Facilitator’s Handbook
Number Sense and Numeration,
Grades 4 to 6
with reference to material in Volumes 2 and 6

Understanding Addition and Subtraction of Whole and Decimal Numbers

The Literacy and Numeracy Secretariat Professional Learning Series
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The effectiveness of traditional professional development seminars and workshops has increasingly been questioned by both educators and researchers (Fullan, 1995; Guskey & Huberman, 1995; Wilson & Berne, 1999). Part of the pressure to rethink traditional PD comes from changes in the teaching profession. The expert panel reports for primary and junior literacy and numeracy (Ministry of Education, 2003, 2004) raise several key issues for today’s teachers:

• Teachers are being asked to teach in ways that they themselves may not have experienced or seen in classroom situations.
• Teachers require a more extensive knowledge of literacy and numeracy than they did previously as teachers or as students.
• Teachers need to develop a deep knowledge of literacy and numeracy pedagogy in order to understand and develop a repertoire of ways to work effectively with a range of students.
• Teachers may experience difficulty allocating sufficient time for students to develop concepts of literacy and numeracy if they themselves do not appreciate the primacy of conceptual understanding.

For professional learning in literacy and numeracy to be meaningful and classroom-applicable, these issues must be addressed. Effective professional learning for today’s teachers should include the following features:

• It must be grounded in inquiry and reflection, be participant-driven, and focus on improving planning and instruction.
• It must be collaborative, involving the sharing of knowledge and focusing on communities of practice rather than on individual teachers.
• It must be ongoing, intensive, and supported by a job-embedded professional learning structure, being focused on the collective solving of specific problems in teaching, so that teachers can implement their new learning and sustain changes in their practice.
• It must be connected to and derived from teachers’ work with students – teaching, assessing, observing, and reflecting on the processes of learning and knowledge production.

Traditionally, teaching has been a very isolated profession. Yet research indicates that the best learning occurs in collaboration with others (Fullan, 1995; Joyce & Showers, 1995; Staub, West & Miller, 1998). Research also shows that teachers’ skills, knowledge, beliefs, and understandings are key factors in improving the achievement of all students.

Job-embedded professional learning addresses teacher isolation by providing opportunities for shared teacher inquiry, study, and classroom-based research. Such collaborative professional
learning motivates teachers to act on issues related to curriculum programming, instruction, assessment, and student learning. It promotes reflective practice and results in teachers working smarter, not harder. Overall, job-embedded professional learning builds capacity for instructional improvement and leadership.

There are numerous approaches to job-embedded professional learning. Some key approaches include: co-teaching, coaching, mentoring, teacher inquiry, and study.

**Aims of Numeracy Professional Learning**

The Literacy and Numeracy Secretariat developed this professional learning series in order to:

- promote the belief that all students have learned some mathematics through their lived experiences in the world and that the math classroom should be a place where students bring that thinking to work;
- build teachers’ expertise in setting classroom conditions in which students can move from their informal mathematics understandings to generalizations and formal mathematical representations;
- assist educators working with teachers of students in the junior division to implement the student-focused instructional methods that are referenced in *Number Sense and Numeration, Grades 4 to 6* to improve student achievement; and
- have teachers experience mathematical problem solving – sharing their thinking and listening; considering the ideas of others; adapting their thoughts; understanding and analysing solutions; comparing and contrasting solutions; and discussing, generalizing, and communicating – as a model of what effective math instruction entails.

**Teaching Mathematics through Problem Solving**

Until quite recently, understanding the thinking and learning that the mind makes possible has remained an elusive quest, in part because of a lack of powerful research tools. In fact, many of us learned mathematics when little was known about learning or about how the brain works. We now know that mathematics instruction can be developmentally appropriate and accessible for today’s learners. Mathematics instruction has to start from contexts that children can relate to – so that they can “see themselves” in the context of the question. Most people learned math procedures first and then solved word problems related to the operations after practising the skills taught to them by the teacher. The idea of teaching through problem solving turns this process on its head.

By starting with a problem in a context (e.g., situational, inquiry-based) that children can relate to, we activate their prior knowledge and lived experiences and facilitate their access to solving mathematical problems. This activation connects children to the problem; when they can make sense of the details, they can engage in problem solving. Classroom instruction needs to provoke students to further develop their informal mathematical knowledge by representing their mathematical thinking in different ways and by adapting their understandings after listening to others. As they examine the work of other students and consider
the teacher’s comments and questions, they begin to: recognize patterns; identify similarities and differences between and among the solutions; and appreciate more formal methods of representing their thinking. Through rich mathematical discourse and argument, students (and the teacher) come to see the mathematical concepts expressed from many points of view. The consolidation that follows from such dynamic discourse makes the mathematical representations explicit and lets students see many aspects and properties of math concepts, resulting in students’ deeper understanding.

**Learning Goals of the Module**

This module is organized to guide facilitators as they engage participants in discussion with colleagues working in junior classrooms. This discourse will focus on important concepts, procedures, and appropriate representations of addition and subtraction of whole and decimal numbers.

During these sessions, participants will:

- develop an understanding of the conceptual models of addition and subtraction with whole and decimal numbers;
- explore conceptual and algorithmic models of addition and subtraction with whole and decimal numbers through problem solving;
- analyse and discuss the role of student-generated strategies and standard algorithms in the teaching of addition and subtraction with whole and decimal numbers; and
- identify the components of an effective mathematics classroom.
Getting Organized

Participants

• Classroom teachers (experienced, new to the grade, new to teaching [NTIP]), resource and special education teachers, numeracy coaches, system curriculum staff, and school leaders will bring a range of experiences – and comfort levels – to the teaching and learning of mathematics. Participants may be organized by grade, division, cross-division, family of school clusters, superintendency regions, coterminous boards, or boards in regions.

• Adult learners benefit from a teaching and learning approach that recognizes their mathematics teaching experiences and knowledge and that provides them with learning experiences that challenge their thinking and introduces them to research-supported methods for teaching and learning mathematics. For example, if time permits, begin each session with 10 minutes for participants to share their mathematics teaching and learning experiences, strategies, dilemmas, and questions.

• Some participants may have prior knowledge through having attended professional development sessions using *The Guide to Effective Instruction in Mathematics, Kindergarten to Grade 3* or *The Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6* through board sessions or Ontario Summer Institutes. These professional learning sessions are intended to deepen numeracy learning, especially for junior teachers.

