MCV4U: Intersection of Planes

	Math Learning Goals • Identify the possible cases for intersections of three planes. • Use equations of planes to identify information about the intersection of planes. • Communicate reasoning to others, using appropriate vocabulary.	**Materials** • cardstock • scissors • graphing calculators **Handouts** • Posing Powerful Questions • Placemat
Minds On…	**Groups → Placemat** Students note the cases for intersection of lines in three dimensions, and information that can be found from the vector equations of the lines. Each group summarizes its work in the centre of the placemat. **Groups → Think Aloud** Each group discusses the possible cases for intersection of two planes. Group members describe their reasoning to the rest of the group then extend the process to describing the possible intersections of three planes.	DI Different learning modalities are addressed through varied activities in the lesson.
Action! A for L	**Groups → Jigsaw** Two groups of four join to make home groups of eight students. Each student then moves to an expert subgroup. Each expert subgroup has three pieces of cardstock labelled with equations of planes. Expert subgroups decide, using information from the equation, which case they are dealing with, and then construct a physical model for their case using the cardstock. Students return to their home groups and describe their case, noting particularly what information from the equations gave information about the physical situation. Students may refer to the physical models that they constructed in their expert subgroups. **Curriculum Expectations/Observation/Mental Note:** Assess student understanding through listening and observation during the jigsaw activities.	**Note:** For each expert subgroup, the equations will produce a different one of the eight intersection cases.
Consolidate Debrief A for L	**Whole Class → Discussion** In chart form, the class summarizes the eight cases, and what information the equation provides for each case. **Individual → Ticket to Leave** Each student writes a brief note explaining the case that they dealt with in their expert subgroup, and how information from the equations helped identify the physical situation. Check student responses for understanding and provide descriptive feedback.	
	Home Activity or Further Classroom Consolidation For the case that you dealt with in your expert subgroup, find the locus/loci of the intersections of the three planes. Be ready to share this information with your home groups in the next class.	

Posing Powerful Questions

Lesson Title MCV4U: Intersection of Planes   **Grade/Program 12 U**

Goals(s) for a Specific Lesson
Students will identify the possible cases for intersections of three planes.
Students will use equations of planes to identify information about the intersection of planes.
Students will communicate their thinking to others, using appropriate reasoning and vocabulary.

Curriculum Expectations
VOE4: represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections.
V4.4 recognize a scalar equation for a plane in three-space to be of the form $Ax + By + Cz + D = O$ whose solution points make up the plane, determine the intersection of three planes represented using scalar equations by solving a system of three linear equations in three unknowns algebraically (e.g., by using elimination or substitution) and make connections between the algebraic solution and the geometric configuration of the three planes.
MP3 Reflecting: demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.
MP7 Communicating: communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.
MP6 Representing: create a variety of representations of mathematical ideas, connect and compare them, and select and apply the appropriate representations to solve problems.
MP5 Connecting: make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts.

Big Idea(s) Addressed by the Expectations
Algebraic representations and models may provide information about physical or geometrical situations or models.

Minds On… Sample Question(s)
In how many different ways can lines intersect in 3 dimensions? (*other*)
How do the vector equations of the lines help you determine information about the ways the lines intersect in 3 dimensions? (*other*)
How do the Cartesian equation of the lines help you determine information about the ways the lines intersect in 3 dimensions? (*other*)
In how many ways can two planes intersect in 3 dimensions?   (*other*)
How do the equations of the planes help you determine how the two planes intersect? (*other*)

Indicate in brackets the type (open, parallel, other).

Action! Sample Question(s)
What do the equations of the planes tell you about the way the planes intersect? Explain.
(*other*)

Scaffolding Questions *(posed to individuals as needed)*

Consolidate/Debrief Sample Question(s)

Write a convincing journal entry to show that you understand the possible cases for intersection of three planes. For each case, identify what information can be found from the Cartesian equations of the planes.
Write a system of Cartesian equations of three planes for three of the possible cases. (*open*)
Identify the similarities and differences between intersections of lines and intersections of planes.(open)
Indicate in brackets the type (open or other)

