

	<p>Math Learning Goals</p> <p>Students will:</p> <ul style="list-style-type: none"> • make connections between various types of payment for work and their graphical representations • represent weekly pay, using equations and graphs • reason about rates of commission and salary from graphical representations of weekly pay • select strategies for computing rates of commission and total pay 	<p>Materials</p> <ul style="list-style-type: none"> • BLMs 1.1, 1.2
<p>Minds On...</p>	<p>Whole Class/Small Groups → Thinking about Earning Money</p> <p>As a class, discuss the different methods (salaries, wages, commissions) that are used to pay employees for their work.</p> <p>Working in groups, students identify various types of jobs they know of that are paid a salary, an hourly wage, or a commission, and discuss the advantages and disadvantages of each type of payment method.</p> <p>Students extend their thinking by responding to the question:</p> <p>“Aside from wages, salaries and commissions, how else can you earn money?”</p>	<p>Tip PPQ provides probing questions to guide students’ discussion.</p> <p>Sample answers:</p> <ul style="list-style-type: none"> • Earn profits from your own business • Create a product or service • Invest in a business (e.g., shares or stock) • Earn interest from money saved
<p>Action!</p> <p>A for L</p>	<p>Pairs → Connecting Linear Graphs and Financial Contexts</p> <p>In pairs, students examine a series of graphs and explain the events illustrated, using their understanding of hourly wages, salaries, and commissions. (BLM 1.1)</p> <p>Student pairs form larger groups to share findings and discuss what they could expect to earn (salaries, hourly wages, rates of commission) for several after-school jobs in their community.</p> <p>Individual → Problem Solving and Financial Literacy</p> <p>Students apply their understanding of linear relations in problem-solving contexts that connect to financial literacy and build on their acquired knowledge of wages and salaries. (BLM 1.2 – Problems 1-2)</p> <p>Mathematical Process/ Selecting Tools and Computational Strategies/ Observation: Encourage students to justify or improve their selection and use of strategies to compute total pay, using wages and commission.</p>	<p>Tip Suggestions for further discussion topics</p> <p>Examples of terms students might use:</p> <ul style="list-style-type: none"> • Step commission based on number of sales • Weekly salary plus commission • Hourly wage with overtime
<p>Consolidate Debrief</p>	<p>Individual → Practice in Solving Financial Literacy Problems</p> <p>Students solve problems, using information gained by reading graphs. (BLM 1.2 – Problems 3-4)</p>	<p>DI</p> <p>Support individual or small groups, using scaffolding questions on PPQ</p>

Lesson 1: Earning Money

<p>Goals(s) for the Lesson</p> <p>Students will:</p> <ul style="list-style-type: none">• make connections between various types of payment for work and their graphical representations• represent weekly pay, using equations and graphs• reason about rates of commission and salary from graphical representations of weekly pay• select strategies for computing rates of commission and total pay
<p>Curriculum Expectations</p> <ul style="list-style-type: none">• Construct tables of values, graphs, and equations, using a variety of tools to represent linear relations derived from descriptions of realistic situations• Determine values of a linear relation, by using a table of values, by using the equation of the relation, and by interpolating and extrapolating from the graph of a relation• Compare the properties of direct variation and partial variation in applications and identify the initial value• Describe a situation that would explain the events illustrated by a given graph of a relationship between two variables• Determine other representations of a linear relation, given one representation• Describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied
<p>Big Idea(s) Addressed by the Questions and Tasks</p> <p>Financial Literacy</p> <ul style="list-style-type: none">• There are various ways that people can earn money (e.g., salaries, wages, and commissions).• There are advantages and disadvantages to each type of payment method. <p>Mathematics</p> <ul style="list-style-type: none">• Different representations of relationships highlight different characteristics or behaviours and can serve different purposes.• Limited information about a relationship can sometimes allow us to predict other information about the relationship.

Considerations for Planning

Discussion Topics

During class discussions there may be opportunities for conversations about topics such as the changing employment market, the opportunities for entrepreneurship, the impact of advertising on consumer spending, and the influences on the wages or salaries that students can anticipate earning.

As students engage in the tasks, capitalize on opportunities to present strategies for saving money (e.g., pre-authorized withdrawals to a savings plan) or for managing the spending of money (e.g., identifying needs vs wants, prioritizing potential spending)

Lesson 1: Earning Money

Posing Powerful Questions

Minds On... Sample Question(s)

What method or methods of payment might be used to pay an employee? (Open-ended question)

Identify a type of job that commonly follows each of these methods of payment. (Open-ended question)

Probing Questions

- What jobs do you know that pay an hourly wage?
- What jobs have a weekly, monthly, or annual salary?
- What jobs do you think pay a commission? Explain your thinking.
- What jobs do you think might pay a combination of salary and commission? Explain your thinking.
- What do you think a “step commission” might be?
- If you owned a business, what are the advantages and disadvantages of paying your employees with hourly wages? With a fixed salary? With commissions?

Action! Sample Question(s)

Probing Questions

- What kinds of jobs pay overtime?
- What do the terms “time and a half” and “double time” mean?
- What information is on the graph to help you decide if this models an hourly wage or a commission?
- What are the characteristics of a graph displaying a salary?

BLM 1.2

Problem 1

Scaffolding Questions *(posed to individuals as needed)*

- How much would Marie earn if she sold nothing in a week? What does this tell you about the equation required to represent her total weekly earnings?
- Sketch the relationship to help you answer the question. What are 2 points that would be on the graph? Explain your thinking.
- Will Maria’s total weekly earnings be more or less than \$350? Why?