Facilitators

Effective professional learning happens daily and over time. These professional learning materials are designed to be used to facilitate teachers’ collaborative study of a particular aspect of mathematics for teaching to improve their instruction. These materials were not designed as presentation material. In fact, these sessions are organized so that they can be used flexibly with teachers (e.g., classroom teachers, coaches, consultants) and school leaders (e.g., vice principals, principals, program coordinators) to plan and facilitate their own professional learning at the school, region, and/or board levels.

It is recommended that the use of these materials is facilitated collaboratively by at least two educators. Co-facilitators have the opportunity to co-plan, co-implement, and make sense of the audience’s responses together, to adjust their use of the materials, and to improve the quality of the professional learning for the audience and themselves. Further, to use these modules, facilitators do not need to be numeracy experts, but facilitators do need to be confident about learning collaboratively with the participants and have some experience and/or professional interest in studying mathematics teaching/learning to improve instruction.

Here are a few ways that facilitators can prepare to use this module effectively:

• Take sufficient time to become familiar with the content and the intended learning process inherent in these sessions.

• Think about the use of the PowerPoint as a visual aid to present the mathematical prompts and questions participants will use.
• Use the Facilitator’s Handbook to determine ways in which to use the slides to generate discussion, mathematical thinking and doing, and reflection about classroom implementation.
• Note specific teaching strategies that are suggested to develop rich mathematical conversation or discourse.
• Highlight the mathematical vocabulary and symbols that need to be made explicit during discussions and sharing of mathematical solutions in the Facilitator’s Handbook.
• Try the problems prior to the sessions to anticipate a variety of possible mathematical solutions.
• As you facilitate the sessions, use the Facilitator’s Handbook to help you and your learning group make sense of the mathematical ideas, representations (e.g., arrays, number lines), and symbols.

**Time Lines**

• This module can be used in different professional learning scenarios: professional learning team meetings, teacher planning time, teacher inquiry/study, parent/community sessions.
• Though the module is designed to be done in its entirety, so that the continuum of mathematics learning can be experienced and made explicit, the sessions can be chosen to meet the specific learning needs of the audience. For example, participants may want to focus on understanding how students develop conceptual understanding through problem solving, so the facilitator may choose to implement only Session B in this module.
• As well, the time frame for implementation is flexible. Three examples are provided below.

<table>
<thead>
<tr>
<th>Module Sessions</th>
<th>One Full Day</th>
<th>Two Half-Days</th>
<th>Four Sessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session A – Activating Prior Knowledge</td>
<td>75 min</td>
<td>Day 1</td>
<td>90 – 120 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120 – 180 min</td>
<td></td>
</tr>
<tr>
<td>Session B – Developing Conceptual Understanding</td>
<td>75 min</td>
<td>Day 2</td>
<td>90 – 120 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120 – 180 min</td>
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<tr>
<td>Session C – Making Sense of Alternative Algorithms</td>
<td>75 min</td>
<td>Day 2</td>
<td>90 – 120 min</td>
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<td>Day 2</td>
<td>90 – 120 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120 – 180 min</td>
<td></td>
</tr>
</tbody>
</table>

If you choose to use these materials during:
• One full day – the time line for each session is tight for implementation; monitor the use of time for mathematical problem solving, discussion, and reflecting.
• Two half days – the time line for each session is tight for implementation; monitor the
use of time for mathematical problem solving, discussion, and reflecting; include time for participants to share the impact of implementing ideas and strategies from the first half-day session.

- Four sessions – the time lines for each session are more generous for implementation; include time for participants to discuss and choose ideas and strategies to implement in their classroom at the end of each session; include time for sharing the impact of implementing ideas and strategies from the previous session at the start of each session

Creating a Professional Learning Environment

- Organize participants into small groups – preferably of 4 to 6 people – to facilitate professional dialogue and problem-solving/thinking experiences.
- Seat participants in same-grade or cross-grade groups, depending on whether you want the discussion to focus on one grade level or across grade levels.
- Ensure that a blackboard or 3 m to 4 m of wall space is cleared, so that mathematical work can be posted and clearly seen.
- Provide a container with the learning materials (e.g., writing implements like markers, paper, sticky notes) on each table before the session. Math manipulatives and materials should be provided for each pair of participants at each table.
- Provide a copy of the agenda and handouts of the PowerPoint for note-taking purposes or tell the participants that the PowerPoint will be e-mailed to them after the session so that they have a record of it.
- Arrange refreshments for breaks and/or lunches, if appropriate.
- If time permits, begin each session with 10 minutes for participants to share their mathematics teaching and learning experiences, strategies, and dilemmas.

Materials Needed

- *Number Sense and Numeration, Grades 4 to 6* (Volumes 2 and 6)
- *Ontario Curriculum, Mathematics: Grades 1–8* (Revised 2005)
- Understanding Addition and Subtraction of Whole and Decimal Numbers, PowerPoint presentation, slides 1 to 33
- computer, LCD projector, and extension cord
- Square grid chart paper (ripped into halves or quarters), markers (6 markers of different colours for each table group), sticky notes (large size), highlighters, pencils, transparencies, and overhead markers (if projector is available), tape for each table of participants
- Base ten blocks
- centimetre ruler
- centicubes
Session A – Activating Prior Knowledge

Aims of Numeracy Professional Learning

Display slides 1 and 2. You may wish to refer to the following statements about students’ understanding of addition and subtraction in the primary and junior grades:

In the primary grades, students develop an understanding of part-whole concepts – they learn that two or more parts can be combined to create a whole (addition), and that a part can be separated from a whole (subtraction).

Instruction in the junior grades should help students to extend their understanding of addition and subtraction concepts and allow them to develop flexible computational strategies for adding and subtracting multidigit whole numbers and decimal numbers.

In the junior grades, instruction should focus on developing students’ understanding of meaningful computational strategies for addition and subtraction, rather than on having students memorize the steps in algorithms . . . The development of computational strategies for addition and subtraction should be rooted in meaningful experiences (e.g., problem-solving contexts, investigations). Students should have opportunities to develop and apply a variety of strategies and to consider the appropriateness of strategies in various situations. (Ministry of Education, 2006a, pp. 11–12)

Display slides 3 and 4. Ask each group of participants to discuss two aims of numeracy professional learning. Ask them to explain how these aims support teachers in their efforts to improve their mathematics knowledge for teaching, their instructional practices, and student achievement.
**About Problem Solving**

Many people learned math procedures first and then solved word problems after practising to recite the number facts and carry out number operations by rote. This information was transmitted from the teacher to students. Unlike rote learning, teaching and learning mathematics through problem solving encourages students to reason their way to a solution or a new understanding.

