

# TIPS4RM

Targeted Implementation  
and Planning Supports for  
Revised Mathematics

*Grades 7, 8, 9 Applied, 10 Applied*

*TIPS4RM for Grades 7, 8, 9 Applied and 10 Applied Math* is designed to be useful to teachers in both Public and Catholic schools, and is intended to support beginning teachers, provide new insights for experienced teachers, and help principals and professional development providers as they work to improve mathematics education.

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TIPS4RM

Leading  
MATH  
Success

**TIPS4RM** Targeted Implementation  
and Planning Supports for  
Revised Mathematics

# Developing Mathematical Literacy

Introduction  
Theoretical Framework for Program Planning  
Research

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# Developing Mathematical Literacy

## A. Introduction

*Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM)* offers ways of thinking about mathematics education, resources, and teacher education for those working with students in Grades 7 to 10.

Students in this age group are at a critical, transitional stage where their perceptions of mathematics will help to shape their success in secondary mathematics and their career decisions.

Mathematics competence and confidence open doors to productive futures. Implementation of this resource will help to enhance the mathematics knowledge and understanding of students in Grades 7 to 10, and help them develop the critical skills identified in the Conference Board of Canada's Employability Skills Profile: communicate, think, continue to learn throughout their lives, demonstrate positive attitudes and behaviours, responsibility, and adaptivity, and work with others.

One way to think of a person's understanding of mathematics is that it exists along a continuum. At one end is a rich set of connections. At the other end of the continuum, ideas are isolated or viewed as disconnected bits of information. (Skemp) A sound understanding of mathematics is one that sees the connections within mathematics and between mathematics and the world. However, to lead students to rich connections, "teachers must be agents of change that they did not experience as students" (Anderson, D. S. & Piazza, J. A., 1996.)

The intent is to help both students and teachers develop the "big picture" of mathematics that includes: competence in mathematical skills, substantive understanding of mathematical concepts, and the application of these skills and understandings in problem-solving situations. This resource package helps teachers to see how, in their program and daily lesson planning, they can address the critical **connections** between:

- instruction and assessment;
- one mathematics topic and the next;
- one strand and the other strands;
- the mathematics done in class and students' sense-making processes;
- learning mathematics and doing mathematics;
- the instructional strategy selected for a specific learning goal and research into how students learn mathematics;
- students' prior learning and new knowledge and understanding;
- existing resources and the envisioned program;
- the mathematics classroom and home;
- mathematics topics and topics in other disciplines.

This resource is intended to:

- support beginning teachers;
- provide new insights for experienced teachers;
- help principals and professional development providers as they work to improve mathematics education for young people;
- assist teachers in implementing the revised Grades 7, 8, 9 Applied and 10 Applied mathematics curriculum.

**"In this changing world, those who understand and can do mathematics have significant opportunities and options for shaping their future."**

NCTM 2000 p. 5

**A sound understanding of mathematics is one that sees the connections within mathematics and between mathematics and the world.**

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## Key Messages

There are important messages in TIPS4RM for the many stakeholders involved in the mathematical education of the young adolescent.

### For Teachers

- Focus on important mathematical concepts or “big ideas” that cluster expectations.
- Include a variety of instructional strategies and assessment strategies.
- Value the abilities and needs of the adolescent learner.
- Provide a positive environment for learning mathematics through problem solving.
- Allow opportunities for students to explore, investigate, and communicate mathematically as well as opportunities to practise skills.
- Encourage a variety of solutions that incorporate different representations, models, and tools.
- Incorporate relevant Ontario Catholic School Graduate Expectations.

### For Principals

- An effective mathematics program should have a variety of instructional and assessment strategies.
- Effective mathematics classrooms are active places where you can hear, see, and touch mathematics.
- Principals are encouraged to provide teachers with necessary support for an effective mathematics program. This can be done through:
  - student and teacher resources;
  - providing time for teachers to collaborate and share ideas;
  - providing in-service opportunities and information about mathematics education for teachers;
  - celebrating mathematics learning for all students, not just the achievements of a select few;
  - showing a positive attitude towards mathematics;
  - demonstrating the importance of life-long learning in mathematics.

### For Coordinators

- Mathematics teachers need support.
- All teachers require information and in-service about mathematics education.
- Teachers need to know how and why these activities promote student learning of mathematics.

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## Program Planning Supports

This package provides grade-specific and cross-grade program planning supports. These are structured to provide opportunities for in-depth, sustained interaction with key mathematical concepts, or “big ideas” that cluster and focus curriculum expectations. The intent is to help students see mathematics as an integrated whole, leading them to make connections and develop conceptual and relational understanding of important mathematics.

## Content and Reporting Targets by Grade

The package for each grade begins with a one-page overview of the mathematics program that includes a suggested sequencing of curriculum expectation clusters, rationale for the suggested sequence, and in Grades 7 and 8, suggested strands for reporting in each term. Detailed alignment of all curriculum expectations with the identified clusters is included in the Appendix for each grade level.

## Continuum and Connections across Grades

*Continuum and Connections* is based on significant themes in mathematics and includes Mathematical Processes and six content-based packages – *Patterning to Algebraic Modelling*, *Solving Equations and Using Variables as Placeholders*, *Perimeter, Area, and Volume*, *Integers, Fractions, and Proportional Reasoning*.

## Summative Tasks

Summative tasks are provided for each of Grade 7, Grade 8, Grade 9 Applied and Grade 10 Applied mathematics. These tasks focus on some of the major concepts of the mathematics program and require facility in several strands that the students have studied throughout the program.

Teachers can use these summative tasks to design other tasks in the program and to recognize some of the important ideas in the program. Teachers can use the summative tasks in planning a program by determining what important mathematical ideas students should understand by the end of the program and then designing the instructional approaches that support those mathematical ideas.

## Lesson Planning Supports

- The pace and structure of lessons will vary depending on the goals of the lesson. Using a single sequence of elements, such as taking up homework, teacher demonstration, student practice can be uninspiring or monotonous.
- Mathematics classes should:
  - provide opportunities for students to become proficient in basic skills;
  - focus on introducing new skills and concepts through problem solving;
  - incorporate a balance and variety of teaching strategies, assessment strategies, student groupings, and types of activities.
- Each lesson should appeal to auditory, kinesthetic, and visual learners. Appealing to all these preferences deepens the understanding of all students.
- Lessons should introduce algorithms only after students have constructed meaning for the procedure, and introduce formulas only after students develop a conceptual understanding of the relationships among measurements.
- Lessons are developed in a template to outline a desired vision for classes.

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## Lesson Planning Template

- The **MATCH** (**M**inds on, **A**ction, **T**iming, **C**onsolidate, **H**ome Activity or Further Classroom Consolidation) acronym reminds teachers that all of these elements must be considered in lesson planning. It suggests that connection/matches should be made in each lesson to:
  - program goals;
  - research and effective instruction;
  - characteristics of adolescent learners.

The following organizers in the template provide for concise communication while allowing the teacher to vary the pace and types of groupings and activities for classes:



**Minds on...** suggests how to get students mentally engaged in the first minutes of the class and establishes a positive classroom climate, making every minute of the math class count for every student.



**Action!** suggests how to group students and what instructional strategy to use. The instructional strategy chosen outlines an effective way to support students in learning the specific concept or skill. These strategies reinforce the connections with literacy and learning for life, as appropriate.

This section maps out what students will do and how the teacher can facilitate and pose thought-provoking questions. Suggestions for time management, scaffolding, and extension are included, as appropriate.



The time clock/circle graphic (Grades 7 and 8) and the time bar graph (Grades 9 and 10) suggest the proportion of class time to spend on various parts of the mathematics class.



**Consolidate/Debrief** suggests ways to ‘pull out the math,’ check for conceptual understanding, and prepare students for the follow-up activity or next lesson. Often this involves whole class discussion and sharing. Students listen and contribute to reflections on alternate approaches, different solutions, extensions, and connections.



**Home Activity or Further Classroom Consolidation** suggests meaningful and appropriate follow-up. These activities provide opportunities:

- to consolidate understanding;
- to build confidence in doing mathematics independently;
- for parents to see the types of math activities students engage in during class and to see connections between the mathematics being taught and life beyond the classroom;
- for giving students some choice through differentiated activities.

# Interpreting the Lesson Outline Template

[Word Outline Template](#)

[FileMaker Pro Outline Template](#)

**Unit 7**  
**Algebraic Models: Making Connections**

**Grade 9 Applied** ← Grade Level

← Lessons Addressing a Theme

**Lesson Outline**

Lessons are planned to help students develop and demonstrate the skills and knowledge detailed in the curriculum expectations.

- To help students value and remember the mathematics they learn, each lesson is connected to and focussed on important mathematics as described in the Big Picture.
- Since students need to be active to develop understanding of these larger ideas, each point begins with a verb.
- Sample starter verbs: represent, relate, investigate, generate, explore, develop, design, create, connect, apply

**BIG PICTURE**

Students will:

- describe relationships between variables graphically;
- use first differences to determine that relationships are linear or non-linear;
- describe linearly related data graphically, in words, and algebraically;
- make connections among number, graphic, and algebraic models;
- look for patterns and use variables to generalize/summarize/simplify.

Day	Lesson Title	Math Learning Goals	Expectations
1	Linear and Non-Linear Investigations	<ul style="list-style-type: none"> <li>Investigate linear and non-linear relationships.</li> <li>Examine first differences and the shape of the graph.</li> <li>Explore the effects of changing the conditions.</li> <li>Write equations for linear relationships and describe non-linear relationships.</li> </ul>	NA2.07, LR3.02, LR3.04, LR4.03, LR4.05 CGE 5a, 7i
2			
3	Building Models <i>Presentation file: Patterns.ppt</i>	<ul style="list-style-type: none"> <li>Use multiple representations (physical, numerical, algebraic).</li> <li>Develop an understanding that simplification is necessary to determine if two algebraic expressions are equivalent.</li> </ul>	NA2.01, NA2.05, LR4.03, LR4.04, LR4.05 CGE 5a, 7i
4	Simplifying Algebraic Models <i>Presentation files: Collecting Terms using Algebra Tiles.ppt, Expanding and Simplifying Algebraic Equations.ppt</i>	<ul style="list-style-type: none"> <li>Use multiple representations (physical, numerical, algebraic).</li> <li>Simplify algebraic expressions.</li> </ul>	NA2.01, NA2.05, NA2.06, LR4.03, LR4.04, LR4.05 CGE 2a, 2b
5	Multiple Representations	<ul style="list-style-type: none"> <li>Use multiple representations (concrete, pictorial, numerical, algebraic, graphical).</li> <li>Simplify algebraic expressions.</li> </ul>	NA2.01, NA2.05, NA2.06, LR4.03 CGE 2b, 5a
6	Use of Variables as Bases of Power	<ul style="list-style-type: none"> <li>Use variables as a base for powers up to degree 3.</li> </ul>	NA2.01, NA2.03, NA2.04, NA2.05 CGE 2a, 2c
7	Algebraic Models in Measurement and Geometry	<ul style="list-style-type: none"> <li>Use variables to make connections between symbolic and concrete models from measurement and geometry application problems.</li> <li>Evaluate expressions after substitution of a value for a variable.</li> <li>Substitute into algebraic equations and solve for one variable in the first degree.</li> </ul>	NA2.01, NA2.03, NA2.04, NA2.08 CGE 5c
8		Instructional Jazz	
9		Instructional Jazz	
10		Assessment	

Two or three points to describe the content of this lesson.

Points begin with a verb.

Individual lesson plans elaborate on how objectives are met.