Problem 2

Scaffolding Questions *(posed to individuals as needed)*

- How does rate of commission connect to the numbers in your chart?
- Sketching the relationship may help you answer the question. How does Marco’s rate of commission connect to the graph? How can you read Marco’s pay from your graph?
- How do your answers in a) and b) help you form the equation?

Consolidate/Debrief Sample Question(s)

BLM 1.2

Problem 3

Scaffolding Questions *(posed to individuals as needed)*

- What is the difference between a wage and a salary?
- If Rene work 46 hours in a week, how many hours would be paid at the overtime rate?
- What do you look for on a graph to determine whether Rene is paid a wage or a salary?
- If Rene is asked to work a lot of overtime, what method of pay would be better, a wage or salary? Explain your thinking.

Problem 4

Scaffolding Questions *(posed to individuals as needed)*

- What information on the graph helps you to identify how Kenji is paid?
- What is commission?
- Why do you think some people are paid commission?
- How would you calculate 5% commission on a sale of \$500?
- Why do you think the rate of commission changes for sales over \$2000?

BLM 1.1—Earning Money

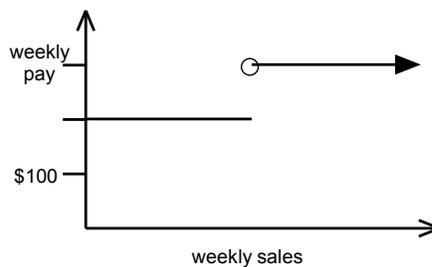
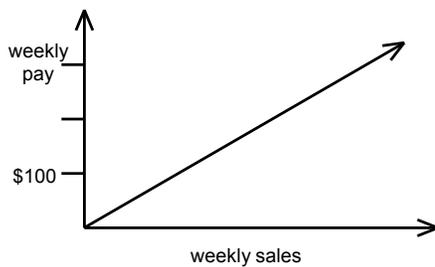
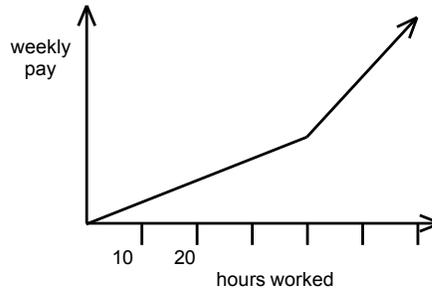
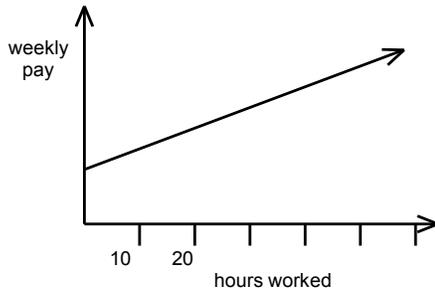
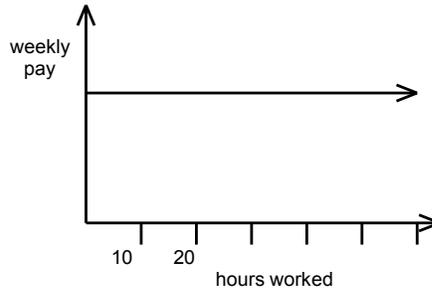
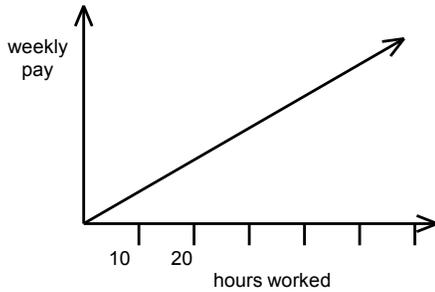
When you work in a job that pays an **hourly wage**, you receive a certain dollar amount for each hour you work. You may receive a higher **overtime** rate for hours you work beyond a stated number of hours per week.

When you work in a job that pays a **salary**, you receive a certain dollar amount per week or per month. You may be expected to be at work between certain hours of the day, but earnings are not directly linked to the number of hours you work.

Commissions are often earned by salespersons and are usually a certain percentage of the value of sales.

Examine the following graphs and describe in words what method or methods of payment a worker is receiving.

Identify a type of job that commonly follows this method.



BLM 1.2—Earning Money

Problem 1

Maria is starting a job at Company XYZ. She is offered 3 options for her pay. Choose one option and answer the questions.

Option 1: Maria earns a weekly salary of \$350 plus a 4% commission on her total weekly sales.

Option 2: Maria earns 10% commission on her total weekly sales.

Option 3: Marie earns a weekly salary of \$350 regardless of her total weekly sales.

- Does this situation represent a direct variation or a partial variation? Explain your answer.
- Create an equation to represent Maria's total weekly earnings E in dollars, if s represents Maria's total weekly sales in dollars.
- What are Maria's total weekly earnings on total sales of \$6000?

Problem 2

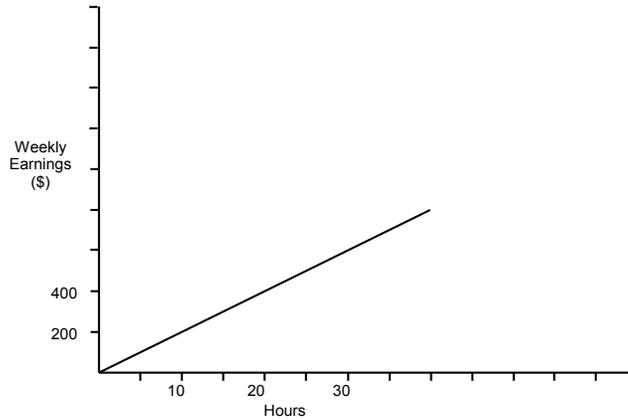
Marco earns a base salary each week and a commission based on his total sales.

