Math Learning Goals

- Construct and carry out a problem-solving plan.
- Recognize that cylinders with a fixed surface area could have different volumes.
- Solve problems involving maximizing volume of cylinders with fixed surface area.

Materials

- various-sized cylindrical cans
- graphing calculators

Handouts

- Posing Powerful Questions
- Building the Ideal Salmon Can

Minds On…

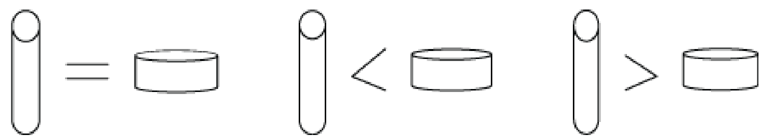
Individual → Construction

Each student constructs two different-shaped cylinders using masking tape and two pieces of 8 1/2 × 11 paper.

DI Different learning modalities are addressed through varied activities in the lesson.

Groups → Three Corners

At each corner post a visual on chart paper for students to capture their discussion:



Each student goes to the corner that represents their conjecture about the volumes of the two cylinders they constructed. Students discuss their reasoning and outline the steps they could take to verify or refute their conjecture.

Whole Class → Discussion

A volunteer from each corner outlines their corner's plan to the whole class.

Action!

Groups → Think Aloud

Each group selects one can on which they will base their investigation. Groups explore the Building the Ideal Salmon Can problem. Group members outline their thinking on the problem, using the Think Aloud Prompts.

A for L

Curriculum Expectations/Observation/Anecdotal Note: Assess student understanding through listening and observation.

Groups → Investigation

Each group formulates and carries out a plan to find the dimensions of the can with the given surface area that will maximize volume.

Provide students with formulas $V = \pi r^2 h$, and $SA = 2\pi^2 + 2\pi rh$, or $SA = 2\pi(r^2 + rh)$.

Allow sufficient time for all groups to complete their investigation. This may require extending the time into the next class.

A for L

Learning Skills/Observation/Rubric: Explicitly assess student learning skills using a rubric.

Consolidate Debrief

Individual → Ticket to Leave

Think about your group's problem-solving plan and write a brief outline of it.

Home Activity or Further Classroom Consolidation

Tomorrow we will summarize everyone's results in a table. (Identify any issues that arose when you were solving the problem.) We will take up questions 5 and 6 from the worksheet as a class.

Posing Powerful Questions

Lesson Title Optimizing Volume for a Given Surface Area **Grade/Program** **9 Academic**

Goals(s) for a Specific Lesson
Students will construct and carry out a problem solving plan
Students will recognize that cylinders with the same surface areas may have different volumes
Students will solve problems involving optimizing volume for a given surface area

Curriculum Expectations
MP1: Problem Solving: develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding.
MP2: Reasoning and Proving: develop and apply reasoning skills to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments.
MP3: Reflecting: demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.
MP7: Communicating: communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.
OM1.3: identify, through investigation with a variety of tools the effect of varying the dimensions on the surface area [or volume], of square-based prisms and cylinders, given a fixed volume or [surface area].
OM1.4: explain the significance of optimal area, surface area or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss).

Big Idea(s) Addressed by the Expectations
There are a variety of ways to solve the same problem. Some methods are more efficient or reveal additional information about the problem.
There is a relationship between volume and surface area of an object. There will exist a set of dimensions that will maximize volume or surface area when the other quantity is fixed.

Minds On… Sample Question(s)
Most pop cans have exactly the same shape. Why do you think they have this shape? (show a pop can**)** (*open*)
Do all cylinders that have the same surface area have the exact same dimensions (shape)? (other)
Do all cylinders with the same surface area have exactly the same volume? (other)
Some items can be bought in bulk. For example, bulk food stores sell large cans of ketchup and large cans of tuna
Describe the shape of these cans. Why do you think these bulk items have this type of shape? (use sample cans that students have supplied) (*open*)
Can you think of other items that come in cylinders that are shaped in a similar way? (*open*) (oil drums)

Action! Sample Question(s)
What is needed to determine the Volume of a cylinder? (*other*)
What is needed to determine the Surface Area of a cylinder? (*other*)
If you know the dimensions of a cylinder, how can you calculate the volume? Surface area? (*other*)
Given the surface area, and a specific height, how can you then calculate the radius of the cylinder? (an example may be required here) (*other*)
Parallel task: Given that Surface Area = 3140cm^2, determine the height if the radius is 5cm OR if the height is 3cm. Then determine the volume of the cylinder. Compare the volumes
How could you determine which height will yield the largest volume for a fixed surface area? (*open*)
How could you organize your data? (*open*)

Scaffolding Questions *(posed to individuals as needed)*
What is the equation that relates surface area, height and radius?
How can this formula help you to determine the height when you know the surface area and radius?
What is the equation that relates volume, height and radius?
How can this formula help you to determine the volume, given the height and the radius?

Consolidate/Debrief Sample Question(s)
Write a journal entry comparing your group's problem-solving method to those outlined by the other groups. Identify strengths and weaknesses of each method.
With a partner, brainstorm other possible applications of the results of your investigation for maximizing volume for a fixed surface area. Be prepared to share your ideas with the class.
Thinking about your group's problem solving plans, how many times will you perform the same calculations?
Describe the shape of the can that had the largest volume.
How does the shape of the can with the largest volume compare to the shape of the bulk ketchup can? Pop can? Oil drum?
If you used a can that had a different surface area, would the can with the largest volume look like the one you found? Why or why not?
Why don't all products come in cans with this shape?
Why is it important to maximize the volume for a fixed surface area? What applications does this have in the world?

Building the Ideal Salmon Can

Dr. W.W. Sawyer, a mathematician at the University of Toronto, once made the following comment:

> "If you have two minutes to solve a problem, the first ninety seconds should be spent in thought."

Dr. Sawyer's problem-solving steps:

1. Understand the problem.
2. Make a plan.
3. Do the plan.
4. Reflect/Look back.

Building the Ideal Salmon Can

Businesses often want to keep packaging costs as low as possible. For example, for canned products, the cost of the can is a major factor. Typically, the business chooses a shape for a can to maximize the volume of product they can put in it, while keeping the total surface area constant.

Your task is to find the dimensions of the can with the greatest volume that can be constructed without changing the total surface area of the material being used to construct the can.

Think Aloud Prompts

- What are we trying to find?
- What stays the same? What changes?
- If we change one dimension, will that affect other dimensions? If so, how?
- Are there upper or lower limits on how far we can change a dimension?
- What formulas might be useful?
- If there are multiple ways of writing a formula, is one version of the formula better than another for this task?
- Does changing any one variable have a larger impact than changing another variable? If so, identify which variable should have the greatest impact.
- Should we approximate with decimals or would it be easier to leave constants exact (e.g., π)?
- How exact do we need the results (e.g., what number of decimal places in the answer)?
- What tools might we need to solve this problem?

1. In your groups, compute the volume and surface area of one of the cans you brought to class.
2. Use the Think Aloud prompts to ensure that everyone in your group understands the problem.
3. Make a plan to solve the problem, using the information about your sample can.
4. Solve the problem.
   - Discuss your proposed solution. Consider:
   - Would your plan work for any other cans, or only for your sample?
   - Are there any other factors, besides total volume, that businesses should consider when deciding on the best shape for a can? If so, list these considerations.
5. Do you think the best shape for a can will depend on the contents, e.g., will the best-shaped fish cans differ from the best-shaped peach cans?
6. Prepare a brief presentation outlining how you solved the problem.