Question

Make sure students understand that they must create at least two story problems for each operation. By suggesting that the problems are different, help students understand that not only might the scenario change, but even the meaning of the operation.

By viewing or listening to student responses, note if they:
• relate real-life situations to fraction operations;
• recognize the different meanings operations can hold when applied to fractions.

Consolidating and Reflecting on the Open Question

◊ How were your subtraction questions different? (e.g., One involved comparing and one involved take-away.)
◊ How were your division questions different? (e.g., One involved a unit rate and one involved how many of one thing fits into another.)
◊ Why did you not use a sharing problem for division? (e.g., I was dividing by $\frac{1}{3}$ rather than by a whole number.)

Solutions

e.g.,

$\frac{2}{3} + \frac{1}{3}$

I read $\frac{2}{3}$ of my book on Monday and $\frac{1}{3}$ of it on Tuesday. How much of the book have I read now? (This involves combining, as addition should.)

I bought $\frac{3}{5}$ of my groceries at one store and $\frac{2}{3}$ of them at the other store. How much of my groceries does that account for? (This involves combining, as addition should.)
\[
\frac{3}{5} - \frac{1}{3}
\]
I read \(\frac{3}{5}\) of my book on Monday and \(\frac{1}{3}\) of it on Tuesday. How much more of the book did I read on Monday than Tuesday? (This involves comparing and that is one of the things subtraction means.)

I finished \(\frac{3}{5}\) of my project, but decided some of it wasn’t good enough. I decided I should revise \(\frac{1}{3}\) of the project. How much of my project is actually ready? (This involves take away and that is one of the things subtraction means.)

\[
\frac{3}{5} \times \frac{1}{3}
\]
We have to turn in \(\frac{3}{5}\) of the money for a field trip by next week. My grandparents and parents and I are each paying \(\frac{1}{3}\). How much of the full amount do I have to pay by next week? (This is multiplication since it is finding \(\frac{1}{3}\) of a group of something.)

\[
\frac{3}{5} \times \frac{1}{3}
\]
Out classroom is \(\frac{3}{5}\) as wide and \(\frac{1}{3}\) as long as the gym. How much of the area of the gym is the area of our classroom? (This is an area problem so it is about multiplying.)

\[
\frac{3}{5} + \frac{1}{3}
\]
You need \(\frac{3}{5}\) of a cup of flour, but only have a \(\frac{1}{3}\) cup measure. How many times will you have to fill it?

\[
\frac{3}{5} + \frac{1}{3}
\]
Small cans of paint are \(\frac{1}{3}\) of the size of large ones. You have \(\frac{3}{5}\) of a large can of paint. How many small ones could you fill? (This is division since it is about how many of something fits in something else.)
**Think Sheet**

### Questions to Ask Before Assigning the Think Sheet

◊ **What might make a problem an addition one?** (e.g., You have to combine things.)
◊ **What might make it a subtraction one?** (e.g., take-away) **Could there be other problems that are not take-away that are subtraction problems?** (e.g., Yes, it could be comparing.)
◊ **What might make a problem a multiplication one?** (e.g., You multiply to get areas or to count a lot of equal groups.)
◊ **What might make a problem a division one?** (e.g., You divide when you are sharing.) **When else might you divide?** (e.g., When you are figuring how many of something fits into something else.)

### Using the Think Sheet

Read through the introductory box with students and make sure students understand the material.

Assign the tasks.

By viewing or listening to student responses, note if they:

• relate real-life situations to fraction operations;
• recognize the different meanings operations can hold when applied to fractions.

### Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ **What made Question 1b a multiplication question?** (e.g., It is multiplication because you are taking a part of a part.)
◊ **Why might you solve Question 1e with either addition or subtraction?** (e.g., You could figure out what to add to get to 7/8 or you can take 1/4 from 7/8.)
◊ **What operation would you use to figure out what fraction of Fred’s money his sister has?** (e.g., Division, since it is like imagining Fred’s amount is 1 whole and then you divide that by 2 1/2.)
◊ **Do you think of division problems where you divide by whole numbers the same way as ones where you divide by fractions?** (e.g., No, since I can use sharing when I divide by whole numbers, but not when I divide by fractions.)

### Solutions

1. a) $1\frac{3}{2} \div \frac{2}{5}$ division
   b) $\frac{1}{4} \times \frac{3}{4}$ multiplication
   c) $\frac{1}{6} + \frac{3}{8}$ addition
   d) $5 \times 1\frac{1}{3}$ multiplication
   e) $\frac{7}{8} - \frac{1}{4}$ subtraction
   f) $\frac{5}{8} + 1\frac{1}{2}$ division

2. a) e.g., You are taking $\frac{3}{4}$ of $2\frac{1}{2}$ and when you take a part of a part, you are multiplying.
   b) e.g., You are finding out how many $\frac{3}{4}$’s are in $5\frac{1}{2}$ and finding how many of one thing is in another is dividing.
   c) e.g., It is a take-away problem.