Sales (\$)	Total Pay (\$)
8 000	710
9 000	730
10 000	750

- Determine Marco's rate of commission.
- Determine Marco's base salary.
- Determine Marco's total pay when sales are \$26 850.
- Create an equation to model Marco's weekly earnings. Clearly identify your variables.

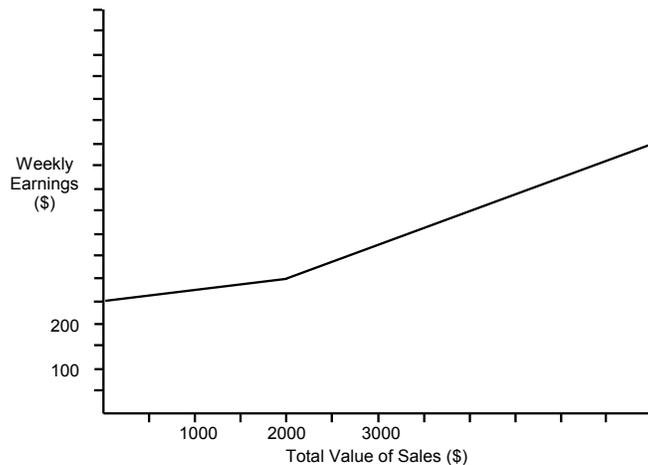
BLM 1.2—Earning Money *continued*

Problem 3



- The graph shows Rene’s weekly earnings.
 - a) Is Rene paid a wage or a salary? Explain how you know.
 - b) Use the information from the graph to determine her rate of pay.
- Rene is paid ‘time and a half’ for hours worked over 40 hours per week. Complete the graph to show weekly earnings if Rene works between 40 hours and 60 hours in a week.
- If E is Rene’s weekly earnings in dollars, and h is the number of hours she worked in a week, create an equation that could be used to calculate Rene’s weekly wage if she were to work over 40 hours in a week.

Problem 4



The graph shows Kenji’s weekly earnings.

- a) Describe in words how Kenji is paid for his work. Explain your reasoning.
- b) What is Kenji’s guaranteed weekly salary?
- c) Using the graph, estimate the value of sales that will provide weekly earnings of approximately \$500.
- d) Determine the rate of commission on sales under \$2000.
- e) Determine the rate of commission on sales over \$2000.
- f) Use information from the graph to calculate Kenji’s exact weekly earnings on sales of \$7238. Justify your answer.
- g) Identify a job that could have a wage plan similar to that shown on the graph. Explain your answer.

	<p>Math Learning Goals</p> <p>Students will:</p> <ul style="list-style-type: none"> • make connections between the mathematical representations used in various types of financial accounts – savings, chequing • represent ways of saving, using equations and graphs • reason about which financial accounts are best for them to use for their savings • communicate features of graphs, using chequing account contexts. 	<p>Materials</p> <ul style="list-style-type: none"> • BLMs 2.1, 2.2, 2.3 — Student, • BLM 2.3— Teacher
<p>Minds On...</p>	<p>Individual/Small Groups → Gathering Data about Making Your Money Work</p> <p>Students gather information from pamphlets from a variety of financial institutions (banks, credit unions, trust companies) that describe the various accounts and services available to help them save money.</p> <p>Individually, they select a type of savings account and a type of chequing account that they think would be best for them. In small groups, they discuss and give the reasons for their selections.</p>	<p>PPQ provides probing questions to guide students' investigation.</p>
<p>Action!</p>	<p>Whole Class → Discussion</p> <p>Discuss the types of fees and charges that consumers pay on savings and chequing accounts.</p> <p>Individual → Connecting Graphs and Financial Contexts</p> <p>Students use accurate terminology and their understanding of chequing accounts to analyze and model linear relations. (BLM 2.1)</p> <p>As students work through the problems, they describe rates of change in terms of transaction fees, and initial values in terms of monthly service charges. They create graphs and algebraic equations that show their understanding of chequing accounts and bank service charges.</p>	<p>Tip</p> <p>Use actual chequing account information to describe and model the plan as a linear relation by creating a graph or an algebraic model.</p>
<p>Consolidate Debrief</p>	<p>Individual → Practising Linear Relations Skills</p> <p>Students examine effective strategies for saving money as they investigate savings accounts. They model their findings, using linear relations. (BLM 2.2)</p> <p>Individual → Investigating Savings and Non-Linear Relations</p> <p>Using a spreadsheet or programmable calculator, students investigate how their money will grow if they make regular deposits into accounts earning interest.</p> <p>Home Activity or Further Classroom Consolidation</p> <p>Students investigate the validity of two important financial strategies - save early in life, and save regularly. They construct a scatter plot for non-linearly related data to determine that by <i>saving early</i> and <i>saving regularly</i>, interest works to make their savings grow. (BLM 2.3— Student)</p>	<p>Suggestions for further investigation</p>

Lesson 2: Saving Money

Goals(s) for the Lesson

Students will:

- make connections between the mathematical representations of various types of financial accounts – savings, chequing
- represent ways of saving, using equations and graphs
- reason about which financial accounts are best for them to use for their savings
- communicate features of graphs, using chequing account contexts.