By starting with a problem within a context that activates students’ prior knowledge and mathematical thinking, students can make sense of the problem details and develop solutions using informal and formal methods. Because each child’s thinking will be different, classroom instruction must focus on provoking students to relate their informal knowledge to the collective ideas and strategies developed by the class during a formal lesson. As students examine the work of other students and the teacher, they recognize similarities and differences between and among the solutions. They also recognize the relationship between informal and formal ways of representing their thinking, so that they can communicate their understanding.

The communication and reflection that occur during and after the process of problem solving help students not only to articulate and refine their thinking, but also to see the problem they are solving from different perspectives. By seeing a range of mathematical ideas and strategies that can be used to develop a solution, students can begin to reflect on their own thinking (a process known as “metacognition”) and the thinking of others, and to consciously adjust their own mathematical thinking in order to improve their solutions.

Through this classroom discourse of questioning, analysis, and argument, students are able to understand the concepts expressed from many points of view. Consolidation through class discussions renders the mathematical representations explicit and lets students experience different aspects and properties of a math concept, resulting in students’ deeper understanding.

**Learning Goals of the Module**

Have participants analyse the learning goals (slide 5) of this professional learning session by identifying and explaining aspects of the learning goals that are:

- familiar (e.g., components of an effective mathematics classroom – accessibility to manipulatives, students organized in small learning groups, a list of what should be in a mathematics classroom);
- unfamiliar (e.g., conceptual models of whole and decimal numbers, such as area, linear, and set models); and
- interesting (e.g., role of student-generated strategies for addition and subtraction of whole and decimal numbers; knowing “student-generated strategies” are important, but not knowing how to get students to share their strategies or how to consolidate them).
In What Ways Does Number Sense and Numeration, Grades 4 to 6 Describe Addition and Subtraction?

Show slides 6 and 7. Introduce Number Sense and Numeration, Grades 4 to 6, Volume 2: Addition and Subtraction and Volume 6: Decimal Numbers by identifying the different sections of the document and noting that the focus of this session is on Volumes 2 and 6.

Provide about five minutes for participants to peruse Volume 2: Addition and Subtraction by having them identify and explain aspects of the guide that are:

• familiar (e.g., junior grades focus on adding and subtracting multidigit whole numbers and decimal numbers);
• unfamiliar (e.g., addition is the joining of parts to make a whole and subtraction is the separation of the whole into parts); and
• interesting (e.g., different types of addition and subtraction problems).

Have participants share their responses to the prompts using a think-pair-share strategy.

Warm Up – What Ways Do We Use Math?

Display slide 8.

1. If participants do not know one another, invite them to introduce themselves (e.g., by name, school, number of years teaching, and current educational responsibility). If participants do know each other, have them share what they are interested in learning during this session to improve their knowledge of mathematics for teaching. Prompt each group to introduce each person in the group to the whole group. Such information can provide facilitators with an overview of the range of participants’ knowledge and experience that can be referred to and built upon during this session.
2. Provide participants with a few examples of ways that you have used addition and subtraction in your daily life, emphasizing that one idea is to be recorded on one sticky note. (e.g., I combined 3 carrots, 500 g of stewing beef, 1 onion, 3 stalks of celery, \( \frac{1}{2} \) package of pot barley, 1 package of onion soup mix, and 10 cups of water to make a pot of beef barley soup.)

3. Have participants record their ideas on sticky notes in their small groups.

4. Provide them with chart paper on which to sort and classify their ideas. Choose two ideas and ask participants if these ideas are similar to or different from their own. If they are similar, organize them into one category; if they are different, organize them into two categories. Represent these categories on a concrete graph, as shown on slide 9. As an option, you might have other groups sort the sticky notes as a concrete circle graph or a Venn diagram.

5. Discuss the sorting criteria or rule used to categorize the sticky notes: for example, sorting by types of mathematics (addition and subtraction of whole numbers, decimals, fractions, or integers) or by types of purchases (food, clothing, services, or household expenses).

**About Informal Mathematical Knowledge**

Having participants describe the different ways they use addition and subtraction provokes them to realize that their daily life experiences include the use of mathematics in order to make observations, to describe their living experiences, to make decisions, and to take action. Such informal mathematical knowledge is rich with experiential purpose and mathematical details, and is learned in ways unlike formal school mathematics.

Teachers should consider ways to activate and build on the range of informal mathematics knowledge that students bring to their mathematics learning situations, so that connections can be made between informal/formal mathematical knowledge and prior/new mathematics knowledge and experiences.
Activating Mathematical Knowledge – Problem #1

Understand the Problem and Make a Plan

Show slide 10.

Provide participants with base ten blocks, square grid chart paper, and metric rulers. Ask participants to identify the information from the problem that they will use to make their plan. Ask the participants to explain what it means to model a number with concrete materials, using a think-pair-share strategy.

Some possible responses include:

• to represent a number in different ways so that flexible thinking about the quantity of a number can be developed; and
• to provide visuals that show how numbers are composed of other numbers; that is, to show part-whole relationships of numbers.

As participants are planning their solutions, ask them to consider how the material actions (i.e., the actions participants use when they work with the concrete materials or manipulatives) that represent addition and subtraction model the idea of joining, separating, and comparing. Have a few participants search Volume 2 of Number Sense and Numeration, Grades 4 – 6 to find out more about the concepts of addition and subtraction. A few examples of addition and subtraction concepts are:

• addition is the joining or composing of parts to make a whole;
• subtraction is separating or decomposing of the parts of a whole; and
• subtraction is comparing two wholes to determine the difference between them.

Further, as the participants are planning ways to use the manipulatives to model their solutions, prompt them to identify the value of the units.
For example, for base ten blocks:

- If the rod is worth 10, is the single unit worth 1?
- If the rod is worth 1, is the single unit worth \( \frac{1}{10} \)?
- If the rod is worth \( \frac{1}{10} \), is the single unit worth \( \frac{1}{100} \)?

