MHF 4U Unit 3 –Trigonometric Functions– Outline

Day	Lesson Title	Specific Expectations
1 (Lesson Included)	Radians and Degrees	B1.1, 1.3
2 (Lesson Included)	Radians and Special Angles	B1.4, 3.1
3 (Lesson Included)	Equivalent Trigonometric Expressions	B1.4, 3.1
4 (Lesson Included)	Sine and Cosine in Radians	B1.2, 1.3, 2.3, C2.1, 2.2
5 (Lesson Included)	Graphs of Sine & Cosine Reciprocals in Radians	B1.2, 1.3, 2.3, C2.1, 2.2
6 (Lesson Included)	Graphs of Tangent and Cotangent	B2.2, 2.3 C1.4, 2.1
7	Trigonometric Functions and Rates of Change	D1.1-1.9 inclusive
8 (Lesson Included)	Trigonometric Rates of Change	D1.1-1.9 inclusive
9-10	JAZZ DAY	
11	SUMMATIVE ASSESSMENT	
TOTAL DAYS:		11

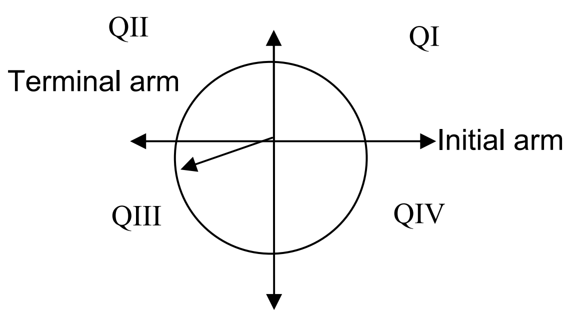
Unit 3: Day 1: Radians and Degrees		MHF4U
Minds On: 10 Action: 50 Consolidate:15 Total=75 min	**Learning Goal**: Explore and define radian measure Develop and apply the relationship between radian and degrees measure Use technology to determine the primary trigonometric ratios, including reciprocals of angles expressed in radians	**Materials** • BLM 3.1.1-3.1.4 • Cartesian Plane of Bristol board with pivoting terminal arm Adhesive

		Assessment Opportunities
Minds On…	**Small Groups – Discussion:** Students will work in groups to identify initial arm/ray, terminal arm/ray, principal angle of 210º, related acute angle, positive coterminal angles, negative coterminal angles, Quadrants I-IV, CAST Rule, unit circle, standard position from teacher-provided cards (BLM 3.1.1). The groups are to post their term on the Cartesian Plane model. Each student group will post the definition for their term on the classroom word wall. (Encourage students to create a Word Wall of the terms for their notes, or create one as a class on chart paper/bulletin board.)	Have adhesive along with Teacher Notes cut into cards Create a Cartesian Plane of Bristol board with a /contrasting, pivoting terminal arm to identify and review key terms
Action!	**Whole Class → Investigation** Teacher and students will work to complete BLM 3.1.2 Students will share with the class how they are converting radians to degrees and degrees to radians. **Whole Class – Discussion:** Discuss and record the rules on BLM 3.1.2 **Pairs – Activity:** Using BLM 3.1.3 each pair of students will find the degree and radian measure of the angle that is graphed on the card. (all angles are multiples of 15º). **Learning Skills/Teamwork/Checkbric: Teacher should circulate among groups to ensure conversations are on-topic, students encourage one another, and everyone in the group contributes** **Mathematical Process Focus: Communicating, Reasoning & Proving: Students communicate within their groups to justify their answers.**	Have adhesive, chart paper and marker for each group's definition Have half moons available for each group of students if desired Have class set of BLM 3.1.1 Cut BLM 3.1.2 into cards and distribute one card to each pair of students
Consolidate Debrief	**Whole Class – Discussion:** Summarize findings from Pairs – Activity. • To convert degrees to radians, multiply by (180º/π) or cross multiply using equivalent fractions. • To convert radians to degrees, multiply by (π/180º) or substitute π = 180º and simplify	
	Home Activity or Further Classroom Consolidation Complete BLM 3.1.4	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
5.6, 5.7	5.1	8.3			

