**Math Content: Fractions**

**RESEARCH STORY**

**Math Content: Fractions**

Understanding Fractions

In this fractions action research project, gaining a deeper understanding of the various conceptions of fractions allowed teachers to better understand and respond to student thinking.

The part-whole relationship of simple fractions is highly emphasized across North American mathematics programs. Most students involved in this project demonstrated a consistently solid understanding of a half of an item, and when presented in the context of sharing food, also identified the need for the two pieces to be of equal size or area. This comfort level was evident when working with students to explore common benchmark fractions, such as $\frac{1}{3}$and $\frac{3}{4}$. However, when asked to represent less common fractions, such as$\frac{2}{5}$or to interpret the fraction $\frac{6}{6}$, many grade 4 through 7 students struggled. Through this research project it was evident that students switched indiscriminately between the use of:

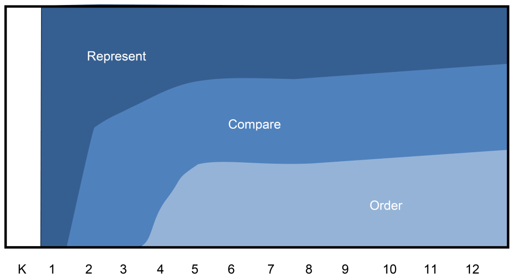
- fractions as part of a whole using a set representation,
- fractions as part of a whole using an area representation, and
- fractions as a ratio (part-part).

These meanings and representations of fractions occur throughout the elementary and secondary Ontario curriculum.

Researchers have identified multiple ways of understanding, perceiving and representing fractions (Lamon, 1999; Marshall, 1993; Mosely & Okamoto, 2007). This research informed the development of the Math for Teaching: Ways We Use Fractions document, which highlights multiple interpretations and representations of a fraction depending on the context or situation. These include interconnected understandings of fractions as:

- Linear Measures: situations in which a rational number's distance from zero is important
- Part-Whole Relationships: situations in which a part is compared to the total amount
- Part-Part Relationships: ratio situations in which separate quantities are compared
- Quotient perceptions: situations that highlight the process or result of a division
- Operator perceptions: situations in which the role of a rational number in enlarging or shrinking a quantity is highlighted

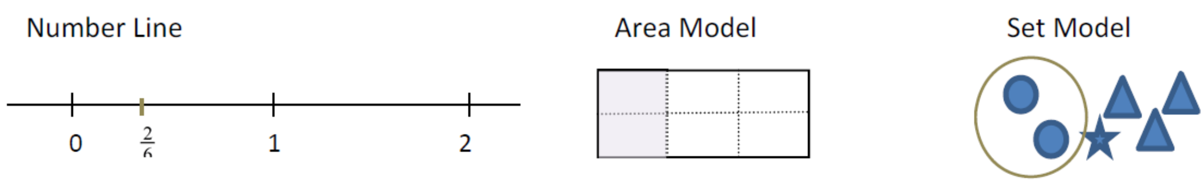
The junior grades represent a key learning period for students with respect to fractions as they move beyond part-whole with continuous models to work with sets as well as part-part relationships. Within the junior curriculum, students are expected to be able to represent, compare, and order fractions. The development of these actions K-12 is shown and further explored below.




In this fraction collaborative action research project, gaining a deeper understanding of the ways we use fractions allowed teachers to better understand and respond to student thinking.

## Representing

**Representing** refers to the process of using symbolic, concrete and pictorial representations as well as words and relevant situations to explore concepts and communicate understanding. There are three general models that Watanabe (2002) illustrates for representations of fractions: linear (number lines), area (partitioned regions), and discrete (set models).



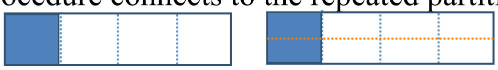

*Note: Within the Ontario curriculum (2005), symbolic notation (e.g.$\frac{2}{3}$) is introduced formally in grade 4. Prior to this, familiarity with fraction terminology such as halves, fourths, fifths supports student understanding.*

Research Findings:

Existing literature and the findings of this this fractions research project indicates that students should be exposed to number lines and rectangular area models in early grades. These representations support students in understanding the notion that a fraction is a number (for example $\frac{1}{3}$ is a number) as well as enable students to create equal partitions using a variety of strategies. Students frequently make incorrect conclusions when comparing fractions that are close in value using a circle model (such as $\frac{4}{10}$ and $\frac{1}{3}$) as it is difficult to accurately partition a circle into ten equal-sized parts. Flexibility and purposefulness with representations enables students to make selections most appropriate to the context (such as a number line for distance).

## Comparing

**Comparing** requires students to determine which fraction is the larger or the smaller. Comparing fractions assumes that the two fractions have the same whole, so is based in the part to whole meaning of fractions. Students should understand a fraction as a number and use benchmarks, such as 0, 1, $\frac{1}{2}$, and $\frac{3}{4}$ to compare fractions. There are a variety of strategies for comparing fractions beyond determining a common denominator, including consideration of the relationship between the numerator and denominator. For example, students could quickly compare $1\,\frac{134}{2025}$ and $1\frac{1963}{2000}$ by realizing that the first fraction is closer to 1 since there is a greater difference between the numerator and denominator than in the second fraction. Older students may use their knowledge of fraction as a quotient to determine decimal equivalencies for comparison.

Research Findings:
In this fractions research project, students frequently doubled the numerator and denominator of a fraction to generate equivalent fractions but demonstrated little understanding of how this procedure connects to the repeated partitioning of a measure, such as area ($\frac{1}{4} = \frac{2}{8}$ or
= 
). Many students did not have consistent success with determining equivalent fractions by merging pieces, such as $\frac{18}{36} = \frac{9}{18}$.

## Ordering

**Ordering** refers to the process of determining the relative size of a number of fractions by placing them in order from smallest to largest or largest to smallest. Students used their understanding of fractions to compare and order fractions in a variety of manners. Ordering fractions on a number line requires students to consider the magnitude of the fraction in comparison to whole numbers, benchmark fractions, and the other fractions they have placed on their number line. This relative comparison engages students in reasoning and proving with each successive placement, allowing them to correct previous errors as necessary.

Research Findings:
In the fractions research project, students who developed a deep conceptual understanding of fractions and flexible processes for comparing fractions were able to successfully order fractions on a number line using a variety of strategies. This extended to decimals, percents, and mixed numbers.