**Carry Out the Plan**

Have participants develop solutions in pairs or in small groups on square grid chart paper. Listen and watch for the mathematics they are using to develop solutions. Use the math vocabulary (Appendix 1) to name the mathematics they are using. Though this task has no context, it is a problem because it is open-routed, requiring at least two or more solutions. Below are some solutions to the problems. Visual support provides a record of the calculation that is reflective of the mathematical thinking. Students can look back at the sketch and discern the process used.

Problem #1 Addition a) \( 13 + 18 = ? \)

**Note About Open Number Lines**

The illustration above uses an open number line in its display. On an open number line, numbers are located relative to each other but not proportionally the appropriate distance from one another. For example, on the illustration above, the arc for 10 and for 1 are not proportional or measured exactly – ten 1s would not match the length of the arc for 10. Open number lines are used as representations to record numerical jumps.
Problem #1 Addition b) \(1.3 + 1.8 = ?\)

\[
\begin{align*}
1.3 + 1.8 &= \\
\square &= 0.1 \\
\square &= 1 = 10 \text{ tenths} = 1 \text{ one}
\end{align*}
\]

Show slide 11.

Problem #1 Subtraction a) \(97 - 58 = ?\)

\[
\begin{align*}
97 - 58 &= \\
\text{Whole: 90 and 7} \\
\square &= 10 \\
\square &= 1
\end{align*}
\]

\[
\begin{align*}
97 - 58 &= \\
\text{Part: 30 and 9} \\
\text{Part: 50 and 8}
\end{align*}
\]

\[
\begin{align*}
97 - 58 \text{ add 3 to each and the difference stays the same} \\
97 - 58 &= 100 - 61 \\
\end{align*}
\]

\[
\begin{align*}
97 - 58 &= 100 - 61 = 39
\end{align*}
\]
Problem #1 Subtraction b) $9.7 - 5.08 = ?$

As you circulate among participants, sort and classify their solutions to the four problems, using criteria, such as:

- different ways to join or compose parts to make a whole (e.g., see examples below in the “Look Back – Reflect and Connect” section);
- different ways to separate or decompose a whole into parts; and
- the use of commutative and associative properties (e.g., see examples below).

Organize the different solutions on a blackboard or whiteboard space. Label the classifications with the operation names. Annotate the solutions using the words from the math vocabulary list in Appendix 1. This organizational strategy is called **bansho**.
**Note about Base Ten Blocks**

Each time you start a new problem, you need to name the base ten block that represents ones.

If a unit cube represents ones, then a rod represents tens and a flat represents hundreds.

However, if in another problem a rod represents ones, then the unit cube represents tenths and the flat represents 10 ones or tens.

The value of the flats, rods, and unit cubes is multiplicative – you determine their relative values by multiplying or dividing by 10.
Look Back – Reflect and Connect

Show slide 12. These prompts were provided to consolidate several key notions about modelling addition and subtraction of whole numbers and decimals using base ten blocks.

Have participants respond to these questions in pairs, then in small groups. Walk about to hear the different responses and choose two or three key ideas to be discussed during the whole group sharing of responses.

Some sample responses are as follows:

1. Ways to solve addition problems:
   - write numbers in expanded form and add same place values:
     e.g., \(13 + 18 = 10 + 10 + ((3+7) + 1) = 30 + 1 = 31\); make groups of tens and ones
   - decompose numbers into place value units and add:
     e.g., \(13 + 18 = 13 + 10 + 8 = 23 + (7 + 1) = 30 + 1 = 31\); decompose 18 to 10 and 8
   - decompose numbers and move parts to make friendly numbers
     e.g., \(13 + 18 = 15 + 15 + 1 = 31\); move 13 up 2 to 15 and 18 down 3 to 15 because you moved up 2 and down 3 and this is not equal compensation, you need to add 1 more to the addends

2. Ways to solve subtraction problems:
   - separate a part, 58 (subtrahend), from the whole, 97 (minuend), to get the other part, 39 (difference)
   - compare two numbers, 97 and 58, to determine the difference, showing that 97 is 39 more than 58.
• rename the numbers and reorder them to get friendly differences:
  – in 9.7 – 5.8, rename 9.7 to be 8 ones and 17 tenths and
  – rename 5.8 to 5 ones and 8 tenths
  – then, 8 ones – 5 ones = 3 ones and
  – 17 tenths – 8 tenths is 9 tenths
  – so, the difference is 3 ones and 9 tenths or 3.9

3a) Similarities in adding whole numbers and decimals:
  • join addends with same number units (place values), add ones with ones, and add
    tenths with tenths, remembering the base ten blocks represent the same value
    throughout a single question (see “Note about Base Ten Blocks” above)

3b) Differences in adding whole numbers and decimals:
  • the blocks can be defined differently for decimal numbers – that is, any base ten
    block can take the value ones and the other blocks take their value in relation to that
    assignment

4a) Similarities in subtracting whole numbers and decimals:
  • decompose the whole 97 (minuend) into two parts showing 58 (subtrahend) as one
    part and the other part, 39 (difference)

4b) Differences in subtracting whole numbers and decimals:
  • two numbers are compared to determine their difference by regrouping to make
    friendly numbers for subtraction
    – 9.7 – 5.08 gets renamed as (9 – 5) and (0.70 – 0.08)
    – 9 – 5 = 4 and 0.70 – 0.08 = 0.62
    – resulting in an answer of 4.62
Session B – Developing Conceptual Understanding

Show slide 13.

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6 highlights the importance of students building conceptual understanding of the operations. It is not enough for students to be able to carry out procedures in a rote fashion. When students are required to use memorized rules, they often forget all or part of the rule and are left stuck without a strategy to solve the problem. To be able to problem solve throughout their lives, students need to acquire a deep understanding of the structures, processes, symbols, and notations in mathematics. They need to understand the representations of numbers and the operations applied to those numbers. They need to develop a conceptual understanding of how the actions on materials (base ten blocks) are related to addition (joining) and subtraction (separating).

This session focuses on the representation of the addition and subtraction of whole and decimal numbers, with an emphasis on using base ten blocks. Students will represent the whole and decimal numbers and then join (compose) or separate (decompose) the base ten materials according to the context set in the problems. When students can confidently demonstrate and explain what they are doing as they compose and decompose, and represent numbers, they will have a deep understanding of the algorithms and be able to use them flexibly when working independently. This is a major goal of mathematics education in the junior grades.
Warm Up – A Knowledge Package for Addition and Subtraction

Show slide 14. Provide participants with the Ontario Curriculum, Mathematics, Grades 1–8 (revised; 2005) and Number Sense and Numeration, Grades 4 to 6 to identify key conceptual and procedural knowledge. To activate participants’ prior knowledge of whole and decimal number addition and subtraction, have participants respond to these questions on chart paper in their small groups. Their ideas could be organized as a list or a concept web to show the interconnections between and among their prior knowledge.

