MATHEMATICAL PROCESS—REASONING AND PROVING

THE ONTARIO CURRICULUM, MATHEMATICS, 2005

Students will develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments.

Students make sense of mathematics through reasoning. An organized, analytical, well-reasoned approach to learning mathematical concepts and processes and to solving problems requires an emphasis on reasoning.

SAMPLE FEEDBACK

- What you have presented is true for the cases you considered. Explain some cases where this situation will not be true.
- Present your solution, showing all the steps so someone else will understand your thinking.
- Your reasoning was good to this point in your argument. Study your next point to see if you can identify the flaw in reasoning.
- Identify the flaw in this argument. How would you correct it?
- How does this reasoning follow from what you said?
- Before you *implied*, and now you are *saying*. How can both be true?

ROLE OF STUDENTS

Hypothesize/make conjectures

- Combine given information with intuition to make a reasoned guess when prompted.
- Refine hypothesis as evidence is gathered.

Make inferences, conclusions and justifications

- Use models and logic to infer/conclude.
- Reason inductively by considering specific cases and identifying patterns.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Present arguments in a logical and organized manner.
- Try multiple examples.
- Look for a case that does not work, i.e., a counter-example.

SAMPLE QUESTIONS

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work, e.g., the procedure for bisecting an angle using compasses?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

SAMPLE FEEDBACK

- Explain how the data you collected to inform your thinking connects to the problem.
- I can follow your thinking up to here. How can you help me understand your next ideas?
- Share your explanation with this group and consider their feedback as you revise your work.
- How does this all make sense together?
- What solution could be more suitable?
- How is this result applicable to the problem?

MATHEMATICAL PROCESS—REFLECTING

THE ONTARIO CURRICULUM, MATHEMATICS, 2005

Students will demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, and by verifying solutions).

Students become good problem solvers when they regularly and consciously reflect on and monitor their own thought processes.

ROLE OF STUDENTS

Consider data collected

- Search for relevant primary and secondary data.
- Check that data being gathered is appropriate to the inquiry.

Reflect on new skills, concepts and questions to see how they connect to prior knowledge

- Apply and extend knowledge to new situations.
- Examine questions and demonstrate flexibility in choice of strategy based on the nature of the question.
- Verify a solution to a problem by using a different method. Consider the reasonableness of their answer.
- Self-monitor progress while problem solving and revise, as necessary.

SAMPLE QUESTIONS

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

MATHEMATICAL PROCESS—SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

THE ONTARIO CURRICULUM, MATHEMATICS, 2005

Students will select and use a variety of concrete, visual and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems.

Students need to develop the ability to select the appropriate electronic tools, manipulatives and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas and to solve problems.

SAMPLE FEEDBACK

- You have chosen a different tool than other students. Why did you choose this one?
- You have selected an appropriate tool but an error has occurred. Review your procedure to identify the error, e.g., key entry on a calculator, improper use of features such as constructing versus drawing in GSP®.
- Share your solution with someone who has used a different tool and discuss the merits of each.
- Use (name a tool) and see how it helps you solve the problem.
- Please model the computational strategy you used for a classmate.
- Please think aloud while you apply the strategy so I can learn why you are getting this answer.
- Try another computational strategy and see if your result is the same.

ROLE OF STUDENTS

Select Tools

- Understand when mental arithmetic or a pencil-and-paper calculation or estimation is more appropriate than technology.
- Use an appropriate tool when:
  - An exact answer is needed.
  - Computation involves several numbers or numbers with more than one digit.
  - The numbers are not easily calculated mentally.
- Use technology to explore, gather, display, manipulate, and present data in a variety of ways.
- Use manipulatives and/or technology to develop understanding.

Select Computational Strategies

- Develop and use a personal set of referents for measurement, e.g., 1 cm is approximately the width of a baby finger.
- Select different computational strategies depending on the numbers involved, e.g., 25 × 16, 23 × 16, 19 × 16.

SAMPLE QUESTIONS

- How did the learning tool you chose contribute to your understanding/solving of the problem? Assist in your communication?
- In what ways would (name a tool) assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Explain why you chose this computational strategy.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use? Was your result sufficiently accurate for the question?

SAMPLE FEEDBACK

- How can you relate your understanding of...to this problem?
- How does your representation (e.g., diagram, sketch, manipulative) connect to..., e.g., the algebraic solution?
- Please describe the connections you see between...and...
- How does this method relate to this problem?
- How is that thinking connected to the question?

MATHEMATICAL PROCESS—CONNECTING

THE ONTARIO CURRICULUM, MATHEMATICS, 2005

Students will make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports).