3.1.1 Angles Review (Teacher Notes)

1. Initial arm	2. Terminal arm	3. Origin	4. Principal angle 210º
5. Related acute angle	6. Positive Coterminal angle	7. Negative Coterminal angle	8. CAST Rule
9. Quadrant I	10. Quadrant II	11. Quadrant III	12. Quadrant IV
13. Standard Position	14. Positive Coterminal angle	15. Negative Coterminal angle	



Principal Angle = 210º

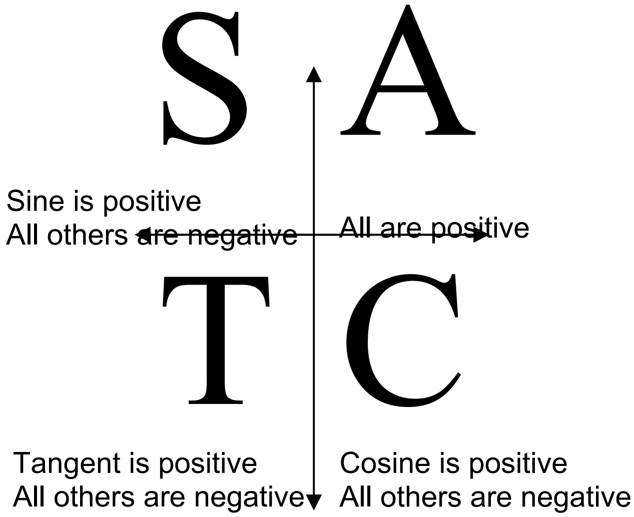
Related Acute Angle = 30º

Positive Co-terminal Angle
210º + 360º = 570º
570º + 360º = 930º

Negative Co-terminal Angle
210º - 360º = -150º
-150º - 360º= -510º

3.1.1 Angles Review (Teacher Notes continued)

CAST Rule:



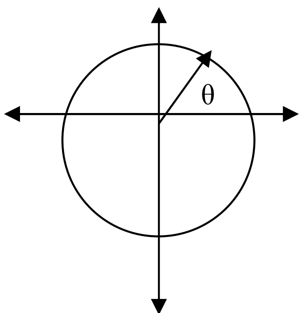
A unit circle is a circle, centred at the origin, with radius = 1 unit.

An angle is in STANDARD POSITION when it is centred at the origin, the initial arm is the positive x-axis and the terminal arm rests anywhere within the four quadrants

3.1.2 Degrees and Radians

Thus far, when you have graphed trigonometric functions or solved trigonometric equations, the domain was defined as degrees. However, there is another unit of measure used in many mathematics and physics formulas. This would be RADIANS.

To understand what a radian is, let's begin with a unit circle.



UNIT CIRCLE –

- Radius = 1 unit
- Centre at origin
- Θ in standard position
- Arc length = 1 unit
- θ= 1radian

1. Calculate the circumference of this unit circle when r = 1 unit?

2. An angle representing one complete revolution of the unit circle measures 2π radians, formerly ______°.

3. Change the following radians to degrees if 2π=360°,

   a) π = ____________

   b) $\frac{\pi}{2} =$ ____________

   c) $\frac{\pi}{4} =$ ____________

   d) $\frac{3\pi}{4} =$ ___________

   e) $\frac{11\pi}{6} =$ __________

4. Change the following degrees to radians if 360°= 2π ,

   a) 270° = __________

   b) 60° = __________

   c) 150° = __________

   d) 30° = __________

   e) 240° = __________

Rules:

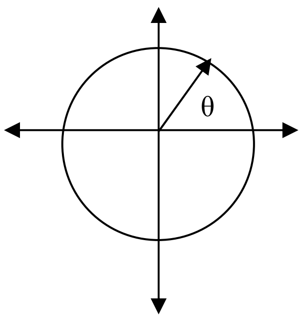
#1

#2

3.1.2 Degrees and Radians (Answers)

Thus far, when you have graphed trigonometric functions or solved trigonometric equations, the domain was defined as degrees. However, there is another unit of measure used in many mathematics and physics formulas. This would be RADIANS.

To understand what a radian is, let's begin with a unit circle.