Curriculum Expectations

- Construct tables of values, graphs, and equations, using a variety of tools to represent linear relations derived from descriptions of realistic situations
- Determine values of a linear relation, by using a table of values, by using the equation of the relation, and by interpolating and extrapolating from the graph of a relation
- Compare the properties of direct variation and partial variation in applications and identify the initial value
- Describe a situation that would explain the events illustrated by a given graph of a relationship between two variables
- Determine other representations of a linear relation, given one representation
- Describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied

Big Idea(s) Addressed by the Questions and Tasks

Financial Literacy

- There are various ways that people can save money (e.g., savings accounts, chequing accounts).
- Starting to save early in life is an effective saving plan.

Mathematics

- Information and relationships can be represented in different ways for different purposes.
- Limited information about a relationship can sometimes allow us to predict other information about the relationship.

Considerations for Planning

Suggestions for further investigation

- Students investigate and define terms such as *debit card*, *internet banking*, *direct deposit*, *minimum balance fees*, *overdraft protection*, and *NSF cheques*.
- Students investigate other ways to save money (Guaranteed Investment Certificates (GIC), Tax-free Savings Accounts (TFSA)) and respond to the question: What are some advantages and disadvantages of GIC's when compared to regular savings accounts? (e.g., higher rate of interest, more difficult to access money)

Lesson 2: Saving Money

Posing Powerful Questions

Minds On... Sample Question(s)

What types of accounts are available to help you save money? (Open-ended question)

Identify a type of savings account and chequing account that you think would be best for you. (Open-ended question)

Probing Questions

- Why should we save money?
- What are different ways you use to save money?
- What types of accounts do financial institutions have to help us save money?
- What are the different transaction fees and service fees?
- What are the factors that may cause someone to choose a type of account? (e.g., how often they use the account, the services that they need, limited awareness of other options that are available)
- What are the different types of services that a chequing account may provide if you pay a monthly service charges (e.g., overdraft protection)?
- Which type of account usually pays more interest?
- Why would you put money into a chequing account if there are fees charged for using the account?
- If you need to make withdrawals from an account every few days, which type of account would be the best to use?
- If you only make a few withdrawals each month and don't need to write cheques, what account might be best for you?

Action! Sample Question(s)

Discuss the types of fees and charges that consumers pay on savings and chequing accounts.

Probing Questions

1. Why do fees and service charges exist?
2. Why might a bank offer reduced fees and service charge packages for students?
3. What are some of the trade-offs among the fees and service charge packages?
4. Describe how having more than one type of bank account can help you save money. Give numerical examples to illustrate your points.

BLM 2.1

Problem 1

Scaffolding Questions *(posed to individuals as needed)*

- Why is the monthly cost of the Thrifty account so much lower than the other two?
- How will a graph help you pick the right chequing account?
- Describe the needs of a person you think should select the Super Chequing Account.

Problem 3

Scaffolding Questions *(posed to individuals as needed)*

- What fees might be related to the different accounts?
- How can you use the graph to match a person to an account choice?
- What conditions would make you recommend one account over the other?

Lesson 2: Saving Money

Posing Powerful Questions

continued

Consolidate/Debrief Sample Question(s)

BLM 2.2

Problem 1

Scaffolding Questions (*posed to individuals as needed*)

- What is the key difference between the two plans?
- What further information would you need to calculate the total savings for each person after one year?
- If Juan's and Gina's pay were increasing each year, which plan would lead to the most savings?

Problem 2

Scaffolding Questions (*posed to individuals as needed*)

- How much is Sharon saving per week?
- How can you estimate how much Changying is saving per week?
- What happens if Henri earns only \$200 per week?
- How much would Changying save after two weeks of earning \$1,000 per week?

Problem 3

Scaffolding Questions (*posed to individuals as needed*)

- Why would a person have a chequing and a savings account?
- What does **pre-authorized** mean?
- Why do they call the PAD method "paying yourself first"?

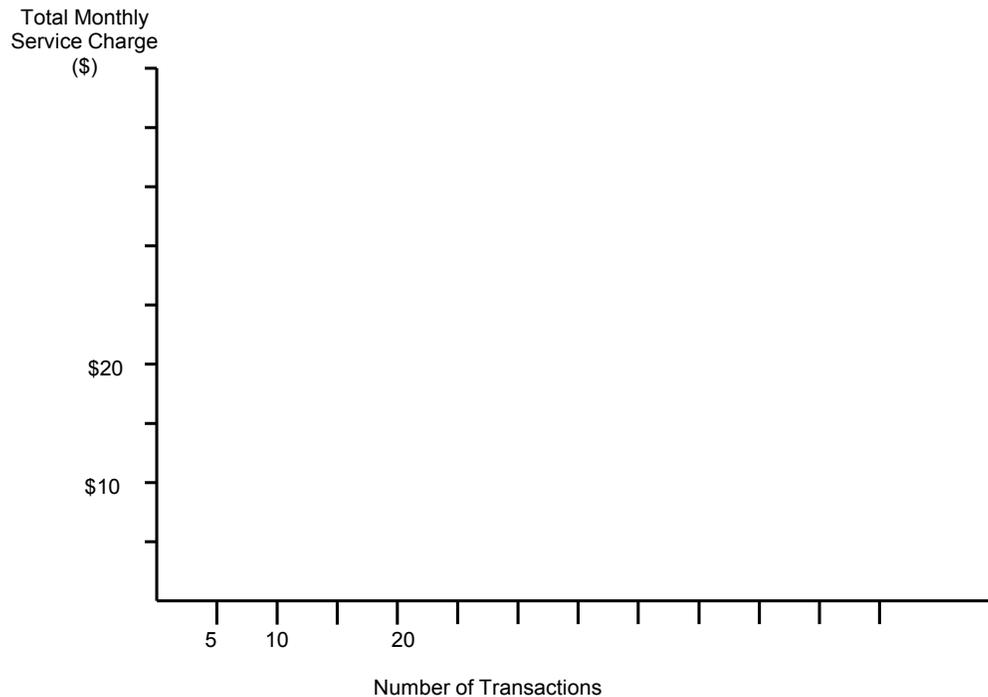
BLM 2:1 — Saving Money

A chequing account is used for money you need now or plan to use soon. Financial institutions offer many different types of chequing accounts with various service fees. These fees are for services and transactions such as paying a bill with a cheque, withdrawing money from an automated bank machine (ABM), or using your debit card at a store.