A brief descriptive lesson title

List of curriculum expectations (and CGEs) by code

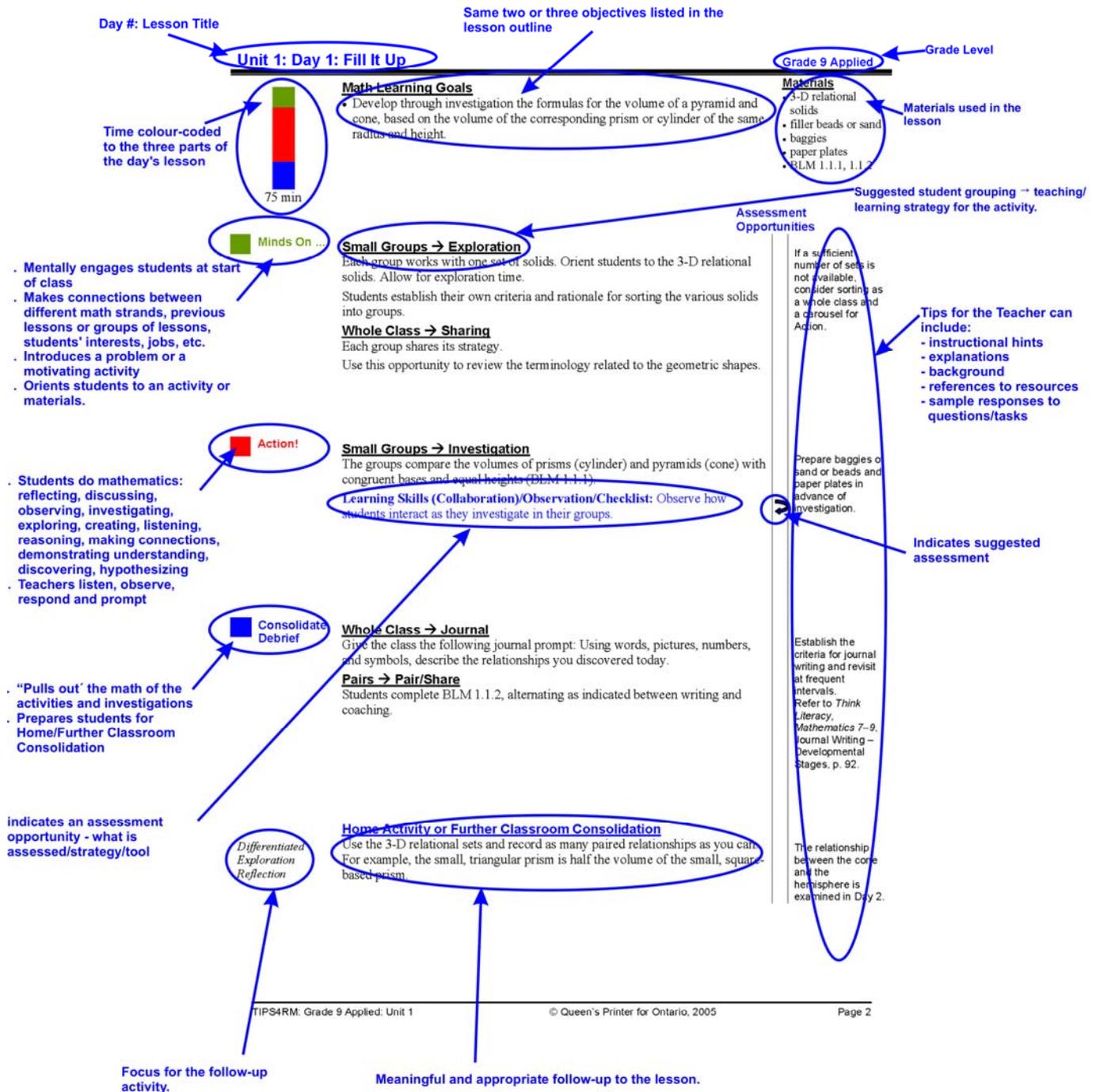
## NOTES

- While planning lessons, teachers must judge whether or not pre-made activities support development of big ideas and provide opportunities for students to understand and communicate connections to the “Big Picture.”
- Ontario Catholic School Graduation Expectations (CGEs) are included for use by teachers in Catholic Schools.
- Consider auditory, kinesthetic, and visual learners in the planning process and create lessons that allow students to learn in different ways.
- The number of lessons in a group will vary.
- Schools vary in the amount of time allocated to the mathematics program. The time clock/circle on completed Grade 7 and 8 lessons suggests the portions of the class to spend on the **Minds On**, **Action!**, and **Consolidate/Debrief** portions. Grade 9 and 10 Applied lessons are based on 75-minute classes.
- The assessments suggested during each lesson are often meant to inform adjustments the teacher will make to later parts of the lesson or to future lessons.

# Interpreting the Lesson Planning Template

[Word Lesson Template](#)

[FileMaker Pro Lesson Template](#)



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## Research Base for Planning

Each of the organizers in *Continuum and Connections* and *Lesson-Planning Templates* is supported by a one-page research synopsis. It is through a thoughtful combination of decisions that teachers proactively establish *Classroom Management* and a community of learners and demonstrate their commitment to equity.

By making thoughtful choices at each decision point in the templates, teachers demonstrate their understanding of *Adolescent Mathematics Learners* (p. 19) as it relates to the students in their class.

It is important to set high standards for all students and to provide the supports and extensions that help all students learn.

## Time

Although the time allotment for mathematics classes may vary within and between the elementary and secondary panels, and between schools, teachers must plan appropriate time for the various components of each lesson. To assist teachers in this stage of planning, research regarding *Japanese Lesson Structure* (p. 26) is provided as an example of an internationally respected model of student inquiry and related time allotment.

## Materials

The contemporary mathematics classroom features a combination of traditional materials and more recently developed tools. *Concrete Materials* (p. 22) provides a description and analysis of effective mathematical manipulatives. *Technology* (p. 34), teachers can read about the various types and applications of technological tools, and important considerations relating to their implementation.

## Type of Learner

Research regarding the complex amalgam of students' abilities and competencies, e.g., Gardner's Multiple Intelligence research, has had an immense impact on modern educational thinking. Students not only demonstrate many different kinds of skills and creative talents, but possess different learning styles including auditory, visual, and kinaesthetic. Teachers can use the lesson template to mindfully interpret each lesson in terms of these considerations.

## Grouping

Research has shown that students benefit from a balance of different learning structures. *Flexible Grouping* (p. 24) informs teachers of the many options available for logistical planning within the classroom. These options include heterogeneous pairs and small groups, homogeneous pairs and small groups, co-operative learning activities, independent research or class work, together with whole-class and small group discussions.

## Instructional Strategies

A variety of effective instructional strategies are explored within this document. *Differentiated Instruction* (p. 36) describes how teachers can orchestrate different activities for different students simultaneously, for one or more instructional purposes, and to the benefit of all involved. *Mental Mathematics and Alternative Algorithms* (p. 27) provides insight into the research surrounding estimation and algorithmic exploration. Further valuable strategies such as *Scaffolding* (p. 31), *Student-Centred Investigations* (p. 32), and *Teacher-Directed Instruction* (p. 33) are also detailed.

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## Communication

Communication contributes to development of mathematical understanding in the classroom.

*Metacognition* (p. 28) describes ways in which teachers can encourage students to analyse and alter their own thinking and learning processes. *Questioning* (p. 30) discusses the complexity involved in, skills required for, and examples of higher-order questioning. In *Communication* (p. 21), a model is presented for a positive classroom environment in which students are encouraged to question, comment, theorize, discuss, and defend ideas.

## Assessment

The purpose of assessment for learning is to improve student learning by providing teachers and students with assessment data that can inform and guide instruction and learning. Assessment embedded in instructional planning connects to student needs and prior knowledge, and promotes achievement. Good assessment practices provide opportunities for students to demonstrate what they “know and can do” and promote their success. Using a variety of assessment strategies and tools assists in providing *Balanced Assessment* that recognizes that students demonstrate their understanding in many different ways and that provides a complete picture of a student’s mathematical understanding. Teachers are reminded of the importance of consistent and detailed feedback regarding student progress to both students and parents throughout the school year. *Evaluation* involves the judging and interpreting of assessment data and the assigning of a grade. Evaluation should be a measure of a student’s current achievement in the important mathematical concepts in the program.

Reference: TIPS4RM: Assessment

## Home Activity or Further Classroom Consolidation

Using the lesson template, teachers can plan a variety of activities for follow-up just as they plan a variety of lesson types, depending on the learning goal.

Students need opportunities to learn, practise, select their own preferred problem-solving methods and strategies, and demonstrate learning in a variety of ways.

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## B. Theoretical Framework for Program Planning

### Vision

#### What is mathematical literacy and why it is important?

Mathematical literacy, as defined by the Programme for International Student Assessment (PISA, [www.pisa.oecd.org/knowledge/summary/b.html](http://www.pisa.oecd.org/knowledge/summary/b.html)) is “measured in terms of students’ capacity to:

- recognise and interpret mathematical problems encountered in everyday life;
- translate these problems into a mathematical context;
- use mathematical knowledge and procedures to solve problems;
- interpret the results in terms of the original problem;
- reflect on the methods applied; and
- formulate and communicate the outcomes.”

Another source suggests that:

“Mathematics literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen.” (*Measuring Up*, OECD PISA Study, 2001, p. 10)

Mathematical literacy is valued for many different reasons. Mathematics provides powerful numeric, spatial, temporal, symbolic, and communicative tools. Mathematics is needed for “everyday life” to assist with decision making. As John Allen Paulos pointed out in his book, *Innumeracy* (2001), “... numeracy is the ability to deal with fundamental notions of number and chance in order to make sense of mathematical information presented in everyday contexts.” Mathematics and problem solving are needed in the workplace for many professions such as health science workers or graphic artists, as well as statisticians and engineers. Mathematics is also, ultimately, a cultural and intellectual achievement of humankind and should be understood in its aesthetic sense (NCTM, 2000). All people have a right of access to the domain of mathematics.

The need for mathematical literacy for all students requires us to describe what an effective mathematics program looks like – one that fosters mathematical literacy in all students.

#### What does effective mathematics teaching and learning look like?

There are several components to sound mathematics teaching and learning:

##### a) A Classroom Climate for Mathematics Learning

Learning does not occur by passive absorption of information. Students approach a new task with prior knowledge, assimilate new information, and construct new understandings through the task (Romberg, 1995). The classroom climate must be conducive to such learning. The following factors need to be considered:

- **Prior knowledge:** Students come to class with significant but varying prior knowledge. Such prior knowledge should be valued and used during instruction.
- **Equity:** All students are able to learn mathematics. Excellence in mathematics requires high expectations and strong support for all students.
- **Technology:** Technology influences the mathematics that we teach and how we teach it. To this end, teachers need to keep up-to-date with technology.
- **Social, emotional, and physical considerations:** Brain research on learning (Given, 2002) suggests that there are five “theatres of the mind” — emotional, social, physical, cognitive, and reflective. The emotional, social, and physical learning systems tend to be the most powerful in terms of their demands. The level of their functioning determines how effectively the cognitive and reflective systems operate.

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- **Attitude and confidence:** Fostering a positive attitude and building student confidence helps students to develop values that do not limit what they can do or can assimilate from a learning experience. Students' emotions, beliefs, and attitudes towards mathematics and learning interact with cognition and are instrumental in empowering learners to take control of their own learning and express confidence in their mathematical decisions.
  - **Valuing Mathematics:** Values include learners' emotions, beliefs, and attitudes towards mathematics and learning. Such affective processes interact with cognition and are instrumental in empowering the learner to express confidence in their mathematical decisions and to take control of their own learning.

A classroom that is conducive to learning takes all of these areas into consideration so that students have a safe and positive environment for learning. Traditionally, mathematics programs have attended to the cognitive learning system with less attention being paid to the other four learning systems described above. However, mathematics teachers are becoming more aware of the importance of the affective domain – emotional, social, and reflective learning systems. For instance, curriculum policy requires that students learn how to communicate effectively in mathematical contexts. This communication provides perspectives into students' feelings and values as well as their thinking.

An appropriate classroom environment in Grade 7, Grade 8, and Grades 9 and 10 Applied math classes also considers the characteristics of the adolescent learner. These characteristics include intellectual development such as the ability to form abstract thought and make judgments. Adolescents are going through many physical changes, and may be experiencing emotional uncertainty. Recognition by peers and social status are extremely important to them. In working with adolescents, it is important to recognize signs of disengaged or at-risk learners and to provide support for them. A more extensive discussion of the adolescent learner is included in *Adolescent Mathematics Learners*, p. 19.

The intent of *Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM)* is to help teachers activate all of the student's learning systems through the delivery of a balanced mathematics program.

### Summary

A classroom environment that supports student learning has the following qualities:

- Students are supported to take risks, explore different problem-solving strategies, and communicate their understanding.
- Mathematics is seen, heard, and felt.
- The teacher models and promotes a spirit of inquiry.
- Students' prior knowledge is valued and built upon.
- Students actively explore, test ideas, make conjectures, and offer explanations.
- Social skills are developed to promote effective teamwork.