Some possible responses are:

- understanding representations of decimal numbers using concrete materials (e.g., base ten blocks, square grid, number line);
- identifying the place value of digits in a decimal number;
- understanding the different types of addition and subtraction problems (which they have used to solve whole number problems);
- adding and subtracting money amounts and making change from $10.00 and $100.00;
- estimating to judge the reasonableness of a solution when solving addition and subtraction problems; and
- using mental math strategies for 2- and 3-digit numbers.

Developing Conceptual Understanding – Problem #2

Understand the Problem and Make a Plan

Show slide 15. Have participants use base ten blocks and a ruler to represent their thinking when solving Problem #2. Before starting the make-a-plan stage, be sure everyone understands the problem. Ask participants to identify the information from the problem that they will use to make their plan (e.g., mass of the
puppy now, amount of mass increase) and ask if this problem can be solved using addition, subtraction, or a combination of both operations. Talk about the importance of making this information public. All thinking about the problem should be made explicit in the classroom, so all students have access to the problem solving.

Remind participants, as they are developing their plans to solve the problem, of these concepts of addition and subtraction:

- there are different ways of composing and decomposing the numbers in a problem to show part-whole relationships of numbers;
- addition can be modelled as the joining or composing of parts to make a whole; and
- subtraction can be modelled as separating or decomposing the parts of a whole and comparing two wholes to determine the difference between them.

This guided lesson design is a problem-solving instructional approach that focuses on the development of different solutions that represent the same concept. For this problem, the solutions show the relationship between addition and subtraction, and how both operations can be used to solve the problem.

**Carrying Out the Plan**

Have participants develop solutions in pairs or in small groups on square grid chart paper. Listen and watch for the mathematics they are using to develop solutions. Use the math vocabulary to name the mathematics they are using. This problem is not open-ended – we refer to it as open-routed as it requires at least 2 solutions.

Identify the value of the base ten blocks.
Sample Solutions

Here are some representations of solutions – they are representations of the mathematical thinking. Visual support provides a record of the calculation that is a reflection of the mathematical thinking. Students can look back at the sketch and discern the process used.

Solution A
So, one flat represents one ten or 10.

Solution B

<table>
<thead>
<tr>
<th>New Mass</th>
<th>Difference</th>
<th>Old Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.6 kg</td>
<td>3.5 kg</td>
<td>??</td>
</tr>
</tbody>
</table>

\[
35.6 - 3.5 = (35 - 3) + (0.6 - 0.5) \\
= 32 + 0.1 \\
= 32.1
\]

Solution C

Count back 3.5 from 35.6 by moving 1, 2, 3 back and then back 0.5

\[
\begin{array}{c}
32.6 \\
33.6 \\
34.6 \\
35.6
\end{array}
\]
Solution D

Look Back – Reflect and Connect

Show slide 16.

Have participants respond to these questions in pairs, then in small groups. Walk about to hear the different responses and choose 2 or 3 key ideas to be shared during the whole group sharing of responses.

Organizing Different Solutions for Class Discussion

You may choose to sort and post all solutions or choose 3 or 4 solutions that exemplify a few key ideas or strategies about addition and subtraction of whole numbers and decimals. Make sure you have clear blackboard or wall space, so that you can sort and post the participants’ responses in a particular order. To organize the solutions for whole-group discussion and for learning, consider the key mathematics ideas that should be made explicit and how you want to order solutions to show similarities and differences.

Some sorting criteria for the solutions are as follows:

- type of representation – array (use of base ten materials), linear (use of number line), computations only
- type of operation and computations – decimal number addition (different ways that numbers are joined (composed)), decimal subtraction (different ways that numbers are separated (decomposed))
Organize the different solutions on a blackboard or whiteboard space. Label the classifications with the operation names. Annotate the solutions using the words from the math vocabulary list. This organizational strategy is called *bansho*.

Display slide 17. Lead a discussion about these questions.

**Ways that Problems Develop Conceptual Understanding**

Some sample responses are as follows:

- sharing addition and subtraction solutions to the same problem makes the numerical relationship between addition and subtraction explicit
- showing multiple solutions for the same problem demonstrates the mathematical power of being flexible in composing and decomposing numbers and allows student to learn from one another
- using the base ten blocks and the number line emphasizes the role of visualization in developing conceptual understanding in mathematics
Session C – Making Sense of Alternative Algorithms

Display slide 18.

Warm Up – Defining an “Algorithm”

Display slide 19 on whole number addition and slide 20 on decimal number addition.

Ask participants if they agree or disagree that the three characteristics of a mathematical algorithm in slide 19 are correct. Then ask them to come up with arguments that would convince the group that the examples in slides 19 and 20 show addition algorithms or that they don’t.

Sample arguments are as follows:

- This algorithm would work with other numbers. It shows digits added from left to right according to their place value columns (i.e., hundreds, tens, and ones) – the written numbers represent the sums of the digits in each of the hundreds, tens, and ones place values; the numbers change when the place value regroupings from tens to hundreds are completed (6 hundreds + 14 tens becomes 7 hundreds + 4 tens) and ones to tens (4 tens + 11 ones becomes 5 tens + 1 one).

- This second addition algorithm shows adding digits in their place value columns (i.e., tens, ones, and tenths) are then added again and the regrouping occurs; the decimal number place values are regrouped from left to right – and the different place value names are used – regrouping from ones to tens (6 tens + 14 ones becomes 7 tens + 4 ones) and tenths to ones (4 ones + 11 tenths becomes 5 ones + 1 tenth). This would work with other numbers and is efficient.
Alternative Algorithms – Problem #3

People from countries all over the world learn and use different algorithms. There is no evidence that one algorithm is more useful or more valued than another algorithm. There is a significant body of evidence showing that understanding how and why an algorithm works indicates deep mathematical understanding.

Understanding the Problem and Making a Plan

The representations of the algorithms in slides 19 and 20 are directly related to the different ways in which Problems #1 and #2 were solved in Sessions A and B. Participants used concrete materials (base ten blocks) and number lines to represent their solutions.