UNIT CIRCLE –

- Radius = 1 unit
- Centre at origin
- Θ in standard position
- Arc length = 1 unit
- θ= 1radian

1. Calculate the circumference of this unit circle when r = 1 unit?
   C = 2π

2. An angle representing one complete revolution of the unit circle measures 2π radians, formerly 360º.

3. Change the following radians to degrees if 2π = 360º,

   a) π = 180º

   b) $\frac{\pi}{2} =$ 90º

   c) $\frac{\pi}{4} =$ 45º

   d) $\frac{3\pi}{4} =$ 135º

   e) $\frac{11\pi}{6} =$ 330º

4. Change the following degrees to radians if 360º= 2π ,

   a) 270º = $\frac{3\pi}{2}$

   b) 60º = $\frac{\pi}{3}$

   c) 150º = $\frac{5\pi}{6}$

   d) 30º = $\frac{\pi}{6}$

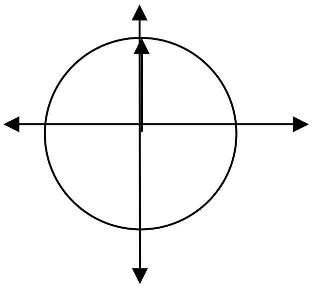
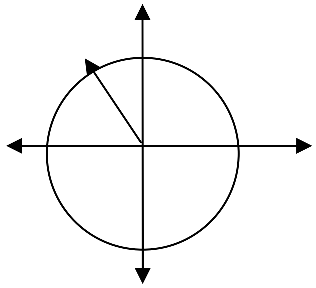
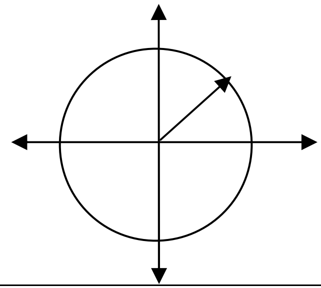
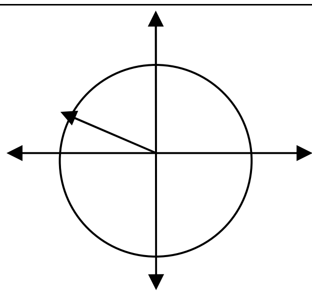
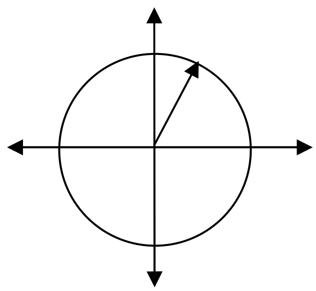
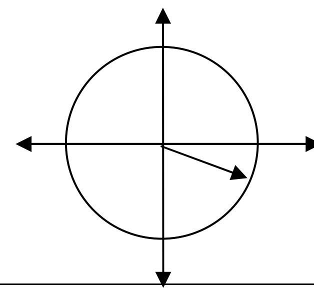
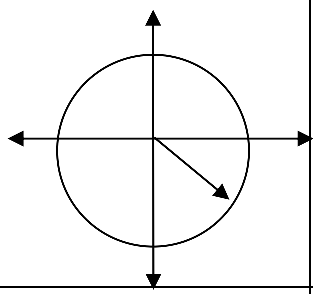
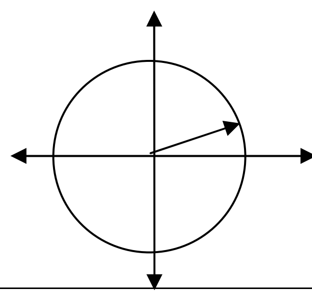
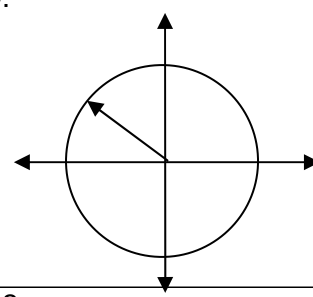
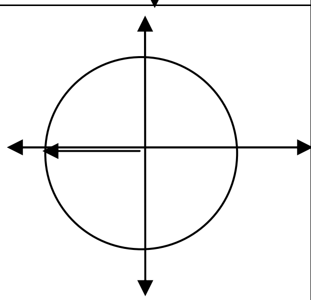
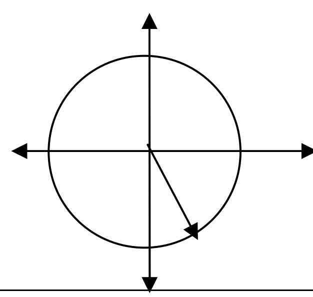
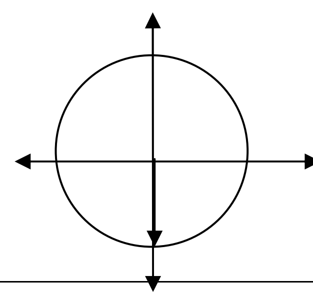
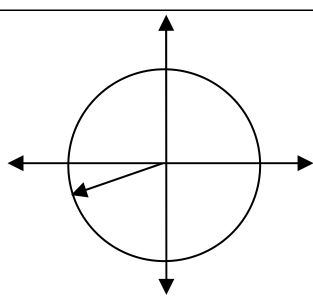
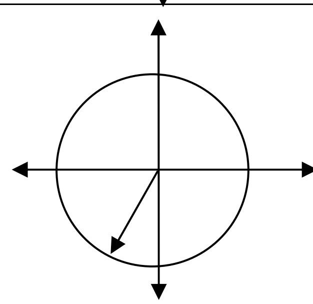
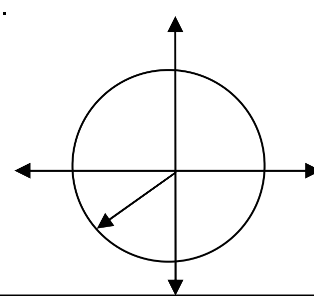
   e) 240º = $\frac{4\pi}{3}$