Problem 1

A credit union offers Sakari three different types of chequing accounts:

- Thrifty Chequing Account is \$4 per month, plus \$1 per transaction with the first 5 transactions free.
- Regular Chequing Account is \$10 per month, plus \$1.50 per transaction with the first 20 transactions free.
- Super Chequing Account is \$16 per month with unlimited transactions.



- Graph the three relations on the same set of axes. Label each type of account clearly.
- If Sakari makes 8 transactions in a month, which account type is cheaper? Justify your answer.
- If Sakari makes 30 transactions in a month, which account type is cheaper? Justify your answer.
- For what range of transactions is the Thrifty Chequing Account less expensive than the Regular Chequing Account?
- For what range of transactions is the Regular Chequing Account less expensive than the Super Chequing Account?
- How many transactions per month are needed for the Super Chequing Account to be the least expensive of all three accounts?
- If S is the total monthly service charge in dollars, and t is the number of transactions per month, create an equation to represent each chequing account. Are your equations valid for all values of t ? Explain.
- Use the equations to verify your answers to d) and e). Show your work.
- Before selecting a chequing account plan, why is it important to carefully consider the service charges? Explain in your thinking.

BLM 2:1 — Saving Money

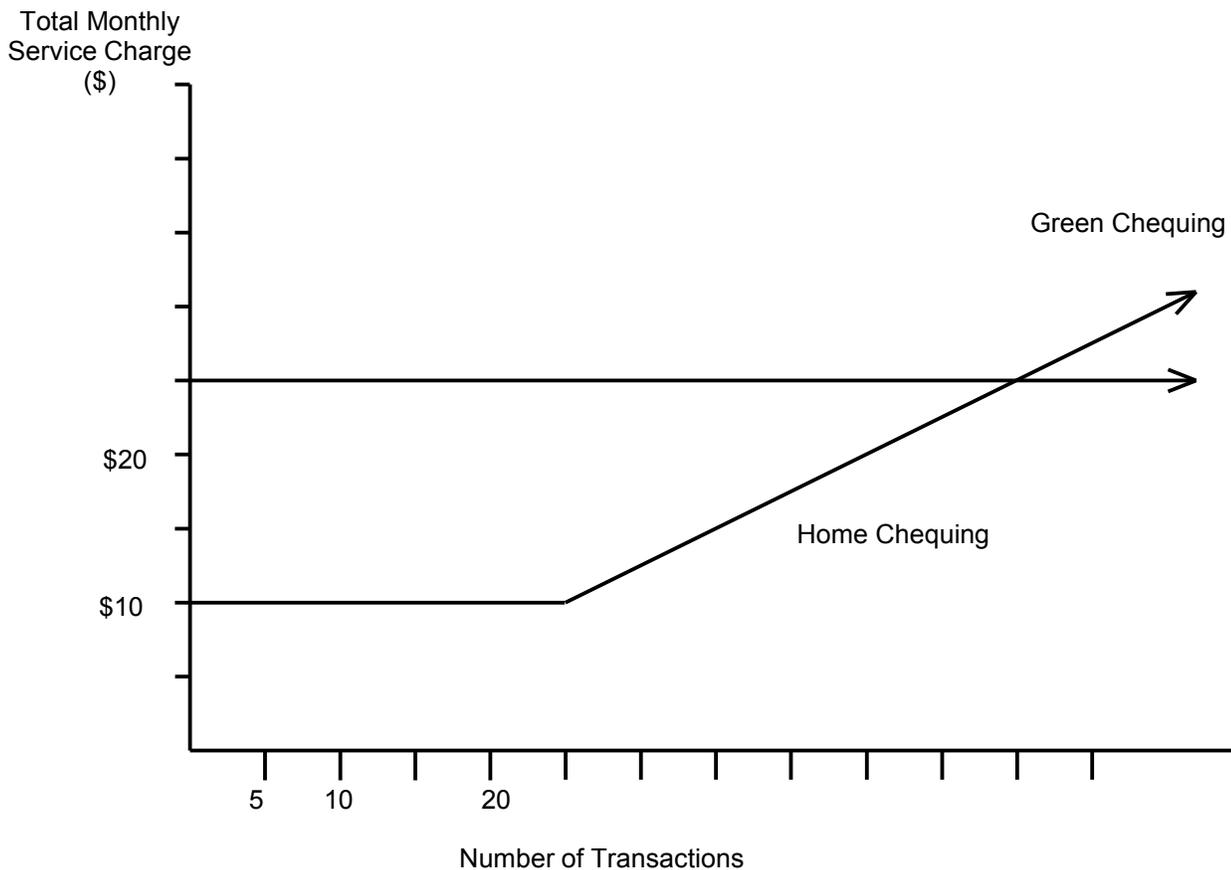
Problem 2

Chequing accounts offer a useful way to help manage money, but they may not be the best way to save money.