Our goal is to have teachers and students become flexible enough in their understanding that they can move between concrete, symbolic, and numeric representations of the processes of addition and subtraction.

The next three slides (21, 22, and 23) present alternative algorithms, including one we call “traditional”. Though the problems in these slides provide no context, this direct or modelled lesson design is a problem-solving approach that focuses on analysing and describing a mathematical solution, algorithm, or strategy. All of these are used to make explicit a wide range of processes that can be used for addition and subtraction. In the end, we want to expand students’ repertoire of mathematical processes and strategies.

In order for students to understand the algorithmic representations of the processes, they need to make sense of how these abstract representations relate to the operations modelled using concrete materials. If they have only learned the algorithm by rote memorization, they often forget or confuse the steps and have no reasoning to fall back on. They need to relate the material actions to the numbers, operations, and symbols. These algorithms will be revisited and used as the basis for the mental math strategies outlined in the last session of this module.

Organize your participants into groups of three or four. Ask them to make sense of the 4 algorithms and to appoint a different member of the group to report on each specific algorithm. Remind them that they are working like mathematicians – articulating why the process works and convincing others of its viability. Make concrete materials available to enable participants to support their arguments using constructed models.
Traditional Addition Algorithm

- Display slide 21. The algorithm represents a series of digit additions and regroupings by place value right to left. Doing the addition with base ten blocks as you record the thinking makes explicit all the reasoning behind the recording. This is a time when someone working on an overhead projector can make the material actions with the base ten blocks explicit while someone else acts as recorder.

- Starting at the smallest unit on the right, add the digits in a single place value
  - \( 3 + 8 = 11 \) or 1 one and 1 ten
  - show 1 one below the line and 1 ten as a smaller 1 above the number in the tens column
  - add the digits in the tens column \((8 + 4 + 1)\) tens = 13 (tens) = 10 tens + 3 tens = 1 hundred and 3 tens
  - write the 3 below the line and the 1 hundred as a smaller 1 above the number in the hundreds column
  - \((5 + 3 + 1)\) hundreds = 9 hundreds
  - write the 9 below the line in the hundreds column

Remember – for ones, you regroup in tens; for tens, you regroup in hundreds, and so on. Addition of decimal numbers follows the same logic: for tenths, you regroup in ones, for ones you regroup in tens, and so on.

Partial-Sums Addition Algorithm

The partial-sums addition algorithm focuses on seeing the numbers in expanded form (e.g., \(348 = 300 + 40 + 8\); \(58.3 = 50 + 8 + 0.3\)) and adding numbers with the same place value to create partial sums. No regrouping is necessary.

- add from left to right (i.e., from hundreds to tens and then ones), acknowledging the place value of
the numbers and writing partial sums: 300 + 500 is 800, then 40 + 80 is 120, and finally 8 + 3 is 11

• then these partial sums are added to reach the final sum

The same logic is represented in the visual below. Digits are considered to have the value defined by their place in the number – so, 348 = 300 + 40 + 8 with the digit 3 in the hundreds place, the digit 4 in the tens place, and the digit 8 in the ones place. Visual support provides a record of the calculation as a reflection of the mathematical thinking. Students can look back at the sketch and discern the process used.

In the decimal example (34.8 + 58.3), the same procedure is applied, except that the numbers added are named with different place values (i.e., tens, ones, and tenths).

• add from left to right (i.e., from tens to ones and then tenths), acknowledging the place value of the numbers and writing partial sums: 30 + 50 is 80, then 4 + 8 is 12, and finally 0.8 + 0.3 is 1.1

• then these partial sums are added to reach the final sum
Adding-Up Subtraction Algorithm

Display slide 22.

This algorithm represents a series of partial sums as steps to “friendly numbers”. It is like counting up from the subtrahend to the minuend to determine the difference. Prior to digital cash registers, this is how change was counted back to customers when they spent $7.95 and gave a $10 bill.

• start with the subtrahend (379), add 1 to get 380 (a friendly number)
• add 300 to get 680 which is closer to 724 (minuend) (Note: If 400 were added, the partial sum would have been 780, which is larger than 724.)
• 680, add 40 to get 720, and then add 4 to get to 724 (minuend)
• the sum of the jumps is the difference from the subtrahend to the minuend (1 + 300 + 40 + 4 = 345), 345 is the difference

The same procedure is applied to decimal number subtraction, except the place value names are different (see diagram on next page). The units are tenths, ones, and tenths and the jumps are 1 tenth or 0.1, 3 tens or 30, 4 ones or 4, and 4 tenths. These jumps sum to 34.5 (0.1 + 30 + 4 + 0.4).
Display slide 23. This algorithm represents a series of partial subtractions carried out by first decomposing the subtrahend 379 to 300 + 70 + 9.

The hundreds part of the minuend is subtracted first (i.e., 300 from 724 to get a difference of 424). Follow the number line illustration.

Then the tens are subtracted by decomposing 70 to 20 + 50, then subtract 20 from 424 to get 404, and 50 from 404 to get 354.

The ones are subtracted by decomposing 9 to 4 + 5. So, 354 – 4 = 350 and 350 – 5 = 345. With all this decomposing, flexible thinking is encouraged.

The same procedure is applied to decimal number subtraction, except the place value names are different – decompose 37.9 to 30 + 7 + 0.9. Subtract 30 from 72.4 to get 42.4, then subtract 7 (decomposed to 2 and 5) to get 40.4 and 35.4, and subtract 0.9 (decomposed to 0.4 and 0.5) to get 35.0 and 34.5.
Have participants develop solutions in pairs or in small groups on square grid chart paper. Listen and watch for the mathematics they are using to develop solutions. Visual support provides a record of the calculation to reflect the mathematical thinking. Students can look back at the sketch and discern the process used. Below are some solutions to the problem.