Rule:#1 To change radians to degrees, multiply by $\frac{180^\circ}{\pi}$.

#2 To change degrees to radians, multiply by $\frac{\pi}{180^\circ}$.

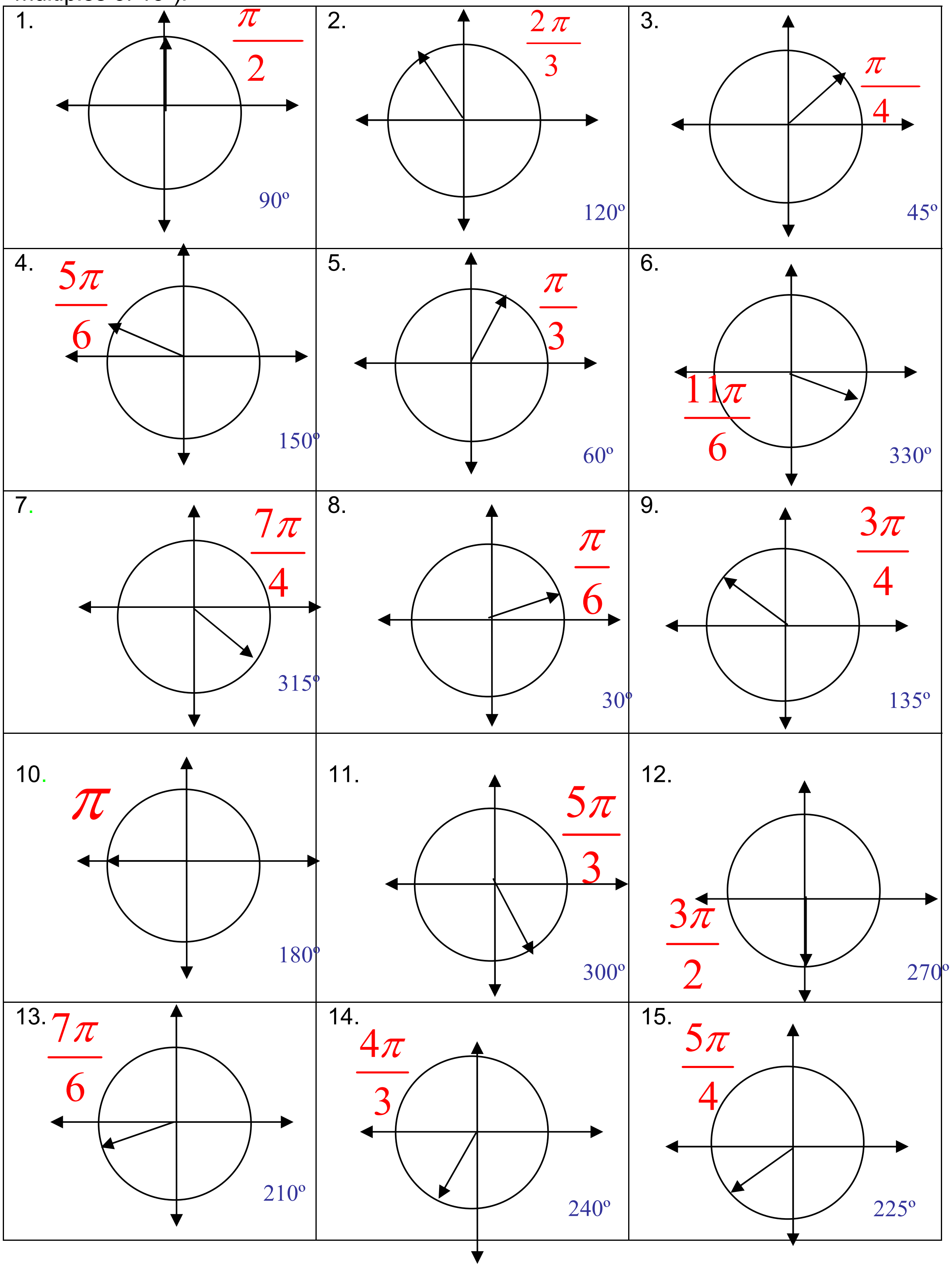
3.1.3 Measuring Angles in Radians and Degrees