- Do you agree or disagree with this statement? Explain.
- What are the advantages of a savings account over a chequing account?
- What are the advantages of a chequing account over a savings account?

Problem 3

A trust company offers two different types of chequing accounts: The Green Chequing account and the Home Chequing account.



- Create a flyer to describe each chequing account. Write a description of each account in your own words to inform the reader of any service charges and transaction fees. Use the graph to identify or calculate these fees. Show your work.
- A customer must decide between the Green Chequing and the Home Chequing accounts. Based on the information in the graph, describe the conditions that would make you recommend the use the Green Chequing account rather than the Home Chequing account.
- If S is the total monthly service charge in dollars, and t is the number of transactions per month, create an equation to represent each chequing account. Are your equations valid for all values of t ? Why or why not?

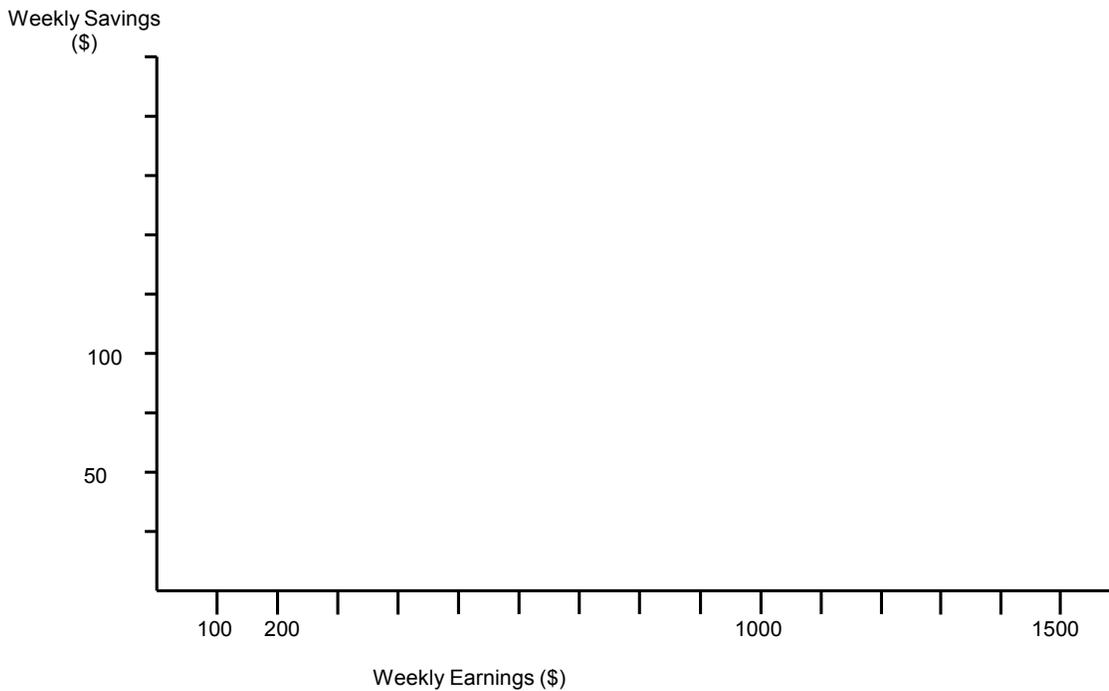
BLM 2:2—Saving Money

Problem 1

Juan works in retail and plans to save 10% of his earnings each week.

Gina works at a trust company and plans to save \$50 plus 5% of her weekly earnings each week.