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## b) Mathematical Activity

Mathematical activity should develop mathematical thinking. The successful teaching of mathematics can be characterized as helping students develop proficiency in mathematics. *In Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001), the concept of mathematical proficiency is described in terms of five intertwining components:

- *conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence* – ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

These five competencies capture what it means for someone to learn mathematics successfully. The integrated and balanced development of all five should guide the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn, on one hand, solely by internalizing what a teacher or book says, or, on the other hand, solely by inventing mathematics on their own. (Kilpatrick et al., 2001, p. 11)

Mathematical activity should be balanced with time to investigate and explore as well as time to practise skills. The seven Mathematical Processes, highlighted in the revised mathematics curriculum, are learned and applied as students work to achieve the knowledge and skills expectations outlined within the strands. Detailed information on the Mathematical Processes is contained in the revised curriculum documents: *The Ontario Curriculum, Grades 1–8, Mathematics*, 2005, pages 11–17 and *The Ontario Curriculum, Grades 9 and 10, Mathematics*, 2005, pages 12–16.

It is important to provide students with opportunities to practise mathematical activities individually, and to provide rich and challenging problems that require students to combine these activities in various ways.

### Summary

Mathematical activity in a mathematics classroom is characterized by:

- Computing
- Recalling facts
- Manipulating
- Using manipulatives and technology
- Exploring
- Hypothesizing
- Inferring/concluding
- Revising/revisiting/reviewing/reflecting
- Making convincing arguments, explanations, and justifications
- Using mathematical language, symbols, forms, and conventions
- Explaining
- Integrating narrative and mathematical forms
- Interpreting mathematical instructions, charts, drawings, graphs
- Representing a situation mathematically
- Selecting and sequencing procedures

### c) A Focus on Problem Solving

Problem solving is fundamental to mathematics. The mathematical processes can be aligned with various problem-solving models, with the mathematical competencies as defined by Kilpatrick, Swafford, and Findell (2001) and with the categories of the Achievement Chart in the Ontario Ministry of Education Mathematics Curriculum. This illustrates that the many different types of organizing structures can be aligned so that teachers see how they fit together and address similar processes of mathematical thinking. The categories in the Achievement Chart – Knowledge and Understanding, Thinking, Communication, and Application – should not be viewed as isolated but rather as component processes of problem solving. Problem solving involves designing a plan and applying a variety of skills to solve the problem. The problem-solving process also requires mathematical communication. Thus, problem solving is seen as the most comprehensive framework in the learning of mathematics. Mathematical activity and learning should be centred on problem solving. NCTM's *Principles and Standards* document (2000) points out the importance of problem solving:

**“Solving problems is not only a goal of learning mathematics but also a major means of doing so.”**

NCTM's *Principles and Standards* (2000)

“Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas ... Good problems will integrate multiple topics and will involve significant mathematics.” (p. 52).

Students need regular opportunities to represent, manipulate, and reason in the context of solving problems and conducting mathematical inquiries. It is expected that students use a variety of tools, including technology to solve problems and develop their understanding. Students should regularly communicate their thinking to peers and to the teacher and reflect on their learning. Problem solving is the most effective way to engage students in all of the mathematical processes. In *Elementary and Middle School Mathematics*, John A. Van de Walle says:

“Most, if not all, important mathematics concepts and procedures can best be taught through problem solving. That is, tasks or problems can and should be posed that engage students in thinking about and developing the important mathematics they need to learn.” (p. 40)

### d) A Balanced Mathematics Program

In a balanced mathematics program, students become proficient with basic skills, develop conceptual understanding, and become adept at problem solving. All three areas are important and need to be included. Students need to develop both skills and conceptual understanding: problem solving is the medium for students to use to make connections between skills and conceptual understanding (*Mathematics Program Advisory*, 1996).

This resource gives teachers examples of lessons and assessments that will allow them and their students to experience a balanced mathematics program. Once experienced, a balanced mathematics program empowers both teachers and students as life-long learners of mathematics.

#### Summary

A balanced mathematics program has:

- students working in groups, pairs, and individually
- a variety of activities for students to engage in all of the mathematical processes
- a variety of assessment data to improve student learning and adjust program
- a focus on developing key mathematical concepts or “Big Ideas”

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## Making It Happen

This section highlights some of the student and teacher roles in a mathematics class, and describes the components of an effective lesson.

As problem solving places the focus on the student's attention to ideas and making sense, student roles are very active. Students are encouraged to:

- make conjectures;
- gather data;
- explore different strategies;
- share their ideas;
- challenge and defend ideas and solutions;
- consolidate and summarize their understanding;
- practise skills that relate to the mathematical concepts being explored.

Teachers' roles are much more than telling and explaining. Teachers need to:

- create a supportive mathematical environment;
- pose worthwhile mathematical tasks;
- recognize when students need more practice with skills and accommodate those needs;
- encourage mathematical discussion and writing;
- promote the justification of student answers;
- pose important questions that help students “pull out the math;”
- listen actively to student questions and responses and react accordingly;
- consolidate and summarize important mathematical concepts and help students make connections.

The structure of a lesson in mathematics can take a variety of forms. In a lesson that focuses on problem solving, the following components incorporate the mathematical processes that have been discussed:

- Teacher poses a complex problem.
- Students work on the problem.
- Students share solutions.
- Teacher consolidates understanding and summarizes discussion.
- Students work on follow-up activities.

In a lesson focusing on skill development, the following sequence should motivate the hard work that is sometimes needed to develop proficiency:

- Teacher poses an interesting problem that will extend students' current skill development.
- The problem exposes the need for a particular skill.
- Building on students' prior knowledge, the teacher instructs, guides, or demonstrates the new skill.
- The teacher guides students through the solution of the original problem by applying the new skill.
- Students practise the skill and apply the skill to similar problems.

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## C. Research – Topic Summaries

<b>Mathematics Reform</b> .....	p. 18
<b>Adolescent Mathematics Learners</b> .....	p. 19
• adolescent students learn mathematics best in environments which allow for physical activity, social interaction, technological investigations, choice, variety, and meaningful input.	
• patience and awareness are key factors in providing adolescent learners with the kind of emotional and pedagogical support that they require during this time of personal change.	
<b>Classroom Management</b> .....	p. 20
• Good classroom management results from well planned lessons and well established routines.	
• A stimulating classroom that engages students through a balanced variety of teaching, learning, and assessment strategies prevents and reduces behaviour problems.	
<b>Communication</b> .....	p. 21
• Communication in mathematics requires the use and interpretation of numbers, symbols, pictures, graphs, and dense text.	
• When explaining their solutions and justifying their reasoning, students clarify their own thinking and provide teachers with windows into students’ depth of understanding.	
<b>Concrete Materials</b> .....	p. 22
• Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.	
• Teachers can diagnose gaps in a student’s conceptual understanding through observing the use of manipulatives and listening to the accompanying narrative explanation.	
<b>Differentiated Instruction</b> .....	p. 23
• Differentiated instruction recognizes and values the wide range of students’ interests, learning styles, and abilities and features a variety of strategies based on individual needs.	
• It is sometimes appropriate to have students in the same class working on different learning tasks.	
<b>Note:</b> More in-depth information on differentiating instruction is provided on pages 36–47.	
<b>Flexible Grouping</b> .....	p. 24
• Flexible groups should be used to maximize learning for all students in the class.	
• Consider prior knowledge, achievement on particular tasks, social skills, learning skills, gender, and exceptionalities when determining a grouping for a particular task.	
<b>Graphic Organizers</b> .....	p. 25
• Graphic organizers are powerful, visual tools that are effective in teaching technical vocabulary, helping students organize what they are learning, and improving recall.	
• Graphic organizers rely heavily on background information and can be particularly helpful for students with learning disabilities.	
<b>Japanese Lesson Structure</b> .....	p. 26
• Japanese lesson structure often features the posing of a complex problem, small-group generation of possible solutions, a classroom discussion and consolidation of ideas, followed by extended practice.	
• In this model, teachers plan lessons collaboratively, anticipate student responses, and watch each other teach.	
<b>Mental Mathematics and Alternative Algorithms</b> .....	p. 27
• Traditional algorithms are an essential part of mathematics learning and should be taught, but only after students have developed understanding of the concept and shared their own approaches to the problem.	
• Teachers and students should share their strategies for mental computation.	

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<b>Metacognition</b> .....	p. 28
<ul style="list-style-type: none"> <li>• Students learn to monitor their own understanding when they are constantly challenged to make sense of the mathematics they are learning and to explain their thinking.</li> <li>• Students who are aware of how they think and learn are better able to apply different strategies to solve problems.</li> </ul>	
<b>Providing Feedback</b> .....	p. 29
<ul style="list-style-type: none"> <li>• Students and parents need regular formative feedback on the students’ cognitive development and achievement of mathematical expectations, as well as on their learning skills.</li> <li>• Feedback should be meaningful and encouraging while allowing students to grapple with problems and should form the basis for future instruction.</li> </ul>	
<b>Questioning</b> .....	p. 30
<ul style="list-style-type: none"> <li>• Higher level (process) and lower level (product) questioning have their place in the mathematics classroom, with teachers employing both types as the need arises.</li> <li>• Successful questioning often involves a teacher asking open-ended questions, encouraging the participation of all students, and patiently allowing appropriate time for student responses.</li> </ul>	
<b>Scaffolding</b> .....	p. 31
<ul style="list-style-type: none"> <li>• Scaffolding works best when students are at a loss as to what they should do, but can accomplish a task that is just outside their level of competency, with assistance.</li> <li>• Successful use of scaffolding requires educators to determine the background knowledge of students, and develop a comfortable, working rapport with each student.</li> </ul>	
<b>Student-Centred Investigations</b> .....	p. 32
<ul style="list-style-type: none"> <li>• Student-centred investigations are learning contexts which require students to use their prior knowledge to explore mathematical ideas through an extended inquiry, discovery, and research process.</li> <li>• Student-centred investigations provide a meaningful context in which students must learn and use these skills to solve more complex problems.</li> </ul>	
<b>Teacher-Directed Instruction</b> .....	p. 33
<ul style="list-style-type: none"> <li>• Teacher-directed instruction is a powerful strategy for teaching mathematical vocabulary, facts, and procedures.</li> <li>• Teachers should plan lessons where students will grapple with problems individually and in small groups, identifying for themselves the need for new skills, before these teacher-directed skills are taught.</li> </ul>	
<b>Technology</b> .....	p. 34
<ul style="list-style-type: none"> <li>• Technology such as graphing calculators, computer software packages (e.g., Geometer’s Sketchpad, Fathom, spreadsheets), motion detectors, and the Internet are powerful tools for learning mathematics.</li> <li>• Technology, when used properly, enhances and extends mathematical thinking by allowing students time and flexibility to focus on decision making, reflection, reasoning, and problem solving.</li> </ul>	
<b>van Hiele Model</b> .....	p. 35
<ul style="list-style-type: none"> <li>• The van Hiele model of geometry learning is comprised of the following sequential conceptual levels: visualization, analysis, informal deduction, formal deduction, and rigor.</li> <li>• To maximize learning and successfully move students through the levels from concrete to visual to abstract, teachers must be aware of students’ prior knowledge and ongoing understandings.</li> </ul>	

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<b>Differentiated Instruction</b> .....	p. 36
Differentiation Based on Student Readiness.....	p. 39
Differentiation Based on Student Interest.....	p. 41
Differentiation Based on a Student’s Learning Profile.....	p. 42
Differentiation Based on Mathematics Content.....	p. 44
Differentiation Based on Process or Sense-Making Activity .....	p. 45
Differentiation Based on Product.....	p. 46

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## Mathematics Reform

Mathematics education is undergoing significant reform in many countries, including Canada and the United States. *Principles and Standards for School Mathematics* (NCTM, 2000) highlighted six key principles—Equity, Curriculum, Teaching, Learning, Assessment, and Technology—that must guide meaningful curricular reform efforts. The vision statement which prefaced the document elaborated on the need for the continued improvement of mathematics education:

Evidence from a variety of sources makes it clear that many students are not learning the mathematics they need or are expected to learn (Kenney and Silver, 1997; Mullis et al., 1997, 1998; Beaton et al., 1996). The reasons for this deficiency are many. In some instances, students have not had the opportunity to learn important mathematics. In other instances, the curriculum offered to students does not engage them. Sometimes students lack a commitment to learning. The quality of mathematics teaching is highly variable. There is no question that the effectiveness of mathematics education in the United States and Canada can be improved substantially. (2000, p. 5)

Similarly, *The Ontario Curriculum: Mathematics* (2005) documents “have been developed to provide a rigorous and challenging curriculum for students” which includes a “broader range of knowledge and skills” (OME, 2005, p. 3), and which “integrates appropriate technologies into the learning and doing of mathematics, while recognizing the continuing importance of students mastering essential arithmetic and algebraic skills” (OME, 2005, p. 3). *The Targeted Implementation and Planning Supports for Revised Math* (TIPS4RM) document has been developed to further assist teachers, coordinators, and administrators as they continue to implement these reforms. This section will specifically provide educators with a variety of teaching strategies, that are supported by contemporary research in mathematics education.