Cut into cards and have pairs of students find each angle in degrees and radians (assume that all angles are drawn to represent multiples of 15º).

1. 	2. 	3. 
4. 	5. 	6. 
7. 	8. 	9. 
10. 	11. 	12. 
13. 	14. 	15. 

3.1.3 Measuring Angles in Radians and Degrees (Answers)

Find each angle in degrees and radians. (Assume that all angles are drawn to represent multiples of 15º).



3.1.4 Angles in Degrees and Radians Practice

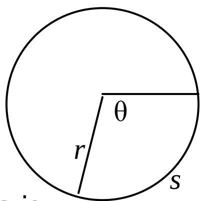
Knowledge

Change each degree to radian measure in terms of π:

1. 18º	2. - 72º
3. 870º	4. 1200º
5. 135º	6. 540º
7. - 315º	8. -225º

Application

9. The earth rotates on its axis once every 24 hours.
   a. How long does it take Earth to rotate through an angle of $\frac{4\pi}{3}$?
   b. How long does it take Earth to rotate through an angle of 120º?
10. The length of any arc, ***s***, can be found using the formula $s = r\theta$, where ***r*** is the radius of the circle, and θ is the radian measure of the central angle that creates the arc. Find the length of the arc for each, to 3 decimal places:
   a. radius of 12cm, central angle 75º
   b. radius of 8m, central angle of 185º
   c. radius of 18mm, central angle of 30º



11. If an object moves along a circle of radius ***r*** units, then its linear velocity, ***v***, is given by $v = r\frac{\theta}{t}$, where $\frac{\theta}{t}$ represents the angular velocity in radians per unit of time. Find the angular velocity for each:
   a. a pulley of radius 8cm turns at 5 revolutions per second.
   b. A bike tire of diameter 26 inches 3 revolutions per second
12. The formula for the area of a sector of a circle ("pie wedge") is given as $A = \frac{1}{2}r^2\theta$, where ***r*** is the radius and θ is the measure of the central angle, expressed in radians. Find the area of each sector described:
   a. θ = 315º, diameter is 20cm.
   b. θ = 135º, radius is 16 ft.

Communication/Thinking

13. When is it beneficial to work with angles measured in radians? Degrees?

14. Explain how to convert between radians and degrees.

3.1.4 Angles in Degrees and Radians Practice (Answers)

1. $\frac{\pi}{10}$
2. $\frac{-2\pi}{5}$
3. $\frac{29\pi}{6}$
4. $\frac{20\pi}{3}$
5. $\frac{3\pi}{4}$
6. 3π
7. $\frac{-7\pi}{4}$
8. $\frac{-5\pi}{4}$
9. a. 16h
   b. 8h
10. a. 15.708cm
    b. 25.831m.
    c. 9.425mm
11. a. 150.796cm/sec
    b. 245.044 in/sec
12. a. 274.889cm^2
    b. 7.069 ft^2

13. It is more beneficial to work in radians if the formula given calls for radians and if working with professionals with a mathematics background. It is more beneficial to work in degrees if the formula given calls for degrees and if working with the general population.