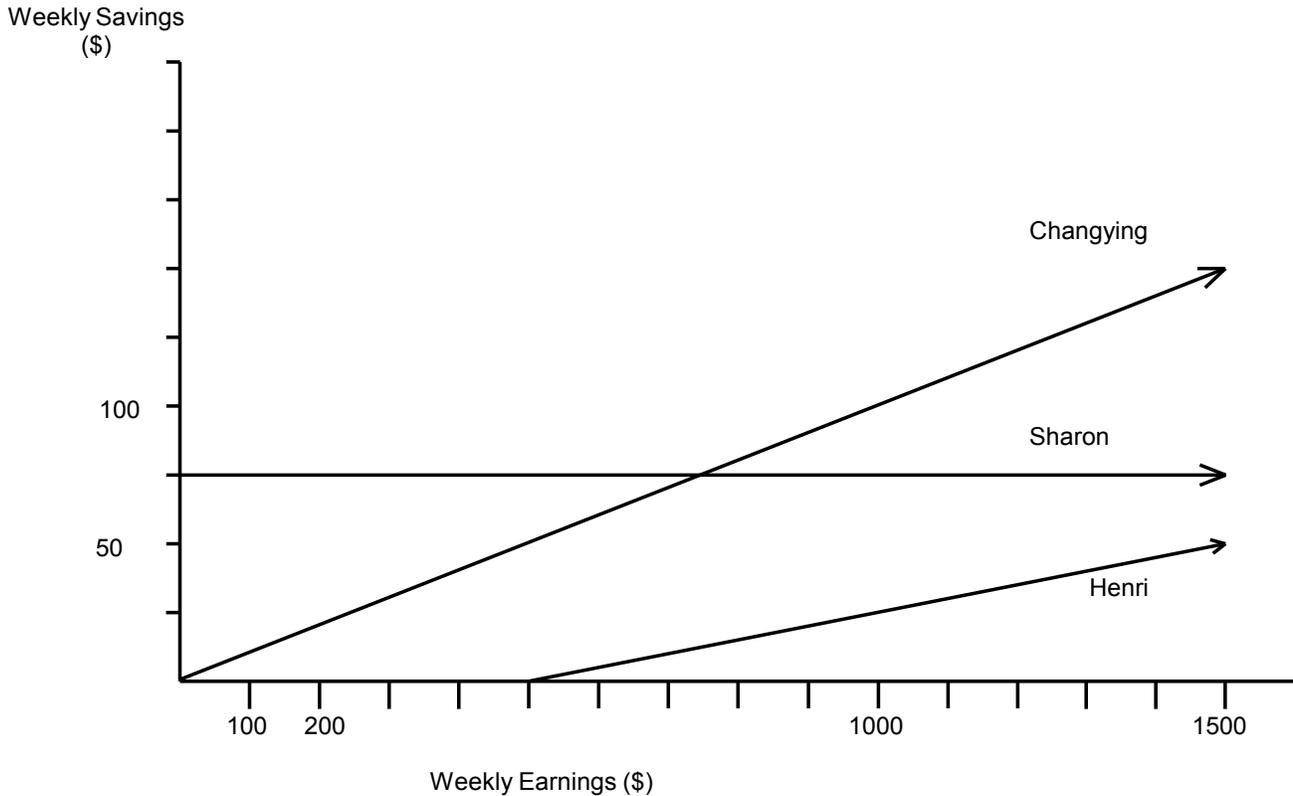
- Is Juan's plan an example of direct variation or indirect variation? Explain how you know.
- Is Gina's plan an example of direct variation or indirect variation? Explain how you know.
- On the same grid, construct a graph to represent the relation of weekly savings and weekly earnings for both Juan and Gina.
- If their weekly savings are S dollars and their weekly earnings are e dollars, express each of their plans as an equation.
- For what weekly earnings will Juan and Gina each have the same weekly saving?
- Under what conditions is Juan's or Gina's saving plan the better plan to use? Explain your thinking.



BLM 2:2—Saving Money *continued*

Problem 2

The following graph shows the savings plan for 3 students. The students are making weekly savings deposits based on their weekly earnings.



- Describe each student's weekly savings plan.
- Write an equation to model each of their savings plans if S is their weekly savings in dollars, and e is weekly earnings in dollars.
- Which savings plan do you think is best for long term saving? Explain your reasoning.

Problem 3

Financial institutions allow their customers to request pre-authorized deposits (PAD) to help with their saving plan. When your paycheck is deposited into your chequing account, the PAD allows the bank to automatically transfer a fixed amount from your chequing account and deposit it to your savings account.

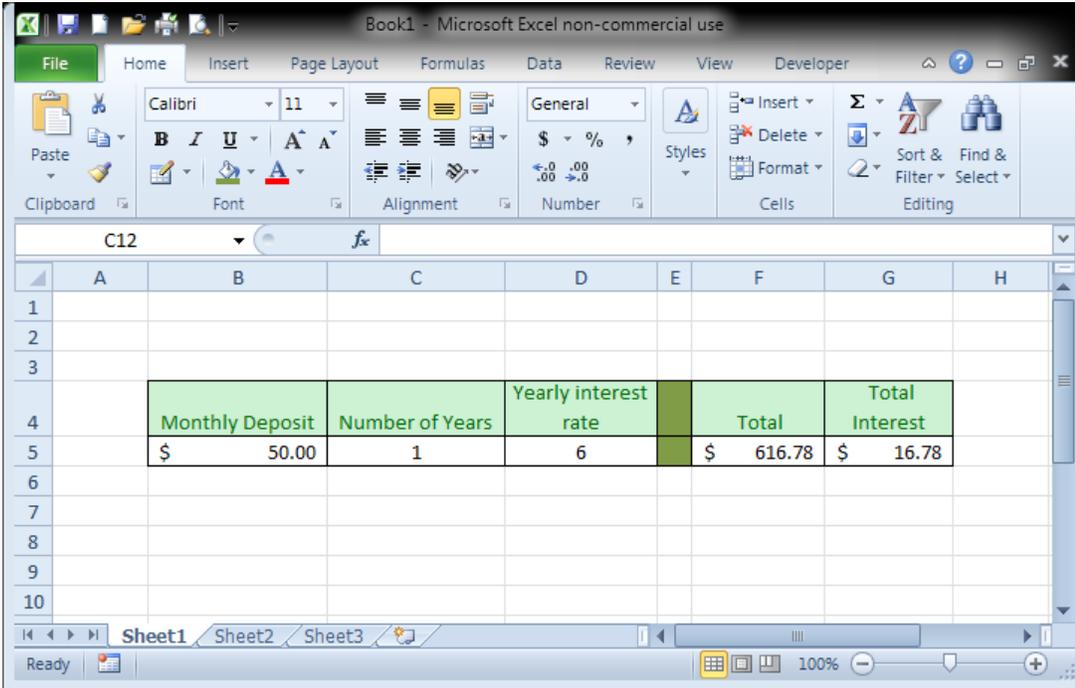
- Many people call this method of saving "paying yourself first." Do you think that this is a good way to save? Why or why not.
- Some people try to save the same amount each week, and others try to save a certain percentage of each paycheck. Which method do you think is the best? Explain your thinking.

BLM 2:3—Student: Saving Money

Watching Money Grow

The best way to save money is to start saving early and to save regularly so your money has time to earn more and more interest. When you leave money in savings investments for long periods of time, you begin to make interest on your interest. This is known as compound interest. In this task, you will investigate how much your savings can grow.