### Traditional Addition

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>439.56</td>
<td>+</td>
<td>88.10</td>
</tr>
<tr>
<td>527.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Adding-Up Subtraction

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>439.56</td>
<td>–</td>
<td>88.1</td>
</tr>
<tr>
<td>351.46</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Left-to-Right Subtraction

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>439.56</td>
<td>–</td>
<td>80.0</td>
</tr>
<tr>
<td>409.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400.00</td>
<td>–</td>
<td>30.00</td>
</tr>
<tr>
<td>379.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>359.56</td>
<td>–</td>
<td>50.00</td>
</tr>
<tr>
<td>309.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300.00</td>
<td>–</td>
<td>8.00</td>
</tr>
<tr>
<td>292.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>284.00</td>
<td>–</td>
<td>0.10</td>
</tr>
<tr>
<td>283.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>283.80</td>
<td>–</td>
<td>0.06</td>
</tr>
<tr>
<td>283.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Partial-Sums Addition

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>439.56</td>
<td>+</td>
<td>80.0</td>
</tr>
<tr>
<td>519.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400.00</td>
<td>+</td>
<td>30.00</td>
</tr>
<tr>
<td>430.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.56</td>
<td>+</td>
<td>8.00</td>
</tr>
<tr>
<td>0.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td>+</td>
<td>0.10</td>
</tr>
<tr>
<td>0.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Count on from 88.1 to 439.56

- 88.1 + 0.9 = 89
- 89 + 300 = 389
- 389 + 50 = 439
- 439 + 0.56 = 439.56
- 300 + 50 + 0.90 + 0.56 = 350 + 1.46 = 351.46
Look Back – Reflect and Connect

Ask participants to respond to these questions in pairs and then in small groups. Walk about to hear the different responses and choose two or three key ideas to be shared during the whole group sharing of responses.

Display slide 25.

What might students have done to add?

Some sample responses are as follows:

A – added from right to left without regrouping, trying to follow the traditional addition algorithm

B – added digits left to right (i.e., regrouping incorrectly, writing the tens digit in the sum and regrouping the ones digit to the column on the right) and recording results of the digit addition incorrectly in terms of place value

C – added the digits and wrote all numbers for the sum without regrouping

What strategies would you use to promote understanding of addition?

Some sample responses are as follows:

• Visual support provides a record of the calculation to reflect the mathematical thinking. Students can look back at the sketch and discern the process used. Use a number line to show adding on, adding to friendly numbers, for example, to get the sum of 439.56 and 88.10.
• Ask the students how they added to get an understanding of the mathematical thinking demonstrated in their inaccurate solution.

• Use base ten blocks to model addition and ask students why the sum is different when we modeled it with materials.

• Use the partial-sums algorithm to show the significance of adding numbers by recognizing their place value.

Display slide 26.

What might students have done to subtract?

Some sample responses are as follows:

A – calculated the difference between the digits (4 – 0 = 4, 8 – 3 = 5, 9 – 8 = 1, 5 – 1 = 4, 6 – 0 = 6), possibly thinking that subtraction is commutative, when only addition and multiplication are commutative

B – subtracted left to right with no regrouping

What strategies would you use to promote understanding of subtraction?

Some sample responses are as follows:

• Ask the students how they subtracted to get an understanding of the thinking demonstrated in their inaccurate solution.
• Use base ten blocks to model subtraction and ask students why the difference we modelled is not the same as their answer.
• Use a number line to show counting up (adding on to friendly numbers) to get from 88.1 to 439.56.

439.56 – 88.1
Count up from 88.1 to 439.56 stopping at friendly numbers.

439.56 – 88.10 = 0.9 + 10 + 1 + 300 + 39 + 0.56 = 350 + 0.90 + 0.56 = 351.46
Session D – Using Mental Math Strategies

Display slide 27.

Warm Up – Sharing Strategies

Display slide 28.

Have participants respond to these questions using a Think-Pair-Share strategy.

Some sample responses are as follows:

For whole numbers:

- add or subtract tens and then ones
  - \(257 + 30 + 9 = 287 + 9 = 287 + 3 + 6 = 290 + 6 = 296\)
  - \(257 - 39 = 257 - 30 - 9 = 227 - 9 = 227 - 7 - 2 = 220 - 2 = 218\)

- when adding
  - whatever you add to one number, you subtract from other to keep the sum the same
    - add 1 to 39 to get 40, so subtract 1 from 257
    - \(257 + 39 = (257 - 1) + (39 + 1) = 256 + 40 = 296\)

- when subtracting
  - whatever you add to one number, you add to the other to keep the difference the same
    - add 1 to 39 to get 40 so add 1 to 257
    - \(257 - 39 = (257 + 1) - (39 + 1) = 258 - 40 = 218\)

For decimal numbers:

- add or subtract tens, ones, and tenths
  - \(25.7 + 3 + 0.9 = 28.7 + 0.9 = 28.7 + 0.3 + 0.6 = 29.0 + 0.6 = 29.6\)
  - \(25.7 - 3 - 0.9 = 22.7 - 0.9 = 22.7 - 0.7 - 0.2 = 22.0 - 0.2 = 21.8\)
• when adding, adjust the numbers to get friendly numbers
  ■ make 25.7 into 26 as follows
    \[ 25.7 + 3.9 = (25.7 + 0.3) + (3.9 - 0.3) = 26.0 + 3.6 = 29.6 \]
  ■ or make 3.9 into 4.0 as follows
    \[ 25.7 + 3.9 = (25.7 - 0.1) + (3.9 + 0.1) = 25.6 + 4.0 = 29.6 \]

• when subtracting adjust the numbers (especially the smaller number, the subtrahend) to get friendly numbers
  ■ change 3.9 to 4 by adding 0.1 as follows
    \[ 25.7 - 3.9 = (25.7 + 0.1) - (3.9 + 0.1) = 25.8 - 4.0 = 21.8 \]

**Mental Math Strategies Made Explicit – Problem #4**

**Understand the Problem and Make a Plan**

Display slide 29.

**Mental Math for Addition**

Ask participants to listen for details of what mental math strategy is represented. Be sure each strategy is made explicit through their oral reasoning about the strategy so that everyone can apply it to a different problem. Use a Think-Pair-Share strategy.

**Adding On Addition**

• decompose an addend to expanded form and create partial sums (i.e., think of 143 as 100 + 40 + 3 and add in 3 steps – hundreds, tens, and then ones)

**Compensation Addition**

• add a number to one addend to get a friendly number (e.g., like a multiple of a hundred or ten) and subtract that number at the end (i.e., 3 is added to 297 then subtracted from the partial sum 536)
Constant Sum Addition

- subtract an amount from one addend and add that amount to the other addend to make friendly numbers (e.g., numbers in the tens or hundreds)

**Mental Math for Subtraction**

Display slide 30.