3. a) $\frac{3}{4} \times 2\frac{1}{2} =$ ?
   b) e.g. $5\frac{1}{2} + \frac{1}{3} =$ ?
   c) e.g., $3\frac{1}{3} - ? = 1\frac{3}{4}$

4. e.g., To divide $3\frac{1}{3}$ by $\frac{2}{3}$ to find out how many $\frac{2}{3}$ are in $3\frac{1}{3}$, you could figure out what to multiply $\frac{2}{3}$ by to get $3\frac{1}{3}$. 

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Fractions (IS)
5. a) e.g., Anne checks the turkey 3 times an hour. How many times does she check it in $5\frac{1}{2}$ hours?

b) e.g., A turkey needs to be in the oven for $5\frac{1}{2}$ hours. It has cooked for $2\frac{3}{4}$ hours so far. How much time does it still need?

6. e.g., Alexander has $3\frac{1}{2}$ dozen eggs. If he is making omelets that use $\frac{1}{4}$ of a carton at a time, how many omelets can he make?

7. e.g., If it is take-away, it is subtraction.
Relating Situations to Fraction Operations (Continued)

Think Sheet

More Than One Option

Sometimes the same situation can be approached using more than one operation. For example, if you can solve a problem by dividing, you can also solve it by multiplying. The last problem above could be solved by thinking: 

- Every division can also be solved as a multiplication.
- Every subtraction can also be solved as an addition.
7. What hints can you suggest to decide if a problem requires subtraction to solve it?

4. Tell what operation or operations you could use to solve each problem. Write the equation to represent the problem.
   a) Cynthia has \( \frac{3}{4} \) containers of juice. Each smaller container holds \( \frac{1}{2} \) as much as a large container. How many small containers can she fill?

   b) About \( \frac{2}{3} \) of the athletes in a school play basketball. About \( \frac{1}{3} \) of those players are in Grade 9. What fraction of the students in the school are Grade 9 basketball players?

   c) Stacey read \( \frac{1}{5} \) of her book yesterday and \( \frac{1}{3} \) of it today. How much of the book has she read?

   d) It takes Lea’s mom about \( \frac{1}{2} \) hours to drive to work in the morning and \( \frac{1}{3} \) hours to drive home every afternoon. If it is Wednesday at noon and she went to work Monday, Tuesday and Wednesday (and is still there), about how many hours has she spent driving to and from work?

   e) The gas tank in Kyle’s car was \( \frac{7}{8} \) full when they started a trip. Later in the day, the tank registered \( \frac{1}{4} \) full. How much of the tank of gas had been used?

2. a) What makes this a multiplication problem?
   Fred has \( \frac{2}{3} \) times as much money as his sister. Aaron has \( \frac{3}{4} \) as much money as Fred. How many times as much money as Fred’s sister does Aaron have?

   b) What makes this a division problem?
   A turkey is in the oven for \( \frac{5}{2} \) hours. You decide to check it every \( \frac{1}{3} \) of an hour. How many times will you check the turkey?

   c) What makes this a subtraction problem?
   Fred has \( \frac{2}{3} \) times as much money as his sister. Aaron has \( \frac{3}{4} \) as much money as Fred. How many times as much money as Fred’s sister does Aaron have?

3. Write an equation you could use to solve each problem in Question 2.
   a)

   b)

   c)

4. Why might you solve this problem using either multiplication or division? You have walked \( \frac{3}{4} \) miles this week. Each workout was \( \frac{1}{2} \) of an hour. How many times did you workout?

5. How would you keep part of the information in Question 2b, but change some of it so that:
   a) it becomes a multiplication question

   b) it becomes a subtraction question

6. Finish this problem so that it is a division problem:
   Alexander has \( \frac{3}{2} \) dozen eggs.

7. What hints can you suggest to decide if a problem requires subtraction to solve it?

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**Fraction Tower**

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1
1/2 1/2
1/3 1/3 1/3
1/4 1/4 1/4 1/4
1/5 1/5 1/5 1/5
1/6 1/6 1/6 1/6
1/9 1/9 1/9 1/9
1/10 1/10 1/10 1/10
1/12 1/12 1/12 1/12
1/15 1/15 1/15 1/15
1/18 1/18 1/18 1/18
1/20 1/20 1/20 1/20
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