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## Adolescent Mathematics Learners

### Characteristics of adolescent mathematics learners

According to Reys et al (2003), students vary greatly in their development and readiness for learning and “teachers play a critical role in judging the developmental stage” of each student and in establishing “rich environments for students to explore mathematics at an appropriate developmental level.” (p. 25) The adolescent mathematics learner (Grades 7–9) is experiencing great changes and challenges in several domains simultaneously. Intellectually, adolescents are refining their ability to form abstract thought, think symbolically, render objective judgments, hypothesize, and combine multiple reactions to a problem to achieve resolution. Wolfe (cited in Franklin, 2003), noted:

By adolescence, students lose about 3 percent of the gray matter in their frontal lobe – this is a natural process where the brain ‘prunes’ away excess materials to make itself more refined and more efficient. Such changes could indicate why adolescents sometimes have difficulty prioritizing tasks or multitasking. . . . Some neuroscientists feel that the cerebellum may be responsible for coordination or cognitive activity in addition to muscle and balance coordination. If that’s true, then it’s possible that physical activity could increase the effectiveness of the brain and learning. (p. 4)

Physically, adolescents tend to mature at varying rates, e.g., girls developing physically earlier than boys; are often concerned about their physical appearance; and may experience fluctuations in metabolism causing extreme restlessness and/or lethargy. Emotionally, many adolescents are sensitive to criticism; exhibit erratic emotions and behaviour; feel self-conscious; often lack self-esteem; search for adult identity and acceptance; and strive for a sense of individual uniqueness. Socially, adolescents may be eager to challenge authority figures and test limits; can be confused and frightened by new school settings that are large and impersonal; are fiercely loyal to peer group values; and are sometimes cruel and insensitive to those outside the peer group.

### Capitalizing on adolescent characteristics

Erlauer (2003) has suggested what she refers to as the *20-2-20 Rule for Reflection and Application at the Middle/High School Levels*. Twenty (20) minutes into the lesson, when students’ attention is waning, the teacher has the students re-explain what they have just learned (e.g., brief class discussion, sharing with partner, or entry in student journals) with some form of feedback (e.g., from teacher or from classmates) to check for understanding. Within two (2) days of initial learning, the teacher requires students to review and apply the new information, e.g., mind-map, piece of writing, developing a related problem for classmate to solve. And within twenty (20) days, usually at the end of a unit, the teacher has the students reflect on what they have learned and apply the concepts/skills they have learned to a more involved project which is then shared with the whole class or a small group of students. (pp. 84–85)

### Considerations regarding adolescent characteristics

Teachers of Grades 7–10 adolescent mathematics learners, should consider the following questions:

- Do students have a role in determining classroom rules and procedures?
- Do students feel safe to take risks and participate during mathematics learning?
- Do students have opportunities to move around and engage in situations kinesthetically?
- Are a variety of groupings used, for particular purposes?
- Do students have opportunities to discuss and investigate different ways of thinking about and doing mathematics?
- Do tasks have multiple entry points to accommodate a range of thinkers in the concrete-abstract continuum?

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## Classroom Management

Classroom management can best be viewed as both an active and proactive phenomenon. Teachers have to make decisions on a daily basis concerning their teaching, learning and assessment processes, as well as the mathematical content they will stress. These and other important teacher choices directly determine student behaviour.

### Important aspects of classroom management

Successful teachers establish a community of learners within a positive learning environment by: gaining the respect and trust of students, establishing clear and consistent routines, developing meaningful relationships with each student, understanding and making adjustments to meet the variety of student needs, and by selecting a full range of materials and strategies to teach mathematics.

Physical settings often convey to students whether their interactions are welcomed or if the teacher is the authority in transmitting the knowledge students are expected to learn. It is the teacher's attitude towards learning, more than the physical setting, which is likely to foster a community of learners. Teachers who want to empower students encourage them to take risks, and appreciate the value of their students *not knowing*, using these occasions as opportunities for growth rather than anxiety. Students are expected to make conjectures, justify their thinking, and respond to alternative perspectives respectfully, rather than memorize arbitrary rules that might not make sense to them.

Teachers present worthwhile tasks that challenge students and hold their interest. These tasks can often be solved in more than one way, and contain flexible entry and exit points, making them suitable for a wide range of students. Teachers who know their students' abilities and attitudes towards mathematics are likely to challenge them and pique their interest. They know what questions to ask and the appropriate time to ask them, when to remain silent, and when to encourage students to meet the challenge without becoming frustrated.

### Considerations regarding classroom management

Kaplan, Gheen and Midgley (2002) conducted research with Grade 9 students and found that the emphasis on mastery and performance goals in the classroom is significantly related to students' patterns of learning and behaviour. Their conclusions are as follows:

This study joins many others (see Urdan, 1997) that have pointed to the benefits of constructing learning environments in which school is thought of as a place where learning, understanding, improvement, and personal and social development are valued and in which social comparison of students' ability is deemphasized. (p. 206)

A balanced and active approach to teaching and learning allows students to become engaged in mathematics and to learn cooperative and self-management skills (Ares & Gorrell, 2002; Boyer, 2002; Brand, Dunn & Greb, 2002). Each lesson in TIPS4RM includes a variety of student groupings and instructional strategies, depending on the learning task. Lesson headers (**Minds On, Action!, Consolidate/Debrief**) are reminders to plan active lessons that appeal to auditory, kinesthetic and visual learners, thereby, having a positive impact on classroom management.

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## Communication

Communication is the process of expressing ideas and mathematical understanding using numbers, pictures, and words, within a variety of audiences including the teacher, a peer, a group, or the class.

### Important aspects of communication

The NCTM *Principles and Standards* (2000) highlights the importance of communication as an “essential part of mathematics and mathematical education” (p. 60). It is through communication that “ideas become objects of reflection, refinement, discussion and amendment” and it is this process that “helps build meaning and permanence for ideas and makes them public” (p. 60). *The Ontario Curriculum* (OME, 2005) also emphasizes the significance of communication in mathematics, describing it as a priority of both the elementary school and the secondary school programs.

Craven (2000) advocates for a strong emphasis on mathematical communication, providing ideas for the recording and sharing of students’ learning (e.g., journal entry/learning log, report, poster, letter, story, three-dimensional model, sketch/drawing with explanation, oral presentation). He concluded by stating:

Children must feel free to explore, talk, create, and write about mathematics in a classroom environment that honours the beauty and importance of the subject. Children must be empowered to take risks and encouraged to explain their thinking. Teachers must construct tasks that will generate discussion and provide an opportunity for students to explain their understanding of mathematical concepts through pictures, words, and numbers. (p. 27)

Whitin and Whitin (2002) conducted research by examining a fourth-grade class under the instruction of a progressive classroom teacher. The authors described how *talking* helped students explore, express their observations, describe patterns, work through difficult concepts, and propose theories; and how *drawing* and *writing* assisted students in recording and clarifying their own thinking. They concluded by noting that communication was “...enhanced, and the children’s understanding developed, when mathematical ideas were represented in different ways, such as through a story, with manipulatives and charts, and through personal metaphors.... In these ways, children can become proficient and articulate in communicating mathematical ideas.” (p. 211)

### Considerations regarding communication

Franks and Jarvis (2001) maintained that new forms of communication, although potentially liberating and motivating, can initially be difficult and uncomfortable for teachers and students to explore. However, they note that both of these groups, given sustained support, had rewarding experiences when asked to become playful yet thoughtful risk-takers. (p. 66)

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## Concrete Materials

Concrete materials (manipulatives) are physical, three-dimensional objects that can be manipulated by students to increase the likelihood of their understanding mathematical concepts.

### Strengths underlying the use of concrete materials

Concrete materials have been widely used in mathematics education, particularly in the K-6 elementary level, but also in the middle school years (see Toliver video series). The results of TIMSS (1996) showed that Grade 8 students from Ontario did less well in the areas of algebra and measurement than other areas. Do Grade 8 students make connections between abstract algebraic concepts and their more *concrete*, previously learned, arithmetic experiences? Chappell and Strutchens (2001) noted that:

Too many adolescents encounter serious challenges as they delve into fundamental ideas that make up this essential mathematical subject [algebra]. Instead of viewing algebra as a natural extension of their arithmetic experiences, significant numbers of adolescents do not connect algebraic concepts with previously learned ideas. (p. 21)

In response to this situation, they recommended the use of concrete models, such as algebra tiles, to assist students in making connections and to facilitate mathematical understanding. Ross and Kurtz (1993) provided the following recommendations for teachers planning to implement a lesson involving manipulatives: (i) choose manipulatives that support the lesson's objectives; (ii) ensure significant plans have been made to orient students to the manipulatives and corresponding classroom procedures; (iii) facilitate the active participation of each student; and (iv) include procedures for evaluation that reflect an emphasis on the development of reasoning and processing skills, (e.g., listening to students talking about mathematics, reading their writings about mathematics, and observing them at work on mathematics. (pp. 256–257)

### Considerations regarding concrete materials

Thompson (1994) maintained that professional development should model selective and reflective use of concrete materials, helping teachers to focus on “what students will come to understand” as a result of using manipulatives, rather than just on “what students will learn to do” (p. 557). Similarly, Moyer (2001) noted that manipulatives can be used in a rote manner with little or no learning of the mathematics, and that the effective use of concrete materials relies heavily on a teacher's background knowledge and understanding of mathematical representations. Other researchers have warned that manipulatives should not be over-used (Ambrose, 2002); should not be treated as a fun reward activity or trivialized through teachers' comments (Moyer, 2001); but, should be used effectively and selectively as a means of facilitating the ongoing transitions from the concrete (physical and visual) information involved in student learning to the more abstract knowledge of mental relationships and deep understandings. (Kamii, Lewis, & Kirkland, 2001)

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## Differentiated Instruction

Differentiated instruction is based on the idea that because students differ significantly in their interests, learning styles, abilities, and experiences, teaching strategies and pace should vary accordingly. See pages 37–48 for more detailed information.

### Strengths underlying differentiated instruction

The September 2000 issue of *Educational Leadership* focused almost exclusively on issues pertaining to differentiated instruction. Pettig (2000) noted that “differentiated instruction represents a proactive approach to improving classroom learning for all students,” and that it “requires from us [teachers] a persistent honing of our teaching skills plus the courage to significantly change our classroom practices.” (pp. 14, 18) Heuser (2000) presented math and science workshops, both teacher- and student-directed, as a viable strategy for facilitating differentiated instruction. He stated that the philosophy behind these workshops is founded on research and theory that support diverse learners’ understanding of math and science, and can be summarized as follows: (i) children learn best when they are actively involved in math and science and physically interact with their environment; (ii) children develop a deeper understanding of math and science when they are encouraged to construct their own knowledge; (iii) children benefit from choice, both as a motivator and as a mechanism to ensure that students are working at an optimal level of understanding and development; (iv) children need time and encouragement to reflect on and communicate their understanding; and (v) children need considerable and varying amounts of time and experiences to construct scientific and mathematical knowledge. (p. 35) Notwithstanding the fact that “teachers feel torn between an external impetus to cover the standards [curriculum] and a desire to address the diverse academic needs,” Tomlinson (2000) maintained:

There is no contradiction between effective standards-based instruction and differentiation. Curriculum tells us what to teach: Differentiation tells us how. Thus, if we elect to teach a standards-based curriculum, differentiation simply suggests ways in which we can make that curriculum work best for varied learners. In other words, differentiation can show us how to teach the same standard to a range of learners by employing a variety of teaching and learning modes. (pp. 8, 9)

### Considerations regarding differentiated instruction

Within differentiated instruction, the successful inclusion of all types of learners is facilitated (Winebrenner, 2000). Karp and Voltz (2000) summarized differentiated instruction in the following way:

As teachers learn and practice various teaching strategies, they expand the possibilities for weaving rich, authentic lessons that are responsive to all students’ needs... the adherence to a single approach will create an instructional situation that will leave some students unravelled and on the fringe. (p. 212)

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## Flexible Grouping

Flexible grouping refers to the practice of varying grouping strategies based on short-term learning goals that are shared with the students, then regrouping once goals are met. Groupings often include individual, partners, student- and teacher-led small groups, and whole-class configurations.

### Strengths underlying flexible grouping

Linchevski and Kutscher (1998) suggest that during whole class discussions, the teacher could: develop conceptions about what mathematics is; create an appropriate learning environment and foster essential norms of classroom behaviour; legitimize errors as part of the learning process; and allow expressions of ambiguity. “These discussions also allow weaker students to participate, albeit many times passively via “legitimate peripheral participation” (Lave & Wenger, 1991) and “cognitive apprenticeship” (Brown, Collins, & Duguid, 1989), in a challenging intellectual atmosphere.”