Partial Subtraction

- subtracting the subtrahend from greatest to least place value (i.e., subtracting 146 by subtracting 100, then 40, and then 6)

Compensation Subtraction

- to make a number friendly (e.g., 300 is a friendly number for 296, so add 4 to make a friendly number and then remember to add 4 to the difference)

Constant Difference Subtraction

- add or subtract the same amount to the subtrahend and minuend to make a friendly number; the difference between the subtrahend and minuend remains constant

**Carry Out the Plan – Apply These Strategies**

Display slide 31.

Have participants work in pairs, so that one person says his or her mental math strategy aloud and the other person records, on a number line, what he or she hears. This enables both participants to practise the use of mental math strategies for addition and subtraction, as well as practise hearing and recording what others say mathematically. Remind the participants that visual support provides a record of the calculation that reflects the mathematical thinking. Students can look back at the sketch and discern the process used.
Adding-On Addition

\[ 637.45 + 219.18 = 856.63 \]
Add 219.18 onto 637.45 using 219.18 flexibly to use friendly numbers.

Compensation Addition

\[ 637.45 + 219.18 = 857.05 \]
Adding 0.82 changes 219.18 to 220. To keep the sum the same, subtract 0.82 at the end. (0.82 = 0.4 + 0.4 + 0.02)

Constant Sum Addition

\[ 637.45 + 219.18 = 800 + 56 + 0.63 = 856.63 \]
Add 1 and 0.02
Subtract 1 and 0.02

\[ 637.45 + 219.18 = 636.43 + 220.20 \]
Partial Subtraction

\[ 637.45 - 219.18 = \]

\[ 637.45 - 219.18 = 418.27 \]

Compensation Subtraction

\[ 637.45 - 219.18 = 6.37.45 - (219.18 + 0.82) \]
(subtracting more than required, so later must add 0.82)
\[ = (637.45 - 220) + 0.82 \]
\[ = 417.45 + 0.82 \]
\[ = 400 + 10 + 7 + (0.40 + 0.80) + (0.05 + 0.02) \]
\[ = 418 + 1.2 + 0.07 \]
\[ = 418.27 \]

Constant Difference Subtraction

\[ \begin{align*}
637.45 - 219.18 & \\
+ 1 & + 1 \\
\downarrow & \downarrow \\
638.45 - 220.18 & \\
+ 0.02 & + 0.02 \\
\downarrow & \downarrow \\
638.47 - 220.20 & \\
\downarrow & \\
= 418.27 & 
\end{align*} \]
Look Back – Reflect and Connect

Display slide 32. Have participants respond to these questions in pairs, then small groups. Walk about to hear the different responses and choose two or three key ideas to be shared during the whole group sharing of responses. Some sample responses are as follows:

Adding On Addition

- useful for adding any set of numbers because you can compose the addends one base ten unit at a time

Compensation Addition

- add a number to one addend to get a friendly number (e.g., like a multiple of a hundred or ten) and subtract that number added

Constant Sum Addition

- subtracting an amount from one addend and adding that amount to the other addend to make friendly numbers (e.g., numbers that have tens, hundreds)

Partial Subtraction

- subtracting the subtrahend from greatest to least place value (e.g., subtracting 146 by subtracting 100, then 40, and then 6)

Compensation Subtraction

- to make a number friendly – for example, 300 is a friendly number for 296, so adding 4 to make a friendly number but realizing you have subtracted 4 too many and then have to add 4 to the difference

Constant Difference Subtraction

- add or subtract the same amount to the subtrahend and minuend to make a friendly number with 5s and 10s; the difference between the subtrahend and minuend remains constant
Professional Learning Opportunities

Display slide 33.

- Share this slide with participants to provide them with some ways that they can further their professional learning of mathematics for teaching.
- Tell participants that ideas about co-teaching, coaching, and teacher inquiry/study can be found at the Literacy and Numeracy Secretariat Coaching Institute website.
- Highlight other e-learning sites, such as e-learning.ca and the numeracy webcast featuring Dr. Deborah Loewenberg Ball at www.curriculum.org.
References


Ministry of Education. (2006b). *Number sense and numeration, Grades 4 to 6: Volumes 1, 2, and 6*. Toronto: Queen’s Printer for Ontario.
Resources to Investigate

Books


E-Resources

Coaching Institute for Literacy and Numeracy Leaders, video on demand available at www.curriculum.org.

eworkshop.on.ca.

### Appendix 1: Math Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>addend</td>
<td>a number that is one of a set of numbers to be added</td>
</tr>
<tr>
<td>addition</td>
<td>the combining of numerical parts to create a whole</td>
</tr>
<tr>
<td>associative property</td>
<td>numbers can be added or combined in different ways without changing the sum (e.g., 4 + 5 + 2 can be added as (4 + 5) + 2 to yield 9 + 2 = 11). The same sum results from 4 + (5 + 2) = 4 + 7 = 11). This property holds true for multiplication as well.</td>
</tr>
<tr>
<td>commutative property</td>
<td>numbers can be added in any order without changing the sum (e.g., 4 + 5 = 5 + 4). This property holds true for multiplication as well.</td>
</tr>
<tr>
<td>compose</td>
<td>putting numbers together or joining them</td>
</tr>
<tr>
<td>decompose</td>
<td>taking numbers apart or separating them</td>
</tr>
<tr>
<td>difference</td>
<td>the amount by which one quantity is greater or less than another quantity</td>
</tr>
<tr>
<td>minuend</td>
<td>the number from which another number, the subtrahend, is being subtracted</td>
</tr>
<tr>
<td>regrouping</td>
<td>a process in which numbers are decomposed and their parts grouped in a variety of ways to facilitate adding or subtracting (e.g., numbers like 45 are represented as 4 tens + 5 ones or 821 is represented as 8 hundreds + 2 tens + 1 one).</td>
</tr>
<tr>
<td>subtraction</td>
<td>partitioning a whole into parts; the difference between two numbers</td>
</tr>
<tr>
<td>subtrahend</td>
<td>a quantity which is subtracted from another number (i.e., the minuend)</td>
</tr>
<tr>
<td>sum</td>
<td>the result of adding two or more quantities or numbers</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{addend} & \quad \text{addend} \\
+ \quad \text{addend} & \quad \text{sum} \\
\text{minuend} & \quad \text{subtrahend} \\
- \quad \text{difference}
\end{align*}
\]
Facilitator’s Handbook
Number Sense and Numeration, Grades 4 to 6
with reference to material in Volumes 2 and 6

Understanding Addition and Subtraction of Whole and Decimal Numbers

The Literacy and Numeracy Secretariat Professional Learning Series

Ontario