14. To convert radians to degrees, multiply by $\frac{180}{\pi}$ or substitute π = 180º and simplify.

    To convert from degrees to radians, multiply by $\frac{\pi}{180}$ or cross multiply using equivalent fractions.

Unit 3: Day 2: Radians and Special Angles

MHF4U

Minds On: 10 Action: 50 Consolidate:15 Total=75 min	**Learning Goal**: Determine the exact values of trigonometric and reciprocal trigonometric ratios for special angles and their multiples using radian measure Recognize equivalent trigonometric expressions and verify equivalence with technology	**Materials** • BLM 3.2.1 • BLM 3.2.2 • BLM 3.2.3 Placemat Activity Sheets included in Teacher Notes Graphing technology

Assessment Opportunities

Minds On…	**Pairs – Activity:** Students will work in pairs to find the value of special angles stated in radians. Students then put their function on an overhead transparency (Teacher Notes) under the appropriate value. Students will prepare to justify their choice and to suggest reasons why there are equivalent trigonometric ratios. **Whole Class – Discussion:** Discuss the entries, looking for/identifying any errors to promote discussion. Review CAST Rule and demonstrate/discuss how technology could be used to verify equivalence Offer reasons why different trigonometric expressions are equivalent	Cut out cards for Minds On activity from first page of BLM 3.2.1 Create transparency from second page of BLM 3.2.1 Have one transparency pen at overhead for students to record answers and name of one group member Cut up Placemat Activity Trigonometric Function Cards Make copies of the 8 pages on BLM 3.2.2 titled Placement Activity: Exact Value of Special Angles
Action!	**Whole Class – Placemat** Students receive a trigonometric function which they evaluate. Students then write their function on the appropriate placemat bearing the value for their function. In those placemat groups, students discuss the validity of their choices using diagrams and appropriate terminology. **Learning Skills/Teamwork/Checkbric: Teacher should circulate among pairs and individuals during the activity to ensure that conversations are on-topic, students are encouraging one another, and everyone in the group is contributing.** **Mathematical Process Focus: Connecting and Representing: Students will make the connection between special right triangles in degrees and radians, then represent their findings on the transparency and Placemat activity.**	
Consolidate Debrief	**Whole Class – Discussion** Summarize findings from Placemat activity .	
Exploration Application	**Home Activity or Further Classroom Consolidation** BLM 3.2.3 Students will submit a journal entry which explains why trigonometric expressions are equivalent and how equivalences can be verified using technology. The journal entry should include diagrams and appropriate use of mathematical terminology as outlined on the word wall.	Ensure that word wall from previous lesson can be seen for reference purposes

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
5.3, 5.7	5.2				

3.2.1 Radians and Special Angles *(Teacher Notes)*

Minds On Pairs Activity

$\sec\frac{\pi}{4}$	$\tan\frac{\pi}{3}$	$\cos\frac{\pi}{4}$
$\sin\frac{\pi}{6}$	$\cot\frac{\pi}{4}$	$\csc\frac{\pi}{3}$
$\sec\frac{\pi}{6}$	$\cos\frac{\pi}{3}$	$\cot\frac{\pi}{3}$
$\tan\frac{\pi}{6}$	$\sin\frac{\pi}{4}$	$\csc\frac{\pi}{6}$
$\tan\frac{\pi}{4}$	$\cos\frac{\pi}{6}$	$\cot\frac{\pi}{6}$
$\sec\frac{\pi}{3}$	$\sin\frac{\pi}{3}$	$\csc\frac{\pi}{4}$

