MCR3U: Transformations of $y = \sin x$

MathGAINS

Math Learning Goals

- Identify as transformations of $y = \sin x$ the effect of $y = A\sin(kx + d) + c$.
- Make connections to transformations of other families of functions.

Materials
- graphing calculators
- sketch of $y = \sin x$
- index cards

Handouts
- Posing Powerful Questions
- Transformational Trigonometry
- Placemat

DI Different student learning preferences are addressed by varying activities and groupings.

Minds On…

Groups → Placemat

On the placemat students identify the differences between $y = x^2$ and equations in the form $y = a(x - p)^2 + q$, using terms such as *vertex, domain, range, intercepts, vertical shift, vertical stretch, horizontal shift, horizontal stretch*.

A for L

Curriculum Expectations/Demonstration/Checklist: Collect placemats to verify student's prior knowledge.

Individual → Review

Given a sketch of $y = \sin x$ (at least two full wavelengths), students identify amplitude, period, phase shift and vertical shift. Students may also identify domain, range, intercepts, which will be used in the next class.

Individual → Anticipation Guide

Students complete the anticipation guide (BLM) and keep it for use in Consolidation.

Action!

Groups → Jigsaw

Students form home groups of 4 and identify an expert for each subgroup. Each expert subgroup explores a different transformation of $y = \sin x$ ($y = A\sin x$; $y = \sin kx$; $y = \sin(x - d)$; $y = \sin x + c$. Provide each expert subgroup with a set of 5 equations of the appropriate type of transformation. For each equation, students conjecture the effect of the transformation, then check using a graphing calculator. (Students should enter $y = \sin x$ in Y_1, then each subsequent equation in Y_2, Y_3, etc.) Students identify amplitude, period, phase shift, and vertical shift for each equation.

Once each expert subgroup is certain of the impact of the transformation, students return to home groups and share their learning.

Home groups practise on four equations involving compound transformations. Students identify amplitude, period, phase shift, and vertical shift for each equation, and use graphing calculators to verify their work.

DI Differentiate process based on student learning preference in order to give student choice. Every student becomes an expert on one transformation; every student teaches that transformation to their home group.

A for L

Curriculum Expectations/Observation/Anecdotal Record: Check for student understanding through observation of group activities.

Consolidate Debrief

Individual → Revisit Anticipation Guide

Students revisit their anticipation guides (BLM) and change as necessary.

Pairs → Think/Pair/Share

Each student writes an equation representing a compound transformation of $y = \sin x$ on an index card. Working in pairs, they write values for amplitude, period, phase shift, and vertical shift on the back of the card.

Whole Class → Inside-Outside Circle

Use the index cards to do an inside-outside circle activity to consolidate students' learning.

A for L

Curriculum Expectations/Observation/Checklist: Verify student understanding through observation during the inside-outside circle activity.

Home Activity or Further Classroom Consolidation

Posing Powerful Questions

Lesson Title Transformations of y=sinx **Grade/Program MCR3U**

Goals(s) for a Specific Lesson

Students will identify as transformations of y=sinx the *effect of changing parameters A,* k, d *and* c *in* y = *Asin(*kx + d*)* + c

Students will ma*ke connections between transformations of y = sinx and transformations of other families of functions • such as* $y = x^2$

Curriculum Expectations

Determine through investigation using technology, and describe the roles of the parameters a, c, and d in functions in the form f(x) = A$\sin x$, f(x) = $\sin x + c$, f(x) = $\sin(x - d)$ in terms of transformations on the graph of f(x) = $\sin x$ with angles expressed in degrees (i.e., translations; reflections in the x-axis; vertical stretches and compressions.

Big Idea(s) Addressed by the Expectations

Changing a parameter in an equation changes the graph of the function in a similar way, no matter what base function is used.

Minds On… Sample Question(s)

In what way(s) is the **equation** $y=x^2$ similar to y=sinx? In what ways are they different? (*open*)
In what way(s) is the **graph** of $y=x^2$ similar to y=sinx? In what ways are they different? (*open*)
$y=x^2$ is the **base quadratic function**. All other quadratics are transformations of $y=x^2$.
What does it mean to be the base function? (*other*)

Action! Sample Question(s)

y=sinx is the **base sine function**. Why do you think this is so? (other)
Action questions for each of the 4 groups: (*parallel*)
In what way(s) is y=2sinx similar to $y=2x^2$?
In what way(s) is y=sinx + 3 similar to $y=x^2 + 3$?
In what way(s) is y=sin2x similar to $y=(2x)^2$?
In what way(s) is y=sin(x+30) similar to y=(x+30)?

Scaffolding Questions

Examine the graphs on the calculator. In what ways does the graph of y=sinx change for each equation that you enter? (open)
What is similar between y=sinx and the other functions? What is different between y=sinx and the other functions? (other) *(posed to individuals as needed)*

Consolidate/Debrief Sample Question(s)

In what ways is the graph of $y = 2\sin(3x - 180°) + 4$ similar to the graph of $y = -2(x -1)_2 + 4$?
In what ways are they different? *(open)*
Compare the graphs of y=sin(3x-180°) to y=sin(3(x-60°)) (*other*)
Compare the graphs of y=sin(x-5°) to y=sinx – 5 (*other*)
For y=Asin(kx+d) +c, write a journal the describes the effects of A,k,d and c on the graph of y=sinx. (*open*)

Transformational Trigonometry (Anticipation Guide)

Use your knowledge of transformations and what you know about $y = \sin x$ to conjecture answers to What I Think below:

	What I Think	What I Know
$y = \sin x$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____
$y = 3\sin x$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____
$y = 3\sin x + 2$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____
$y = 3\ \sin(x - 45^\circ) + 2$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____
$y = 3\sin 4x + 2$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____
$y = 3\ \sin4(x - 60^\circ) + 2$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____
$y = 3\sin(4x - 180^\circ) + 2$	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____	Amplitude _____ Phase Shift _____ Period _____ Vertical Shift _____

Anticipation Guide

$$y = A\sin k(x - d) + c$$

Summarize your conjectures:

The A affects the _____, by _____

The k affects the _____, by _____

The c affects the _____, by _____

The d affects the _____, by _____

What I Now Know

The A affects the _____, by _____

The k affects the _____, by _____

The c affects the _____, by _____

The d affects the _____, by _____

Placemat