*The Early Math Strategy* (2003) suggests that a reason for independent mathematics, as well as shared and guided mathematics, is that “children demonstrate their understanding, practise a skill, or consolidate learning in a developmentally appropriate manner through independent work...Students need time to consolidate ideas for and by themselves.” (p. 37)

Students need opportunities to learn from each other in small groups and pairs, to try ideas, practise new vocabulary, and construct their own mathematical understanding with others. Large homogeneous groups can engage in differentiated tasks including enrichment topics, remediation, and filling gaps for groups of students who were absent when a concept was taught.

### Considerations regarding flexible grouping

In reporting the results of three studies that focused on the effects of teaching mathematics in a mixed-ability setting on students’ achievements and teachers’ attitudes, researchers Linchevski and Kutscher (1998) concluded that “it is possible for students of all ability levels to learn mathematics effectively in a heterogeneous class, to the satisfaction of the teacher.” (p. 59) To examine high-achieving students’ interactions and performances on complex mathematics tasks as a function of homogeneous versus heterogeneous pairings, Fuchs, Fuchs, et al. (1998) videotaped ten high achievers working with a high-achieving and a low-achieving classmate on performance assessments. Based on their findings, they recommended that “high achievers, when working on complex material, should have ample opportunity to work with fellow high achievers so that collaborative thinking as well as cognitive conflict and resolution can occur.” (p. 251) However, they also noted that heterogeneous groupings can prove valuable when “used appropriately with less complex tasks, by providing maximal opportunities for high achievers to construct and low achievers to profit from well-reasoned explanations.” (p. 251) Flexible grouping has much to offer, yet also demands careful planning.

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## Graphic Organizers

Graphic organizers are visual representations, models, or illustrations that depict relationships among the key concepts involved in a lesson, unit, or learning task.

### Strengths underlying the use of graphic organizers

According to Braselton and Decker (1994), mathematics is the most difficult content area material to read because “there are more concepts per word, per sentence, and per paragraph than in any other subject” and because “the mixture of words, numerals, letters, symbols, and graphics require the reader to shift from one type of vocabulary to another.” (p. 276) They concluded by stating, “One strategy that is effective in improving content area reading comprehension is the use of graphic organizers. (Clarke, 1991; Flood, Lapp, & Farnan, 1986; Piccolo, 1987)” (p. 276) Similarly, DiCecco and Gleason (2002) noted that many students with learning disabilities struggle to learn in content area classes, particularly when reading expository text. Since, in their opinion, content textbooks often do not make important connections/relationships adequately explicit for these students, the authors recommended the use of graphic organizers to fill this perceived gap.

Graphic organizers are one method that might achieve what textbooks fail to do. ...They include labels that link concepts in order to highlight relationships (Novak & Gowin, 1984). Once these relationships are understood by a learner, that understanding can be referred to as relational knowledge. ...Logically, if the source of relational knowledge is structured and organized, it will be more accessible to the learner (Ausubel, 1968). (p. 306)

Monroe and Orme (2002) presented two general methods for teaching mathematical vocabulary: meaningful context and direct teaching. Described as a powerful example of the latter method, the use of graphic organizers was further highlighted as being “closely aligned with current theory about how the brain organizes information” and as one of the more promising approaches regarding the recall of background knowledge of mathematical concepts. (p. 141)

### Considerations regarding the use of graphic organizers

Monroe and Orme note that the effectiveness of a graphic organizer is limited by its dependence on background knowledge of a concept (Dunston, 1992). As an example, they explain that “if students have not encountered the concept of rhombus, a graphic organizer for the word will not help them to develop meaning.” (p. 141) Also noteworthy on this topic is the fact that many software programs and websites are now being developed which provide students with digital versions of graphic organizers and *virtual manipulatives*, e.g., interactive animations of 3-dimensional, mathematical learning objects, an increasing number of which are becoming available free of charge for educators and parents.

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## Japanese Lesson Structure

A typical Japanese mathematics lesson features the following components: (i) the posing of a complex, thought-provoking problem to the class; (ii) individual and/or small group generation of possible approaches for solving the problem; (iii) the communication of strategies and methods by various students to the class; (iv) classroom discussion and collaborative development of the mathematical concepts/understandings; (v) summary and clarification of the findings by the teacher; and (vi) consolidation of understanding through the practice of similar and/or more complex problems.

### Strengths underlying the Japanese lesson structure

Since the release of the Third International Math and Science Study (TIMSS) results in 1996, nations have been devoting a great deal of time, resources, and research to the process of unravelling the diverse findings, and exploring various methods for national mathematical and scientific reform. Since Japanese mathematics students outperformed their global peers, attention has focused on the methods of instruction and classroom management that have been adopted by Japanese educators. Lessons in Japanese classrooms were found to be remarkably different from those in Germany and the U.S., promoting students' understanding, while U.S. and German teachers seemed to focus more exclusively on the development of skills. (Martinez, 2001; Roulet, 2000; Stigler & Hiebert, 1997)

The considerable time spent in Japanese classrooms on inventing new solutions, engaging in conceptual thinking about mathematics, and communicating ideas has apparently paid rich dividends in terms of students' understanding and achievement. It is somewhat ironic to note that in light of the fact that TIMSS has been criticized as being, overly skill-based as opposed to featuring more problem-solving content, Japanese students, who have experienced the types of reforms promoted by groups such as the National Council of Teachers of Mathematics, also outperformed the world on more *traditional* mathematics questions. It appears that the deeper understandings cultivated through these methodologies are not at the expense of technical prowess; rather this form of instruction seems to strengthen both procedural and conceptual knowledge.

### Considerations regarding Japanese lesson planning

Japanese teachers regularly take part in lesson study and inquiry groups, producing gradual but continual improvement in teaching. (Stigler & Hiebert, 1999) Watanabe (2002) noted that "a wide range of activities characterizes this kind of professional development, offering teachers opportunities to examine all aspects of their teaching – curriculum, lesson plans, instructional materials, and content." (p. 36) He further recommended that in order to learn from Japanese lesson study, North American teachers should: (i) develop a culture through regular and collective participation; (ii) develop the habit of writing an instruction plan for others; (iii) develop a unit perspective; (iv) anticipate students' thinking; and (v) learn to observe lessons well.

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## Mental Mathematics and Alternative Algorithms

People who are mathematically literate can generally compose (put together) and decompose (take apart) numbers in a variety of ways. Having flexible thinking skills empowers students and frees them to invent strategies that make sense to them.

Example:

Students who use the traditional subtraction algorithm for  $6000 - 1$ , often get the answer wrong, and generally don't have a good sense of number or a positive attitude towards mathematics.

When asked to multiply 24 by 25 many people who are comfortable playing with numbers use strategies such as divide 24 by 4 (recognizing 4 quarters make one dollar and so 24 quarters equals \$6 or 600 cents); or add  $240 + 240 + 120$  (decomposing:  $24 \times 10 + 24 \times 10 + 24 \times 5$ ); or  $625 - 25$  (they know  $25 \times 25$  then subtract 25).

### Strengths underlying mental mathematics and alternative algorithms

Research projects have shown that young people who invent their own alternative algorithms to solve computation questions generally have a firm sense of number and place value (Kamii & Dominick, 1997; Van de Walle, 2001). When students invent their own strategies for solving computation questions, the strategies tend to be developed from knowledge and understanding rather than the rote memorization of the teacher's method. As Kamii and Dominick (1997) explain, knowledge is developed from within and young people can then trust their own powers of reasoning. Kamii and others have shown that students who invent their own algorithms tend to do as well as other students on standardized computation tests, and also have a far better understanding of what is happening mathematically.

Many computational errors and misconceptions are based on students' misunderstandings of traditional algorithms. When subtracting 48 from 72 they often write 36, subtracting the smaller digits from the larger ones. When asked why zeros are included when using the traditional algorithm to multiply 25 by 33, few students can explain that they multiplying 25 by 3 and 25 by 30. They have an even more difficult time explaining why these two sums are added together. Misconceptions are often the result of following procedures without understanding. Alternative algorithms encourage students to make sense of what they are doing rather than accepting rules based on faith.

### Considerations regarding alternative algorithms

Many mathematics educators believe that students should be taught traditional algorithms after or along with the opportunity to invent their own algorithms. Traditional algorithms are an essential part of mathematics learning and should be taught, but only after students have developed understanding of the concept and shared their own approaches to the problem.

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## Metacognition

Metacognition is the awareness and understanding of one's own thought processes; in terms of mathematics education, the ability to apply, evaluate, justify, and adjust one's thinking strategies.

### Important aspects of metacognition

Metacognition and its implications for mathematics education are described in the National Council of Teachers of Mathematics *Principles and Standards* (2000) document:

Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al., 1999). Such reflective skills (called *metacognition*) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as “Before we go on, are we sure we understand this?” “What are our options?” “Do we have a plan?” “Are we making progress or should we reconsider what we are doing?” “Why do we think this is true?” Such questions help students get in the habit of checking their understanding as they go along. (p. 54)

Kramarski, Mevarech and Lieberman (2001) noted that “For more than a decade, metacognition researchers have sought instructional methods that use metacognitive processes to enhance mathematical reasoning.” (p. 292) For example, Pugalee (2001) described how metacognition has been shown to complement problem solving, enhancing decisions and strategies such as predicting, planning, revising, selecting, checking, guessing, and classifying. (p. 237)

### Considerations regarding metacognition

Although many students are able to discover thinking and problem-solving strategies independently or detect unannounced strategies that they see others use, some students who have “metacognitive deficits may not even be aware that others are using strategies to successfully complete the task at hand.” (Allsopp et al., 2003, p. 310) Moreover, these deficits “become more evident as students are expected to apply strategies they have learned to new situations, concepts, or skills.” (p. 310) In light of this reality, and the fact that young people typically like to talk about their thinking in the classroom, researchers Stright and Supplee (2002) recommend that small-group instructional contexts be used often to encourage *math talk* and to facilitate the sharing of varied models of thinking among students. (p. 237) Part of consolidation and debriefing should provide opportunities for students to reflect on the lesson, through writing in journals and/or talking in small groups, or whole-class discussions.

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## Providing Feedback

Providing feedback involves the comprehensive and consistent communication of information regarding a student's progress in mathematics learning, to both the student and the parent.

### Important aspects of providing feedback

Mathematics educators are responsible for not only preparing and presenting quality instruction, but also for developing and implementing effective methods of assessment and reporting. Providing regular formative and summative feedback to both students and parents is of importance to help students adjust to the demands of the mathematics program. There is perhaps nothing worse than arriving at the end of a term or semester only to find that one's marks, or one's child's marks, in mathematics are dismally below average, yet without any prior feedback being received.

One difficult area for teachers is to know how much assistance to offer students on a particular mathematics problem or task, without giving away too much information. Chatterley and Peck (1995), in their article entitled, "We're Crippling our Kids with Kindness!", noted the following:

The key is experience. We cannot, even though with the most kindly of intentions, exclude students from those experiences that come from struggling with a problem. . . . We cannot cripple our students mentally by taking away from them the struggles that must come before understanding is brought to fruition. If we understand the process necessary to provide the referents within the minds of our students, we will cease to mentally cripple them by being overly kind and sympathetic and by helping too much and often far too soon. (pp. 435–436)

Kewley (1998) emphasized the importance of this notion of *cognitive dissonance*, or psychological discomfort, by referring to the work of several key educational theorists:

Both Piaget (Ginsburg & Opper, 1979) and Vygotsky (Wertsch & Stone, 1985) believed that disequilibrium was a process necessary to learning, because if everything goes according to plan, nothing rises to the level of consciousness. Disequilibrium, or cognitive imbalance, is a state that occurs when the learner is unable to assimilate an experience or achieve a goal. It motivates the student's search for better knowledge and a valid solution. (pp. 30–31)

Assistance during the lesson or task must be meaningful and encouraging, but not overstated; teachers must carefully provide students with enough feedback to help them move forward by themselves (see Scaffolding).

### Considerations regarding providing feedback

Both the NCTM *Principles and Standards* (2000) and *The Ontario Curriculum* (2005) highlight the importance of parental involvement in mathematics education. Because teachers, students, and parents/guardians are considered partners in schooling, the provision of consistent feedback regarding student progress becomes a vital link connecting all members of this team. As Onslow (1992) noted:

If we want parents to understand mathematics as more than the procedural drills of arithmetic then they must be provided with opportunities to explore their children's mathematics programme. In this way, parents will be in a position to understand not only what we are teaching but why we are teaching the way we do. (p. 25)

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## Questioning

Teacher questioning and teacher listening are closely linked skills that are employed daily in the mathematics classroom to guide both teaching and learning, facilitate participation, and to stimulate higher-order thought.