Placemat
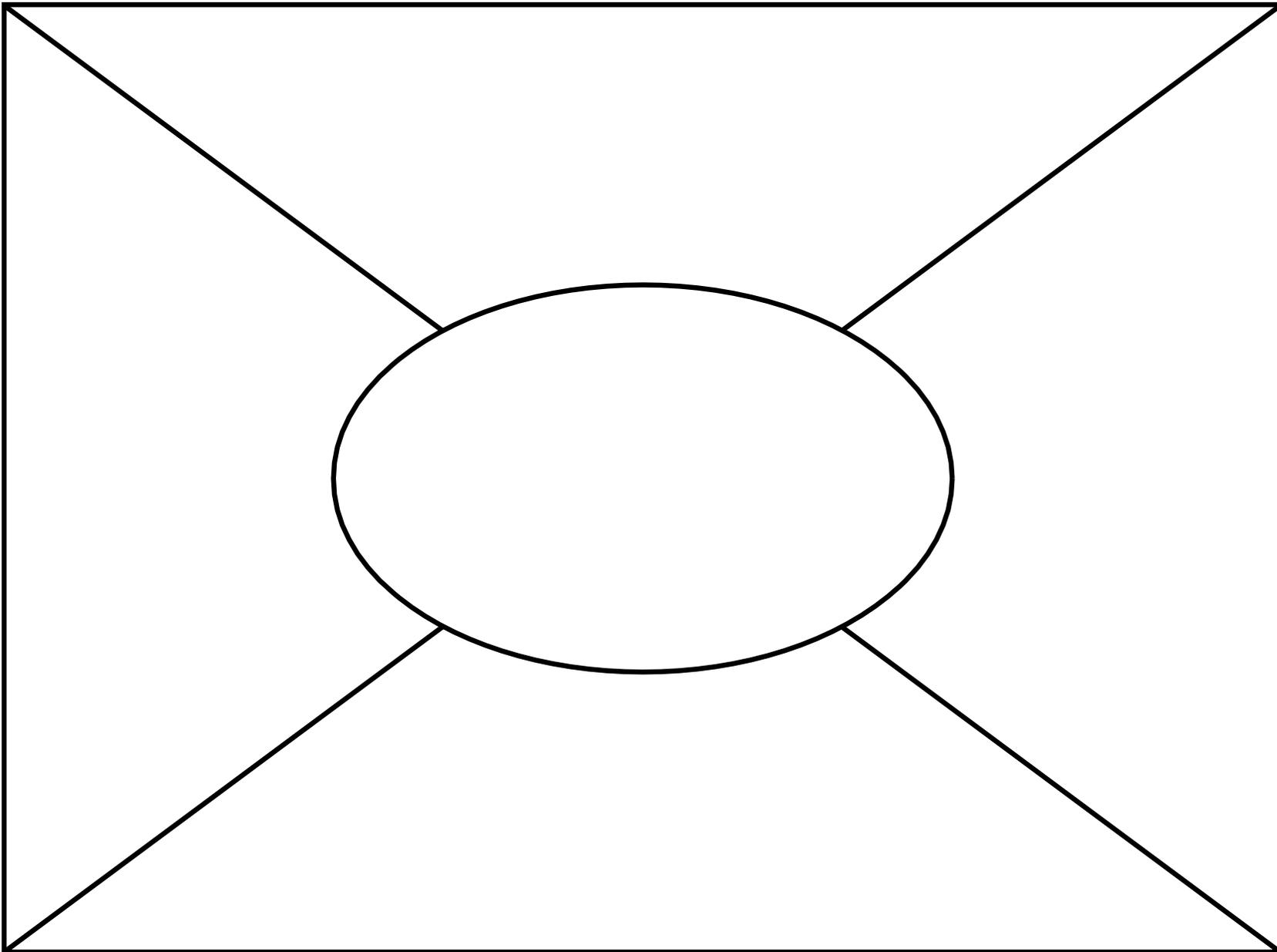
Sets of Equations for Jigsaw Activity

Set A
$2x+y+6z=7$
$x-2y-4z=9$
$3x+4y+3z=-8$

Set B
$8x+5y+z=20$
$x-5y+2z=10$
$x+7y-2z=-6$

Set C
$3x-9y+6z=2$
$2x-6y+4z=7$
$2x-y+z=1$

Set D
$-3x+9y-6z=-12$
$2x-6y+4z=8$
$5x-15y+10z=20$

Set E
$2x+6y-2z=12$
$-5x-15y+10z=-20$
$X+3y-z=6$

Set F
$x+3y-z=-10$
$x-2y+2z=-4$
$2x-y+z=1$

Set G
$3x-y+2z=8$
$9x-3y+6z=30$
$-6x+2y-4z=-20$

Set H
$6x-3y+9z=12$
$4x-2y+6z=35$
$-10x+5y-15z=20$