

Actions to Develop Fractions Understanding

The seven mathematics processes – problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating – are integral to meaningful learning of fractions.

In this section, we will focus on learning and teaching fractions through the following three processes:

- ❖ representing
- ❖ reasoning and proving
- ❖ selecting tools and computational strategies

Representing

Students of all ages need to represent fraction ideas and relationships by using concrete materials, pictures, diagrams, words and symbols. Often, symbols are privileged at the junior and intermediate/senior grades (students may use mathematical notation without knowing what it means). However, it is essential that all students develop the ability to represent mathematics in a variety of ways, as this skill allows them to make sense of the initial task, predict a reasonable answer, determine one or more possible strategies, check to see that their answer makes sense and communicate their thinking. Sometimes, models can be rough sketches that are approximate; at other times, they need to be precise. Students who have significant experience with constructing diagrams or models will be better able to discern when an approximate representation is sufficient and when an precise one is necessary. Constructing models will initially involve composing and decomposing fractions by using unit fractions. As students learn about equivalence, comparing and operations, it is essential that they can construct appropriate diagrams or models. This ability supports students in senior mathematics as they construct models or diagrams to solve for an unknown value (e.g., in a trigonometry question).

In the Classroom: Consider asking students to share a pan of brownies equally among four people. When students solve this task by using concrete materials, such as paper folded into four equal regions, they are able to connect the physical solution with a symbolic notation indicating that each person would receive one *one-fourth* of the pan. By asking students how they might share this same pan of brownies with an additional four people, a number of different paper-folding strategies will be used, which generate different-looking yet equivalent solutions. Students justify that the solutions are equivalent by comparing the sizes of the regions. In being able to do so, students understand much more than how to generate a correct answer: they are able to see the difference between fourths and eighths, able to connect the symbolic notation to the concrete representation and able to consider how to generalize this strategy to other situations. Students quickly notice, for example, that other friendly numbers for sharing are 16 and 32 but that 10 and 12 would be more difficult. By using the concrete representation and the symbolic together, students can see that *one-eighth* is half of *one-fourth* and experience fractions across constructs in a meaningful and interconnected context.



Reasoning and Proving

In the tasks described below, students are constantly engaged in reasoning about and proving both the strategies being used and the solutions being generated. Students who are given opportunities to make conjectures about fractions and explore the conjectures to refine or refute them will have a more solid understanding of fractions. Such opportunities frequently arise from an unexpected strategy or solution in a lesson.

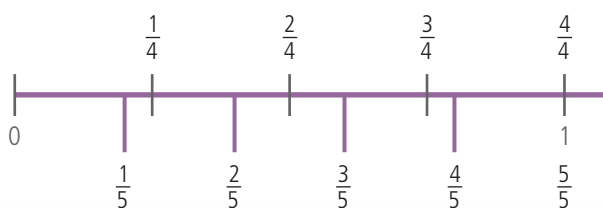
In the Classroom: In one classroom, Grade 4 students were asked to identify examples and non-examples of fractions. Although the task seemed straightforward initially, it resulted in a number of questions being generated by the students, such as "Can any number be a fraction?" and "Can we put fractions in a number sentence?" which the students explored further in a subsequent class. In a Grade 6 classroom, where students were using pattern blocks to create part-whole and part-part fractions by using set and area models, one student suggested that the fraction $\frac{1}{6}$ could be used to represent one vertex on the hexagonal pattern block but wondered if that was an example of a set model. The teacher, rather than answering the question, asked students to discuss this in small groups and then engaged in a whole class discussion to share ideas. In this way, students were required to reason about the new question based on their understanding of set and area models, allowing them to extend their knowledge to more general situations.



Selecting Tools and Computational Strategies

The learning of fractions is rife with opportunities to build students' ability to thoughtfully select tools and computational strategies. A range of tasks allow for students to make, discuss and reflect on decisions for tool and calculation strategies. In some intermediate classrooms, students are hesitant to use a manipulative to solve a mathematical problem. Sometimes, a model can provide a simpler solution. Also, engaging students in selecting manipulatives and tools to solve a task requires them to consider all aspects of the question, including what the whole is and what the fractional units are.