If you were able to make a deposit of \$50 every month into a savings account, how much money would you have after 1 year? After 5 years? After 20 years?



- Use your spreadsheet to complete the following table.
 - Enter \$50 as your monthly payment, 1 for the number of years, 6% as your interest rate.
 - The total amount in the account, \$616.78 appears in column F and total interest earned, \$16.78, appears in column G.
 - Record these values in the table below.
 - Complete this table. Notice that the number of years that you make monthly payments changes, but the monthly deposit and interest rate remain the same

Monthly Deposit	Number of Years	Yearly Interest Rate	Total in Account	Interest Earned
\$50	1	6%	616.78	16.78
\$50	2	6%		
\$50	5	6%		
\$50	10	6%		
\$50	15	6%		
\$50	20	6%		
\$50	25	6%		
\$50	30	6%		
\$50	35	6%		
\$50	40	6%		

BLM 2:3—Student: Saving Money *continued*

Watching Money Grow

2. Make a scatter plot of the total money in the account versus number of years in the account.
 - a) Is this a linear relationship? Why or why not?
 - b) Describe the shape of the graph and explain how regular savings can make your money grow.

3. Complete the table for regular monthly deposits of \$50 with the interest rate of 12% each year.

Monthly Deposit	Number of Years	Yearly Interest Rate	Total in Account	Interest Earned
\$50	1	12%		
\$50	2	12%		
\$50	5	12%		
\$50	10	12%		
\$50	15	12%		
\$50	20	12%		
\$50	25	12%		
\$50	30	12%		
\$50	35	12%		
\$50	40	12%		

- a) How does this change the totals in your account and the shape of your graph?
- b) How has doubling the interest rate changed the total in your account?
- c) Examine the interest that you have earned from your savings. Why do financial planners suggest that we start saving early and save regularly?

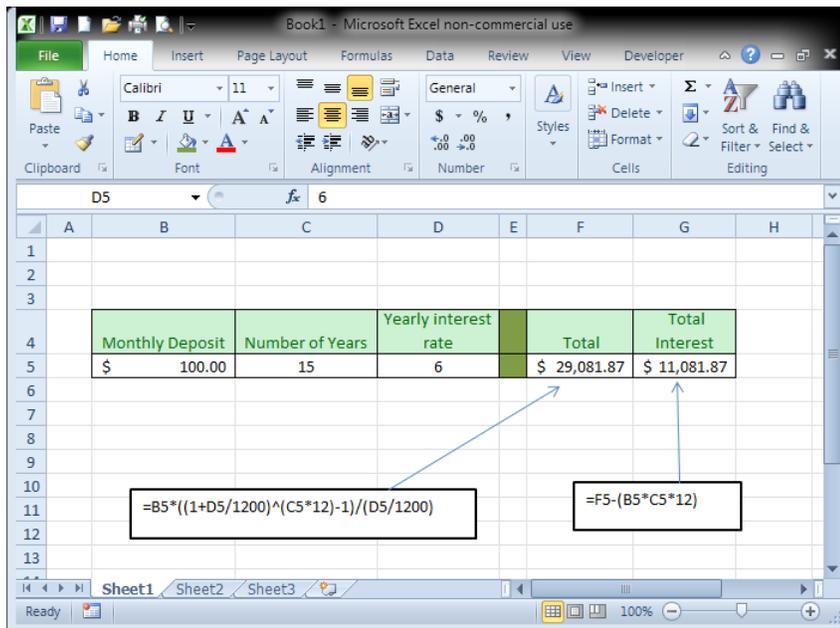
BLM 2:3—Teacher: Saving Money

Rather than introducing the compound interest formula, students can use spreadsheets or graphing calculators with required formulas pre-entered by the teacher. The required formulas are highlighted in the screen capture below. The formula for F5 is for an ordinary annuity.

Students enter monthly deposit, number of years and yearly interest rate and then record the total value (cell F5) and total interest (G5) on their recording sheet.

Students use a spreadsheet, graphing software, or graph paper to plot the points derived from their investigation. They identify the scatter plot as a non-linear relation and analyze the dramatic growth of their savings plan.

Example:



Tip

If spreadsheets are not available, there are websites with annuity calculators that can also be used.

Suggestions for Adapting the Task

- The sample handout uses a monthly deposit of \$50. Choose regular monthly deposits that are realistic for your students based on their life experience.
- Use interest rates that are consistent with the current market.
- Introduce the concept of exponential growth, and how some savings may grow slowly in the beginning, but compounding allows savings to grow more rapidly the longer that you keep saving.
- Explain that it is not realistic to have an interest rate and monthly payment remain constant for 40 years as shown in the table. Emphasize that the task is to illustrate how money can grow. If interest rates increase, your money will grow faster and if interest rates decrease then your money will grow slower. Also, as you earn money you may increase your monthly deposit, which allows your money to grow that much faster.
- Encourage students to explore additional possibilities, using their spreadsheet and graphs. Change the annual interest rates, the monthly payment, and the number of years to show the impact of allowing money to grow over time.

BLM 2:3—Teacher: Saving Money *continued*

- Interested students could explore the task using the ordinary annuity formula

$$A = R \times \left(\frac{(1 + i)^n - 1}{i} \right)$$

where A is amount, R the regular payment, n the number of payments, and i the interest per payment. Depending on their skills, students could complete the entire table in a spreadsheet and use the spreadsheet graphing features to draw their scatter plot. A sample of a completed table and scatter plot are shown for a regular monthly deposit of \$100.