3.2.1 Radians and Special Angles *(Teacher Notes continued)*

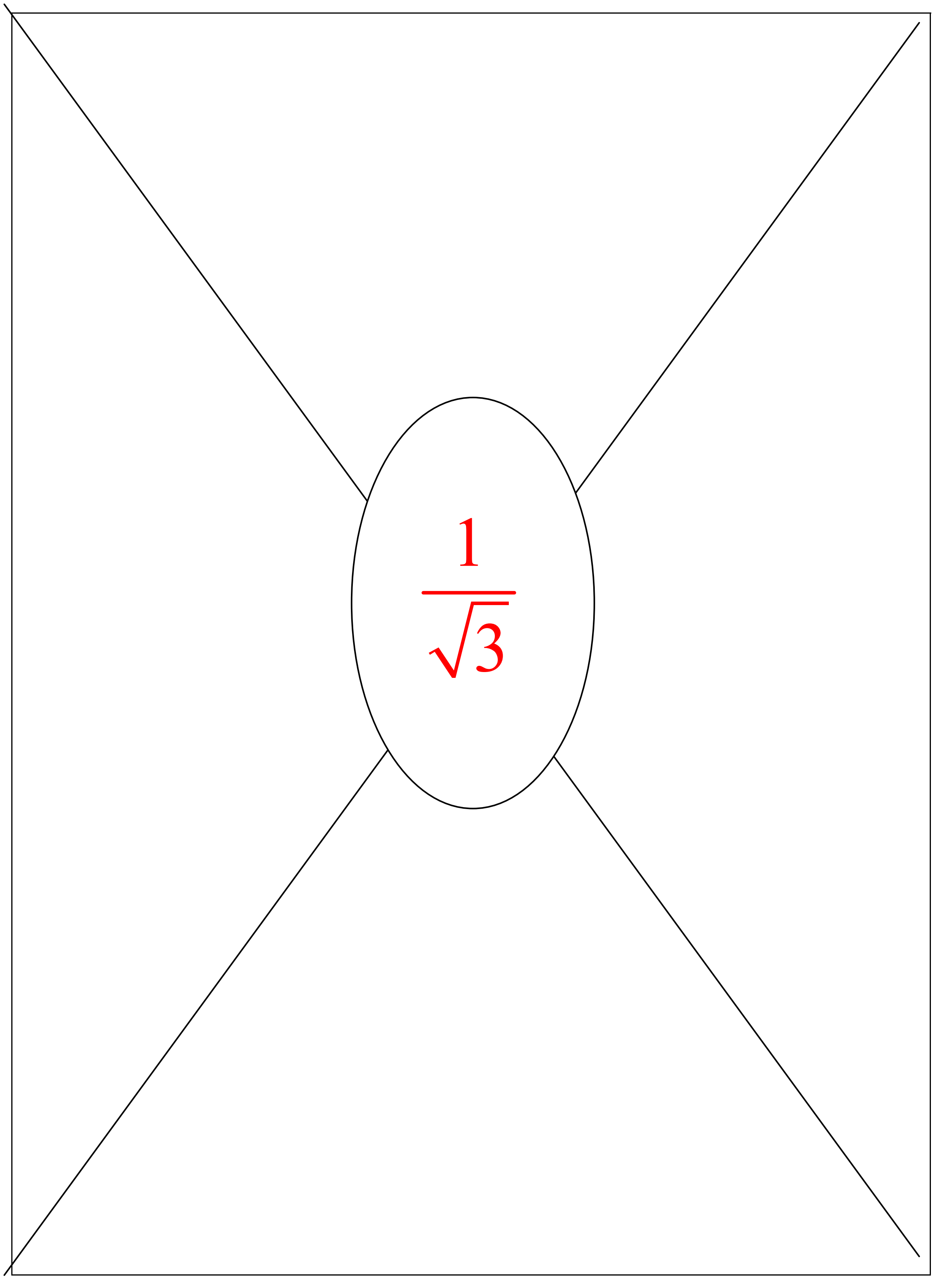
Overhead Transparency for Minds On Activity

$\frac{1}{2}$	$\sqrt{2}$	2
1	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3}}{2}$
$\sqrt{3}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{\sqrt{3}}$

3.2.1 Radians and Special Angles (Answers)

$\sec\frac{\pi}{4}=\sqrt{2}$	$\tan\frac{\pi}{3}=\sqrt{3}$	$\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$
$\sin\frac{\pi}{6}=\frac{1}{2}$	$\cot\frac{\pi}{4}=1$	$\csc\frac{\pi}{3}=\frac{2}{\sqrt{3}}$
$\sec\frac{\pi}{6}=\frac{2}{\sqrt{3}}$	$\cos\frac{\pi}{3}=\frac{1}{2}$	$\cot\frac{\pi}{3}=\frac{1}{\sqrt{3}}$
$\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$	$\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$	$\csc\frac{\pi}{6}=2$
$\tan\frac{\pi}{4}=1$	$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$	$\cot\frac{\pi}{6}=\sqrt{3}$
$\sec\frac{\pi}{3}=2$	$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$	$\csc\frac{\pi}{4}=\sqrt{2}$