In the Classroom: For example, if students are selecting a tool to represent a fraction task that includes fractional units of fourths and fifths, a number line or a rectangle area model will be much friendlier than a hand-drawn circular model (it is difficult to accurately partition circles into fifths).

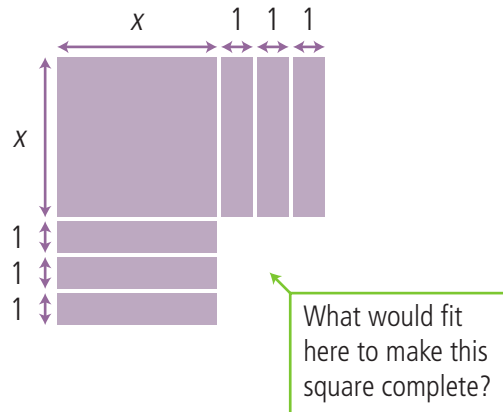


A student who has experience with selecting from various tools will be able to quickly dismiss those that are less appropriate for given fractional units. Secondary students are frequently required to complete calculations involving fractions. Sometimes an estimate is sufficient, so a student who has a strong sense of fractions could correctly estimate the sum of $\frac{1}{15} + \frac{16}{17}$ to be approximately equal to 1. This knowledge could aid in checking an algebraic solution, such as $\frac{1}{15}x + \frac{16}{17}x$ as being close to $1x$.

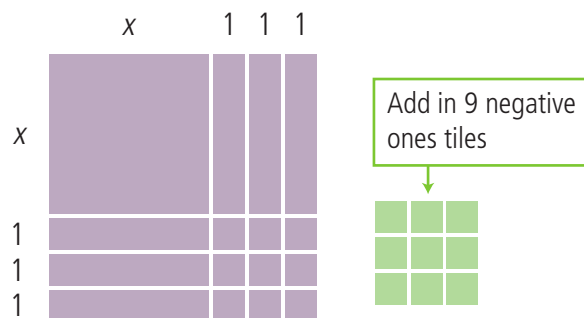
Secondary students benefit from using a range of manipulatives, including algebra tiles to represent the algebraic action of “completing the square,” which allows students to extend their understanding to expressions involving fractions. It is helpful for students to recognize that algebraically completing the square correlates to completing the physical model in which the tiles are arranged to create a square.

For example, to complete the square for $x^2 + 6x$, students might

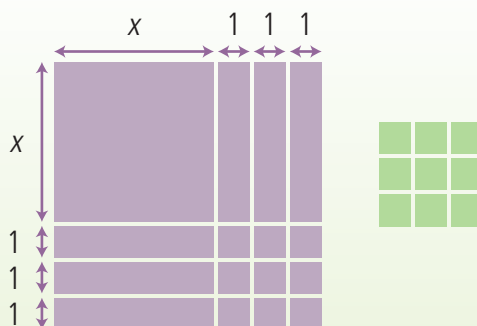
- ❖ construct $x^2 + 6x$ by using algebraic tiles



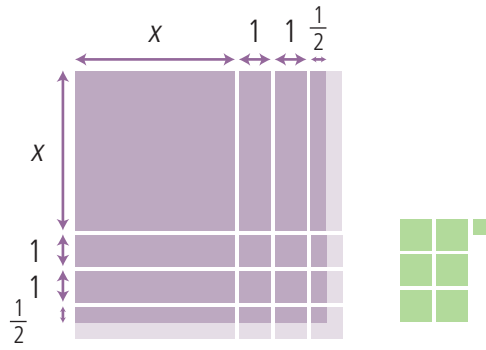
- ❖ identify that the square is missing 9 ones tiles and add them in



- ❖ recognize that they need to keep the new image equivalent to the original expression, so add in 9 negative ones tiles
- ❖ write the expression by using the dimensions of the square $(x + 3)^2 - 9$



Note that for fractional situations, such as completing the square for $x^2 + 5x$, the diagram would be as follows (since the $5x$ has to be split to create a square):



And the expression would be $(x + 2\frac{1}{2})^2 - 6\frac{1}{4}$.