### Important aspects of questioning

Nicol (1999) conducted research regarding questioning with pre-service teachers. She found that prospective teachers seemed to “struggle with not only how they might ask students questions but also what they might ask and for what purpose.” (p. 53) She concluded that “As prospective teachers... began to consider students’ thinking and create spaces for inquiry through the kinds of questions posed, they began to see and hear possibilities for mathematical exploration that evolved as their relationship with mathematics and with students changed.” (p. 62) Defining a higher order question as a query that asks students to respond at a higher level than factual knowledge, Wimer et al. (2001) conducted research surrounding the *higher order* questioning of boys and girls in elementary mathematics classrooms. They noted in summary that “higher level and lower level questions have their place in the classroom; vigilant teachers employ both types when the need arises.” (p. 91)

After observing the low levels of achievement of many of his middle school students, Reinhart (2000) decided to implement changes in his teaching methods. He noted, “It was not enough to teach better mathematics; I also had to teach mathematics better. Making changes in instruction proved difficult because I had to learn to teach in ways that I had never observed or experienced, challenging many of the old teaching paradigms.” (p. 478) Understanding his students proved helpful in this process:

Getting middle school students to explain their thinking and become actively involved in classroom discussions can be a challenge. By nature, these students are self-conscious and insecure. This insecurity and the effects of negative peer pressure tend to discourage involvement. To get beyond these and other roadblocks, I have learned to ask the best possible questions and to apply strategies that require all students to participate. (pp. 478–79)

Reinhart offered five suggestions for implementing these positive, yet difficult changes: (i) never say anything a kid can say; (ii) ask good questions (i.e., that require more than recalling a fact or reproducing a skill) the best questions are open-ended; (iii) use more process questions (i.e., that require the student to reflect, analyse, and explain his/her thinking and reasoning) than product questions (i.e., that require short answers, yes/no responses, or rely almost completely on memory); (iv) replace lectures with sets of questions; and (v) be patient (i.e., wait time is very important; increasing it to five seconds or longer results in better responses). (p. 480)

### Considerations regarding questioning

Although the Socratic method of questioning and answering has existed as a longstanding and effective mathematics teaching strategy, it is best used as one method among several. Open-ended questioning, some of it creating disequilibrium, and informal classroom discussions regarding mathematical thinking and processes are valuable.

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## Scaffolding

Scaffolding is a metaphor that is used as a framework to describe how teachers can guide students through a learning task. Scaffolds may be tools, such as cue cards, analogies, and models; or techniques, such as teacher modeling, prompting, or thinking aloud.

### Strengths underlying scaffolding

Research has shown that scaffolding is most useful in situations where students are at a loss as to what they should do, but can proceed, and accomplish a task that is just outside their level of competency, with assistance. The successful use of scaffolding requires the educator to determine the background knowledge of each student, and to develop a comfortable, working rapport with each student. One research study in which a mathematics teacher used a “thinking aloud” strategy as a form of scaffolding is described in the following vignette:

In a mathematics study by Schoenfeld (1985), the teacher thought aloud as he went through the steps in solving procedures he was using (for example, making diagrams, breaking the problem into parts). Thus, as Schoenfeld points out, thinking aloud may also provide labels that students can use to call up the same processes in their own thinking. ...Through modeling and thinking aloud, he applied problem-solving procedures and revealed his reasoning about the problems he encountered. Students saw the flexibility of the strategies as they were applied to a range of problems and observed that the use of a strategy did not guarantee success. ...As individual students accepted more responsibility in the completion of a task, they often modeled and thought aloud for their less capable classmates. Not only did student modeling and think-alouds involve the students actively in the process, but it allowed the teacher to better assess student progress in the use of the strategy. Thinking aloud by the teacher and more capable students provided novice learners with a way to observe “expert thinking” usually hidden from the student. (Rosenshine & Meister, 1992, p. 28)

Although scaffolding was developed as an educational theory particularly in response to the specific needs of students with learning disabilities (Stone, 1998), it has more recently seen application in both general and adult education, as an affective strategy for all learners (Graves et al., 1996). Furthermore, this strategy has also been found effective in teaching higher-order cognitive skills. Rosenshine and Meister (1992) described six components that they believe comprise the successful teaching of higher-order cognitive skills via scaffolding: (i) presenting a new cognitive strategy; (ii) regulating difficulty during guided practice; (iii) varying the context for practice; (iv) providing feedback; (v) increasing student responsibility; and (vi) providing student responsibility. (pp. 26–32)

### Considerations regarding scaffolding

Stone (1998) documented some cautionary notes. Teachers should be aware that scaffolding is meant to be a temporary, as opposed to long-term strategy, and that scaffolding requires adaptation for individual learners. (pp. 349–350) Scaffolding should not focus exclusively on teacher-directed methods that provide more information than is necessary.

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## Student-Centred Investigations

Student-centred investigations, referred to as rich investigations or rich tasks, are learning contexts which require students to explore mathematics through inquiry, discovery, and research.

### Strengths underlying student-centred investigations

Chapin (1998) defined a mathematical investigation as a “multidimensional exploration of a meaningful topic, the goal of which is to discover new ways of thinking about the mathematics inherent in the situation rather than to discover particular answers.” (p. 333) She elaborated on their significance:

Investigations afford students an opportunity for sustained, in-depth study of a topic; students must make sense of their observations and synthesize and analyze their conclusions. ...In addition, connections among different areas of mathematics (e.g., algebra, geometry, statistics) can be made, since students tend to bump into related ideas and concepts while pursuing the investigative questions. The “bumping” phenomenon enables a teacher to pursue related topics in parallel with the mathematical investigation – presenting a context for connecting ideas, for clarifying concepts, or for teaching new material. Finally, mathematical investigations can assist in establishing a classroom environment that supports inquiry. Students are expected to explore questions and engage in relevant discourse. By working slowly through many levels of questions and responses, students begin to experience the importance of careful reasoning and disciplined understanding. (pp. 333–34)

Flewelling and Higginson (2000) have developed similar ideas in their work regarding rich tasks.

Creativity is presented as a salient feature of these explorations, in contrast to many traditional tasks which “ask students to follow given recipes to expected end-points, giving students little opportunity to consider alternatives and be creative.” (p. 18) Rich learning tasks are designed in such a way that “different students are able to demonstrate (very) different kinds and levels of performance.”

(p. 18) Researchers Ares and Gorrell (2002) interviewed teachers and students from five middle schools to gain insight into perceptions surrounding meaningful learning experiences. They reported that: “The overriding message from students is that active learning, rather than passive listening, reading, and note-taking, draws them into subjects and deepens their understanding and appreciation of what they are learning.” (p. 270) Based on their study, they concluded that, “...the assumption underlying management techniques should be that students and teachers value the same things: productive interactions centred on substantive learning.” (p. 275)

### Considerations regarding student-centred investigations

Some educators advocate the use of student-centred investigations throughout the entire curriculum, introducing *traditional* mathematics skills, when appropriate, to support the rich task explorations. Others view investigations as one successful strategy among many, to be used several times throughout the term, semester, or course. Teachers new to the idea are encouraged to plan and try one investigation of interest, reflect on and discuss the task with colleagues, and then make changes to the investigation before using it again, based on this professional dialogue.

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## Teacher-Directed Instruction

Teacher-directed instruction involves teaching rules, concepts, principles, and problem-solving strategies in an explicit fashion, providing a wide range of examples with extensive review and practice.

### Strengths underlying teacher-directed instruction

According to Baker, Gersten, and Lee (2002), who synthesized empirical research on teaching mathematics to low-achieving students, “research suggests that principles of direct or explicit instruction can be useful in teaching mathematical concepts and procedures.” (p. 68) Monroe and Orme (2002), focusing on the development of mathematical vocabulary, stated the following:

Direct teaching of selected vocabulary has been advocated for many years (Gray & Holmes, 1938, cited in Chall, 1987; Moore, Readence, & Rickelman, 1989) and is supported by Vacca and Vacca (1996) and Klein (1988). Vacca and Vacca (1996) assert that the most important vocabulary words “need to be taught directly and taught well.” (p. 136) Klein expresses the idea that direct teaching of vocabulary will guide students to assign deeper meaning to words. (p. 140)

The authors also noted the existence of poor and ineffectual methods of teaching vocabulary via direct-instruction, such as the “definition-only” approach. Therefore, they advocated a balanced approach for teaching mathematical vocabulary “that combines meaningful context and direct teaching through the use of a graphic organizer.” (p. 141) Wilson, Majsterek, and Simmons (1996) compared the effects of computer-assisted versus teacher-directed instruction on the multiplication performance of elementary students with learning disabilities. Although their findings suggested that for these students teacher-directed procedures were the more efficient and effective method of achieving basic fact mastery, they also recommended a balanced and context-sensitive approach, involving both teacher-directed and computer-assisted instruction. (p. 389) Whereas the results of Stright and Supplee’s (2002) research suggested that students are more self-regulated learners in small group and seat work settings, the authors also concluded by stating, “In order for children to truly become self-regulated learners, the classroom should include all three contexts [teacher-directed, individual seat work, and group work] to provide direct instruction, independent practice, and the opportunity to practice metacognitive skills in a social context (Slavin, 1987).” (p. 242)

### Considerations regarding teacher-directed instruction

Kewley (1998) pointed out that, although research indicates that teacher-directed instruction promotes learning, the method also has inherent problems. For example, the “superior adult mentality tends to dominate the proceedings, suppressing the reciprocity of ideas and their coordination. The children are then less stimulated to clarify their own ideas, an essential element in gaining understanding and eventually taking ownership of a concept.” (p. 31) The author asserted that “by varying instructional methods there is a greater possibility that teachers will meet the needs of all students, since some may learn better through one set of mechanisms coming into play than another.” (p. 31)

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## Technology

Technology, a term derived from the Greek *teknologia* meaning “systematic treatment,” encompasses both a wide range of products, e.g., calculators, software, hardware, and related systematic processes.

### Strengths underlying the use of technology

Gilliland (2002) noted that although machines do not think, calculators can “take the drudgery out of computation by performing low-level tasks in mathematics.” (p. 50) In referring to related research, she further explained:

Students who learn paper-and-pencil techniques in conjunction with the use of four-function calculators or other technology, and are tested without calculators, perform as well as, or better than, those who do not use technology in class. Appropriate use of calculators does not result in the atrophy of computational skills; instead, it provides an impetus and opportunity for students to focus on conceptual learning (Heid, 1997). (p. 50)

Despite this type of research, only 18% of Grade 3 students and fewer than 10% of Grade 6 students use calculators on a regular basis in Ontario schools (EQAO, *Report of Provincial Results*, 2002, pp. 30, 39).

Being aware of the increased emphasis on mathematics problem solving and the poor performance of students with learning disabilities, Babbitt and Miller (1996) documented and explored appropriate methods for teaching these skills with greater effectiveness and efficiency. (p. 392) The researchers recommended the use of computers, and specifically hypermedia, i.e., digital environments that go beyond text and traditional computer-assisted instruction to incorporate sound, animation, photographic images, and video clips in sophisticated ways, to teach mathematical problem solving to students with learning disabilities. (pp. 393–395)

### Considerations regarding the use of technology

The advent of increased technology in the mathematics classroom does not guarantee improved teaching or increased learning. Although complex in nature, and possessing expansive possibilities, technological products such as calculators and software packages are only as effective as the teacher using them. Among the many considerations regarding the implementation of technology that face administrators and teachers in the 21st Century, the two that are of perhaps greatest import are resources and training. Extended and innovative teacher training programs (Woolley, 1998) are most desirable. Dedicated teachers should be encouraged to continue exploring these strategies and tools, keeping in mind the reality that often the students themselves are a rich resource when it comes to technology in the classroom.

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## van Hiele Model

The van Hiele level approach is based on the 1950's work of a Dutch couple, Dina van Hiele-Geldof and Pierre van Hiele, who developed a model that described five distinctive geometry learning levels: visualization, analysis, informal deduction, formal deduction, and rigor. Being neither age-dependent nor content-based, the van Hiele levels describe how students think about geometry and how this thinking changes over time as students become increasingly competent.

### Strengths underlying the use of the van Hiele model

Van de Walle (2001) described the five levels in the following way:

**Visualization:** Students recognize and name figures based on the global, visual characteristics of the figure and are able to make measurements and talk about simple properties of shapes.