Students need to see the connections and the relationships between mathematical concepts and skills from one strand of mathematics to another. As they continue to make such connections, students begin to see that mathematics is more than a series of isolated skills and concepts, and they can use their learning in one area of mathematics to understand another. Seeing connections among procedures and concepts also helps deepen students' mathematical understanding. Further, making connections between the mathematics they study and its application in their everyday lives helps students see how useful and relevant it is in the world beyond the classroom.

ROLE OF STUDENTS

- Apply a strategy or reference system that draws on previous learning in another context.
- Make connections between new and prior knowledge to make sense of what they are learning.
- Apply mathematics to contexts outside of mathematics.
- Use different models to best convey mathematical information and demonstrate their conceptual understanding of a procedure.
- Make connections between different representations, e.g., numeric, graphical and/or algebraic.

SAMPLE QUESTIONS

- What other math have you studied that has some of the same principles, properties or procedures as this, e.g., how does knowing the formula for the volume of the rectangular prism help us to find the formula for the volume of a triangular prism? How is adding "like terms" similar to adding integers?
- How do these different representations connect to one another, e.g., what seems to be the connection between the horizontal intercept of the graph and the numeric table of values?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

MATHEMATICAL PROCESS—REPRESENTING

THE ONTARIO CURRICULUM, MATHEMATICS, 2005

Students will create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial; on-screen dynamic representations), connect and compare representations, and select and apply the appropriate representations to solve problems.

Students represent mathematical ideas and relationships and model situations using concrete materials, pictures, diagrams, graphs, tables, numbers, words, and symbols. Learning various forms of representation helps students to make connections and develop flexibility in their thinking about mathematics.

SAMPLE FEEDBACK

- Show how you can represent this situation more efficiently.
- How can your representation of the data include your outliers?
- In what other way(s) can you represent this problem?

ROLE OF STUDENTS

- Select an appropriate representation and defend their choice:
  - Physical/concrete/manipulative
  - Electronically generated, e.g., graphs, dynamic geometry representation
  - Mental image
  - Numerical, e.g., table of values
  - Graphical
  - Scale drawing
  - Diagram
  - Graphic organizers, e.g., Venn diagram, T-chart, concept map
  - Equation/algebraic expression/formula
  - Algorithm/logic model
- Understand that various representations can be used to represent the same situation appropriately.
- Understand that there may be different variations of one representation, e.g., algebraic expressions may be equivalent yet appear different.

SAMPLE QUESTIONS

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? Graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

MATHEMATICAL PROCESS—COMMUNICATING

THE ONTARIO CURRICULUM, MATHEMATICS, 2005

Students will communicate mathematical thinking orally, visually and in writing, using mathematical vocabulary and a variety of appropriate representations and observing mathematical conventions.

Students…[use] a variety of appropriate representations including numbers, symbols, pictures, graphs, diagrams, and words. Through communication, students are able to reflect upon and clarify ideas, relationships and mathematical arguments.

SAMPLE FEEDBACK

- How would you show the steps to your solution so someone else can follow your thinking?
- If I follow your equal signs, it looks like 8 = 1. Since we know this is not true, how can we express this differently?
- Use *Smart Ideas*® to help you connect your ideas and communicate them more clearly.
- Provide a more complete, organized outline of your ideas so that your reasoning can be followed.
- How might you present this argument and data to the Principal?
- Be more specific and give more details.
- Express this in a different way.

ROLE OF STUDENTS

- Respond to instructions orally, visually and in writing as appropriate, e.g., explain, discuss, describe, justify, compare, suggest, write, tell, read, share, demonstrate, and present.
- Use correct mathematical language and vocabulary in explanations, e.g., interpolate, extrapolate, draw a line of best fit, evaluate, factor, expand, simplify, solve, rearrange, drag a vertex, transform by reflection.
- Respond clearly with sufficient detail so that their thinking can be understood.
- Interpret and summarize information from charts and graphs, providing appropriate detail, e.g., describe patterns and contrasts.
- Use the symbolic language of mathematics correctly.
- Read and reread all of the given information and instructions to ensure understanding, e.g., identify key information needed to solve the problem.
- Communicate mathematical learning by combining various representations, e.g., words with diagrams, charts or graphs with verbal descriptions.

SAMPLE QUESTIONS

- How can you express (explain, describe) this in a different way?
- What is a definition for…?
- How can you reword this question (answer)?
- What mathematical symbols could you use to communicate this statement?
- What mathematical operations are implied by the wording of this problem?