**Analysis:** Students are able to consider all shapes within a class rather than a single shape, but are unable to understand the intricacies of categorical definitions.

**Informal Deduction:** As students begin to be able to think about properties of geometric objects without the constraints of a particular object, they are able to develop relationships between and among these properties.

**Formal Deduction:** Students are able to examine more than just the properties of a shape, recognizing the significance of an axiomatic system and able to construct geometric proofs.

**Rigor:** Students become increasingly capable of understanding a complex system complete with axioms, definitions, theorems, corollaries, and postulates. (pp. 309–310)

Because the van Hiele approach assumes that students progress through the five levels sequentially, and that lessons must be delivered at a level that matches this progress, the necessity for teachers to develop a clear picture of prior knowledge and ongoing student understanding is underscored. According to Malloy (1999): “By the time that students enter the middle grades, most of them are between the concrete [first and second] and informal deduction [third] levels defined by the van Hieles. (p. 87) However, teachers often talk about geometry using third or fourth level language that students cannot understand, leading to a mismatch in teaching and learning. An awareness of this disjuncture allows teachers to adjust their presentation of geometric concepts in order to appropriately engage students.

### Considerations regarding the use of the van Hiele model

To illustrate how educators can help students progress from one van Hiele level to another, Malloy (1999) provided the following recommendations:

The van Hiele model suggests using five phases of instruction to help students in this progression. Students first gather information by working with examples (e.g., finding the perimeter of shapes), then they complete tasks that are related to the information, such as adding tiles to the figure to increase perimeter. The students become aware of relationships and are able to explain them. Finally, students are challenged to move to more complex tasks and to summarize and reflect on what they have learned. The language used by teachers and students is important for students' progression through the levels from concrete to visual to abstract (Fuys, Geddes, and Tischler, 1988. p. 89)

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## Differentiated Instruction

(see page 23)

Within an era of standards-based, international mathematics education reform *and* standardized provincial assessment, differentiated instruction represents a promising teaching method that may facilitate both high levels of student *engagement* and curricular *achievement*. Carol Ann Tomlinson, a leading proponent of differentiated instruction, maintains that these goals are indeed consistent:

There is no contradiction between effective standards-based instruction and differentiation. Curriculum tells us *what* to teach: Differentiation tells us *how*. Thus, if we elect to teach a standards-based curriculum, differentiation simply suggests ways in which we can make that curriculum work best for varied learners. In other words, differentiation can show us how to teach the same standards to a range of learners by employing a variety of teaching and learning modes. (2000, pp. 8–9)

Differentiated instruction is based on the idea that because students differ significantly in their interests, learning styles, and readiness, teaching strategies and decisions involving issues of content, process, and product should vary accordingly (see Figure 1. Tomlinson’s Differentiation of Instruction Model, 1999, p. 3). These ideas are in keeping with the National Council of Teachers of Mathematics’ *Principles and Standards for School Mathematics* (2000), as explained by Pierce (2004):

The tenets of differentiated instruction support both the Equity Principle and the Teaching Principle of the Principles and Standards for School Mathematics (NCTM, 2000). These principles direct us to select and adapt content and curricula to meet the interests, abilities, and learning styles of our students; to recognize our students’ diversity; and to encourage them to reach their full potential in mathematics. (p. 60)

This concept echoes our own provincial curriculum mathematics documents:

Teachers are responsible for developing appropriate instructional strategies to help students achieve the curriculum expectations, and for developing appropriate methods for assessing and evaluating student learning. Teachers also support students in developing the reading, writing, and oral communication skills needed for success in learning mathematics. Teachers bring enthusiasm and varied teaching and assessment approaches to the classroom, addressing different student needs and ensuring sound learning opportunities for every student. (Ontario Ministry of Education, *The Ontario Curriculum, Mathematics*, 2005)

A multi-strategy approach to mathematics education from Grades 7–12 has been highlighted and elaborated upon within the messages and examples of both the resource document *Targeted Implementation and Planning Strategies for Revised Mathematics (TIPS4RM): Grades 7, 8, 9 Applied, and 10 Applied* (Consortium of Ontario School Boards, 2005) and the recently-released expert panel report on student success, *Leading Math Success: Mathematical Literacy, Grades 7–12* (Ontario Ministry of Education, 2004).

### Differentiated instruction includes:

- Using a variety of groupings to meet student needs;
- Providing alternative instruction/assessment activities;
- Challenging students at an appropriate level, in light of their readiness, interests, and learning profiles.

### Differentiated instruction does not include:

- Doing something different for every student in the class;
- Disorderly and undisciplined student activity;
- Using groups that never change, or isolating struggling students within the class;
- Never engaging in whole-class activities with all students participating in the same endeavour.

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Much of the planning and preparation can be done before a teacher begins to teach a new class or course. Diagnostic and formative assessment tools, tiered assignment structures, flexible grouping strategies, etc., can all be carefully designed well in advance. This organization offers opportunities for teachers to focus on student learning and understanding during the early part of the course/year, and to effectively differentiate curriculum and activities. Differentiated instruction does not negate the need for activities in which all students are working on the same learning task at the same time, whether individually, in small groups, or as a whole class. However, within a differentiated framework, the teacher will frequently choose to assign different tasks to different students (individually or small groups) based on observations of the students in the class. For example, some students will require remediation activities while others may need extensions. With practice, the teacher can respond to such diverse needs on the spot.

### Low-Prep Examples

There are aspects of differentiated instruction that require minimal preparation by the teacher. The teacher has different students work on different tasks using materials that are already available or using strategies that are reasonably easy to implement (e.g., increasing ‘wait time’ when posing questions to the whole class). In many cases, it may be for just a few minutes that different students work on these different tasks.

### High-Prep Examples

Some aspects of differentiated instruction require significant preparation by the teacher—developing new instructional and assessment materials, and using strategies that depend on students’ having acquired specific social skills, e.g., Jigsaw. When teachers build up their repertoires and share resource materials, high-prep examples become lower-prep.

### Research and Literature Related to Differentiated Instruction

A review of the relevant research and literature regarding differentiated instruction reveals that while few empirical studies have yet been undertaken in this particular area, much is available for analysis by way of qualitative research and testimonial accounts of differentiated instruction in progress (Hall, 2003).

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## Differentiation of Instruction

is a teacher's response to learner's needs

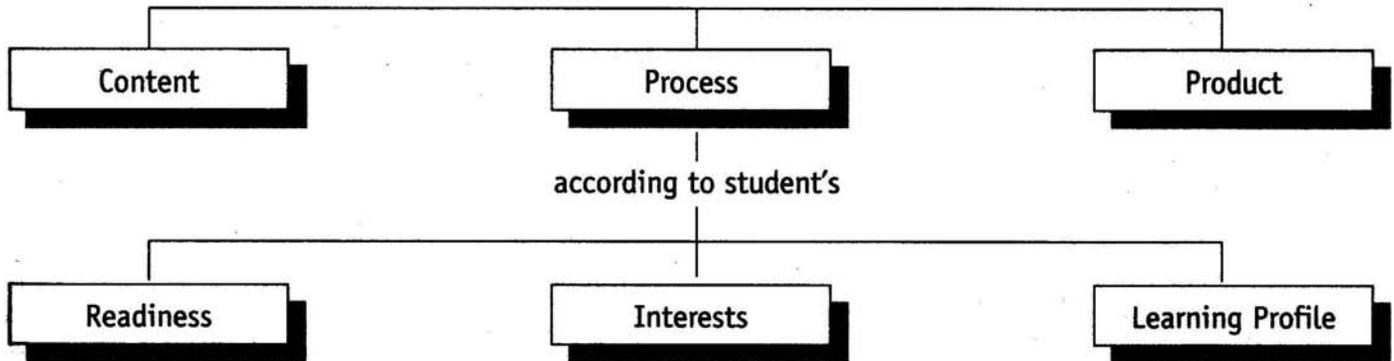
↓  
guided by general principles of differentiation,  
such as

respectful  
tasks

flexible grouping

ongoing assessment  
and adjustment

Teachers can differentiate



through a range of instructional and management strategies such as

<p>multiple intelligences jigsaw taped material anchor activities varying organizers varied texts varied supplementary materials literature circles</p>	<p>tiered lessons tiered centers tiered products learning contracts small-group instruction group investigation orbitals independent study</p>	<p>4MAT varied questioning strategies interest centers interest groups varied homework compacting varied journal prompts complex instruction</p>
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Figure 1. Tomlinson's Differentiation of Instruction Model (Tomlinson, 1999b, p. 15)

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## Differentiation Based on Student Readiness

Equality in education does not require that all students have exactly the same experiences. Rather, education in a democracy promises that everyone will have an equal opportunity to actualize their potential, to learn as much as they can. (Fiedler, 2002, p. 111)

Readiness is the current knowledge, understanding, and skill level a student has related to a particular sequence of learning. It is important to note that readiness is not merely a synonym for general ability level, but rather, “it reflects what a student knows, understands, and can do *today* in light of what the teacher is planning to teach *today*” (Tomlinson & Eidson, 2003, p. 3; emphasis added). Differentiating instruction based on student readiness involves knowing where particular students are on the learning continuum of important mathematics, then planning program features to move them along this continuum.

### Important aspects of differentiation based on student readiness

Struggling students have to overcome many conceptual obstacles and are often not at the conceptual developmental level that their age might suggest. To help these students move forward, the teacher needs to determine the particular conceptual stumbling block(s); search out strategies to help the students overcome the learning obstacles; and provide the needed materials, time, and support that will help the students develop conceptual understanding with practice.

Differentiated instruction draws upon the work of Russian psychologist and educational theorist Lev Vygotsky. One of the theories that Vygotsky (1930/1978) developed was the Zone of Proximal Development (ZPD)—the difference between the learner’s capacity to solve problems on his/her own, and his/her capacity to solve them with assistance. According to this theory, the teacher’s role is to provide appropriate instructional scaffolding and non-intrusive, relational support in order to maximize student achievement within his/her ZPD. In differentiated instruction, teachers scaffold and tailor learning episodes to individual students’ needs and understanding, while providing emotional support.

### Low-prep examples

Select two sets of home or follow-up consolidation questions from the students’ textbook where some questions are common to both sets: target one set of questions for students who think they need more time to develop the concept; target the other set for students who think they are ready to apply the concept in new contexts.

### High-prep examples

Provide all students, including the struggling and the gifted, with challenging and different assignments; ‘compact the curriculum’ for advanced learners; and develop and complete learning contracts.

### Considerations regarding differentiation based on student readiness

The recent resurgence of interest in differentiated instruction began with a focus on the needs of the gifted learner within a mixed-ability classroom (Fiedler, 2002; Winebrenner, 2000). However, it is significant to note that differentiation, when applied globally, reaps benefits for all students, including the gifted, low-achieving, special needs, and average, grade-level learners (Bender, 2002). Karp and Voltz note that special educators can be viewed as particularly valuable resources, in terms of their skills and experiences in weaving instructional strategies in inclusive settings (2000, p. 207).

Kapusnick and Hauslein explain how the brain requires appropriate levels of stimulation:

When a student experiences a learning situation, the brain responds with the release of the chemical noradrenaline. Students, who feel intimidated and rejected because their level of readiness is over-challenged, experience an overproduction of noradrenaline, causing the brain to be over-stimulated. Attention is diverted from learning and focused on self-protection, resulting in misbehaviour or withdrawal, with more time being spent on learning to cope rather than learning concepts. Conversely, if student readiness is beyond what is needed for a particular task, the brain is, quite literally, not engaged, releasing fewer neurochemicals. The advanced student often feels apathetic because his or her brain is under-stimulated (Tomlinson, 1999). Diverse learning styles, interests, and abilities act as filters for student experiences, while emotional safety, challenges, and self-constructed meaning determine how students make sense of information. (2001, p. 156)

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## Differentiation Based on Student Interest

To differentiate instruction is to recognize students' varying background knowledge, readiness, language, preferences in learning and interests, and to react responsively. (Hall, 2003, pp. 2–3)

Interest is that which a student enjoys learning about, thinking about, and doing. Differentiating instruction based on student interest involves the variation of themes, examples, and projects accordingly.

### Important aspects of differentiation based on student interest

Adolescents are undergoing tremendous personal changes during the middle years of schooling. Physical, emotional, and social changes make concentrating on classroom activities and taking responsibility for their own learning all the more difficult for the adolescent learner (*TIPS4RM*, Consortium of Ontario School Boards, 2005). In differentiating instruction according to students' interests, a teacher attempts to increase the likelihood that any given lesson or project is at once highly engaging and personally meaningful for each student in the class.

### Low-prep example

Use the names of students in the class when creating problems in contexts that appeal to these particular students; situate mathematics problems in contexts that will appeal to individual students in the class.

### High-prep example

Prepare multiple versions of assignments that allow students to have choice and pursue their own interests.

### Considerations regarding use of differentiation based on student interest

Brain research is one of the most exciting areas of educational research being developed now at the beginning of the 21st century. As we discover more about how the human brain functions, we will be better able to design and tailor curriculum to take full advantage of these insights. Tomlinson and Kalbfleisch note:

Brain research suggests three broad and interrelated principles that point clearly to the need for differentiated classrooms, that is, classrooms responsive to students' varying readiness levels, varying interests, and varying learning profiles: (i) Learning environments must feel emotionally safe for learning to take place; (ii) To learn, students must experience appropriate levels of challenge; and (iii) Each brain needs to make its own meaning of ideas and skills. (2001, pp. 53–54)

Safe learning environments, appropriate challenges, and meaning-making are indeed paramount if teachers are to be successful in using differentiated instruction to engage all learners.

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## Differentiation Based on a Student's Learning Profile

Individuals learn in different ways and at different rates; it is the major responsibility of schools to accommodate these differences to maximize each student's education. Rather than assuming that all students can and do learn in the same way and at the same rate, it is imperative for educators to acknowledge those differences. (Wang & Walberg, cited in Tieso, 2003, p. 34)

A learning profile is a student's preferred mode of learning. Individual learning profiles are influenced by learning style, intelligence preference, gender, and culture (Tomlinson & Eidson, 2003). Differentiating instruction based on a student's profile requires that teachers: be aware of the demographics of their school population and classroom; think of understanding in terms of the multiple ways in which individual students learn best; and ensure that classroom teaching and learning reflect this diversity in cognitive mechanics.

### Important aspects of differentiation based on student learning profile

Gardner's *Theory of Multiple Intelligences* (1993) emphasizes the unique learning capabilities and aptitudes of individual students. At its core is the validation of students' divergent talents and areas of expertise. Gardner has listed eight of these areas that he claims represent the complex workings of the human brain and how these complexities translate into thinking, learning, and doing in the classroom: verbal/linguistic; logical/mathematical; visual/spatial; musical/rhythmic; bodily/kinesthetic; interpersonal; intrapersonal; and naturalist. Learning styles are also important to consider when differentiating instruction—Do students prefer auditory, visual, tactile, or kinesthetic modes of internalizing, processing, and communicating information?

Although this standard list of learning styles is perhaps most familiar, others have been considered and researched, including one recent project in mathematics education. Strong, Thomas, Perini, and Silver (2004, pp. 74–75), interested in helping mathematics teachers find the proper balance between unity of instructional purpose and models of differentiation, created a series of research workshops. Based on their findings, they presented the following four types of learning styles: Mastery (people in this category tend to work step-by-step); Understanding (tend to search for patterns, categories, and reasons); Interpersonal (tend to learn through conversation and personal relationship and association); and the Self-Expressive (tend to visualize/create images and pursue multiple strategies). These researchers draw the following conclusions:

These different mathematical learning styles provide a map of cognitive diversity among mathematics students. Understanding these styles helps teachers address student strengths and weaknesses as learners. If teachers incorporate all four styles into a math unit, they will build in computation skills (Mastery), explanations and proofs (Understanding), collaboration and real-world application (Interpersonal), and non-routine problem solving (Self-Expressive). (pp. 74–75)

### Low-prep examples

Use pre-made TIPS4RM lessons, developed to appeal to a wide range of learning profiles, and provide a balanced variety of activities across them.

### High-prep examples

Prepare multiple versions of certain lessons for students with differing styles to explore multiple paths.

### Considerations regarding use of differentiation based on student learning profile

The Strong et al. (2004) learning styles mentioned above correspond well with the *Mathematical Processes* as discussed and modelled repeatedly within the *TIPS4RM* (Consortium of Ontario School Boards, 2005) document: Knowledge; Reasoning and Proving; Making Connections; and Communicating, respectively. Students require support and challenge in areas of difficulty and strength.

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## Differentiation Based on Mathematics Content

It is sometimes necessary for a teacher to have different students in the class work on different content, e.g., students in combined grade classes work on different curriculum expectations for the different grades; students in a regular classroom focus on different curriculum expectations based on readiness to move from concrete to abstract.

Content is what students should know, understand, and be able to do as a result of a section of study. It is typically derived from a combination of sources: international standards; provincial curriculum documents; local curriculum guides and textbooks; and the teacher's knowledge of the subject and of pedagogy. Differentiating instruction based on mathematics content requires an understanding of multiple developmental continua and each student's placement along these various continua. Further, it calls for instructional repertoires and materials that help move each student along each separate continuum of mathematical learning and understanding.

### Important aspects of differentiation based on mathematics content

In Ontario elementary schools and secondary schools where teachers are assigned to combined grades or have repeating students, classrooms are often comprised of multi-age learners. Not entirely unlike the one-room school dynamics of the past, differentiated instruction provides contemporary educators with a powerful approach to meet the needs of multi-age students and capitalize upon the unique benefits of combined configurations. One key strategy used to achieve these ends is that of flexible grouping.

The flexible grouping strategies utilized by multi-age classroom teachers allow opportunities for students to form small groups based on common interests and shared tasks. Peer learning takes place in structured, purposely planned instruction, as well as in less-structured situations that occur in the classroom every day as students are flexibly grouped for instruction. ...A multi-age classroom is an ideal environment for capitalizing on peer learning opportunities; in fact, a hallmark of multi-age classrooms is their collaborative environments. (Hoffman, 2002, pp. 48–52)

In differentiated classrooms, the teacher coordinates what students learn (curricular content) by engaging in such practices as providing the broad content framework; delineating between essential learning in which *all* students will be involved and areas of learning in which students can *choose* to be involved; considering the impact of varying student needs, abilities, strengths, and interests on the identified content areas; locating and securing a variety of resources; and, when appropriate, integrating the curricular content across subject areas. (Kronberg et al., 1997, p. 37)

### Low-prep examples

Use commercial materials that support a different content focus for combined grades. Use commercial diagnostic and formative assessment tasks to match students to materials developed to move students forward from various points along learning continua.

### High-prep examples

Create custom-made materials to match different readiness levels, interests, and learning profiles. Use course compacting for high-level and gifted students while engaging them in whole-class learning activities on a regular basis. Modify curriculum according to students' IEPs and collaborate regularly with other teachers supporting struggling students in order to optimize student success on content.

### Considerations regarding use of differentiation based on mathematics content

Assessment and evaluation must be based on the provincial curriculum expectations and the achievement levels outlined in the curriculum documents. Some exceptional students and students who have not been identified as exceptional but who are receiving special education programs and services may need to have the curriculum expectations modified in keeping with their special needs. Such students may be provided with modified curriculum expectations (Ontario Ministry of Education, 1999a, pp. 24–25).

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## Differentiation Based on Process or Sense-Making Activity

I soon realized that if I were willing to let go of ineffective practices refuted by research, I could find time for activities that allowed me to reach a wide range of students, representing a wide range of needs and abilities. (Cronin, 2003, p. 49)

Process refers to the activities, both mental and physical, in which a student strives to make sense out of the new learning, whether it be procedural skill attainment or deeper conceptual understanding. Differentiating instruction based on process or sense-making activity involves the use of a combination of teaching strategies.

*Processes* can be differentiated by allowing students to learn in their preferred learning style. Mathematical *models* can be differentiated by teaching students how to work with alternatives such as numerical models, verbal descriptions, physical models, graphical organizers, scale drawings, and not-to-scale diagrams, as well as algebraic models (*Leading Math Success*, Ontario Ministry of Education, 2004). Working with these different types of models allows students to make sense of the mathematics in different ways.

### Important aspects of differentiation based on process or sense-making activity

To teach mathematics using a wide variety of strategies, a generous mix of mathematical models and manipulatives, and with due deference to learning styles and multiple intelligences is to significantly increase the likelihood of high student comprehension—in terms of both the number of students affected and the level of individual student understanding.

One specific strategy for differentiating mathematics curriculum is the *Math Workshop* (Heuser, 2000). The format of the math workshop is similar to that of the writing workshop; consisting of a mini-lesson, an activity period, and reflection, with workshops in two varieties: teacher-directed and student-directed.

During the activity period, children can follow their abilities and interests. Each period is a self-differentiated inquiry session... Like the teacher-directed variety, student-directed workshops end with a reflection period. Before they clean up their objects, students share their creations with others. (Heuser, 2000, p. 36)

### Low-prep examples

Use *TIPS4RM* lessons developed with the MATCH (**M**inds **O**n, **A**ction!, Time, **C**onsolidation, **H**ome **A**ctivity) framework (Consortium of Ontario School Boards, 2005); create a positive classroom environment in which students feel safe and encouraged to participate and take risks.

### High-prep examples

When some students have not learned something important, reteach it with another strategy while other students work on extensions.

### Considerations regarding use of differentiation based on process or sense-making activity

Using a textile metaphor, authors Karp and Voltz (2000) describe a creation and a caveat:

As teachers learn and practice various teaching strategies, they expand the possibilities for weaving rich, authentic lessons that are responsive to all students' needs. ...The combination of subject matter knowledge, pedagogical knowledge, and knowledge of learner characteristics gives strength to the weave. Then as each individual student is considered, the pattern and texture of the cloth emerge. In many paradigms, the adherence to a single approach will create an instructional situation that will leave some students unravelled and on the fringe. (p. 212)

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## Differentiation Based on Product

In differentiated classrooms, assessment and instruction are interwoven and assessment is viewed as an ongoing process of feedback that occurs throughout a unit or course. (Kronberg et al., 1997, p. 53)

A product is a means by which students demonstrate what they have come to know, understand, and be able to do. Products are often considered as major or culminating demonstrations of student learning. Differentiating instruction based on product involves the generation (by teacher and/or students) of various assessment tools and strategies that will be used to demonstrate new learning that has taken place in the lesson, unit, or course.

### Important aspects of differentiation based on product

The purpose of assessment is to improve student learning by providing feedback and generating data to inform and guide instruction. These ideas are described in detail *Leading Math Success*, Ontario Ministry of Education:

Multiple strategies—such as observations, portfolios, journals, rubrics, tests, projects, self-assessments, and peer assessments—tell students that the teacher appreciates their daily contributions and does not base evaluations solely on test results (Consortium of Ontario School Boards, 2003). Information gathered through assessment helps teachers determine students' strengths and weaknesses in achieving the curriculum expectations. As part of assessment, teachers provide students with descriptive feedback that guides their efforts to improve. ...Assessment *for* learning puts the focus on using diagnostic assessment immediately before learning and formative assessment in the middle of learning to plan and adjust instruction. The emphasis moves from making judgments to coaching students and planning the next steps in teaching and learning. (pp. 33, 50)

When individuals are viewed in light of differentiated instruction, multiple forms of assessment are simply extended to pairs or groups of students, simultaneously. Not only can this make for a more meaningful and appropriate assessment experience for each student, but the classroom itself can become the repository of multiple exciting and stimulating projects, made only more so through presentations of student work to peers. Hall, Strangman, & Meyer (2003) describe some of the salient features of differentiated product assessment:

Initial and on-going assessment of student readiness and growth are essential. Meaningful pre-assessment naturally leads to functional and successful differentiation. Incorporating pre- and ongoing assessment informs teachers so that they can better provide a menu of approaches, choices, and scaffolds for the varying needs, interests and abilities that exist in classrooms of diverse students. Students are active and responsible explorers. Teachers respect that each task put before the learner will be interesting, engaging, and accessible to essential understanding and skills. Each child should feel challenged most of the time. Vary expectations and requirements for student responses. Items to which students respond may be differentiated so that different students can demonstrate or express their knowledge and understanding in different ways. A well-designed student product allows varied means of expression and alternative procedures and offers varying degrees of difficulty, types of evaluation, and scoring. (pp. 3–4)

### Low-prep example

Spend one-on-one time with students who have not done well on a particular assessment task, giving them different ways to demonstrate their knowledge and skill.

### High-prep example

Create a variety of project types that allow students to choose how they will demonstrate required knowledge and skills, e.g., photo essay, musical rap, poster display, performance task.

### Considerations regarding use of differentiation based on product

Clear expectations must be identified and communicated to students for *all* variations of assessment methods. Students and parents need to feel confident that all students in the class are being held to the same high standard.

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