3.2.2 Placemat Activity: Exact values of special angles



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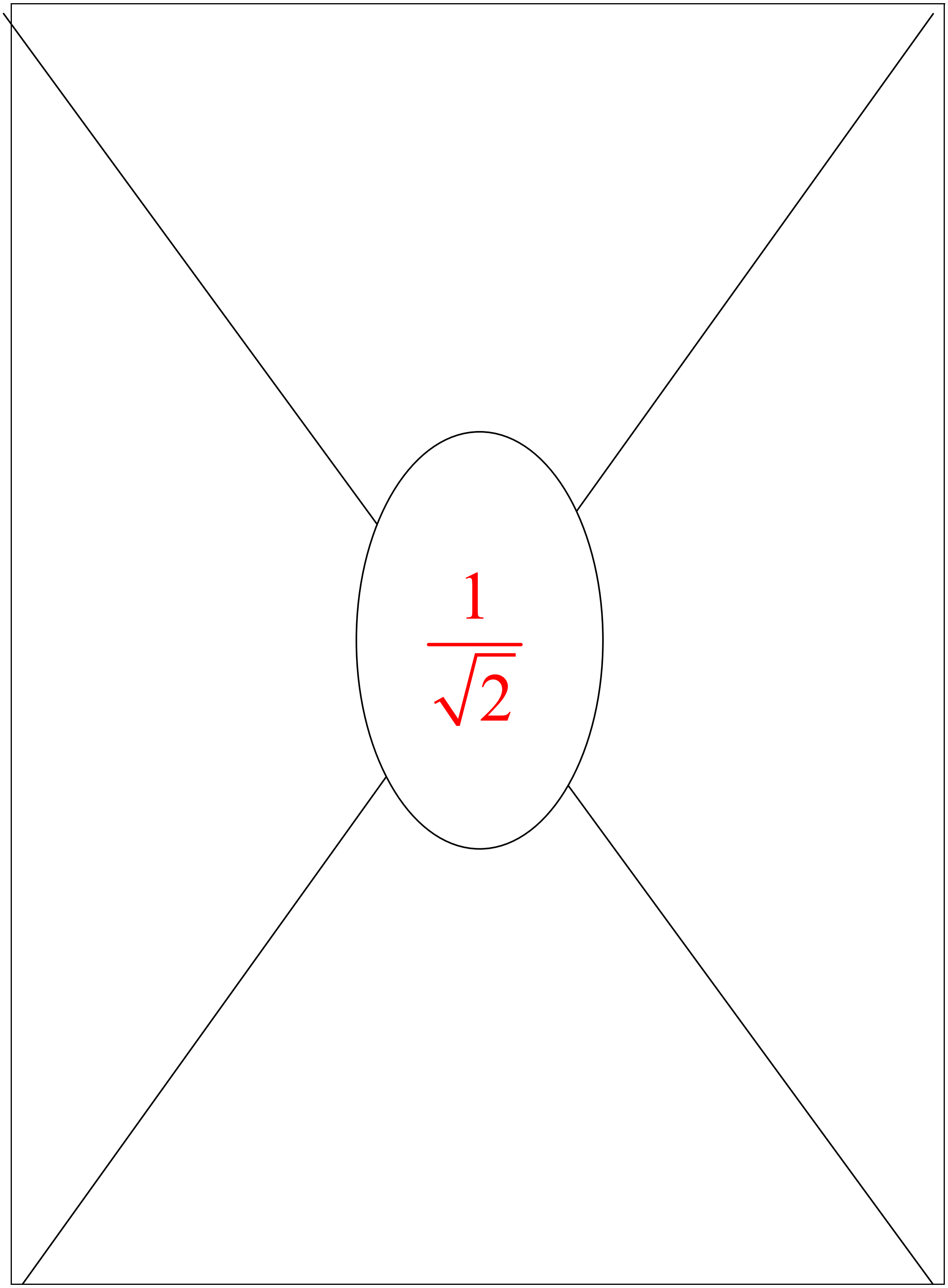

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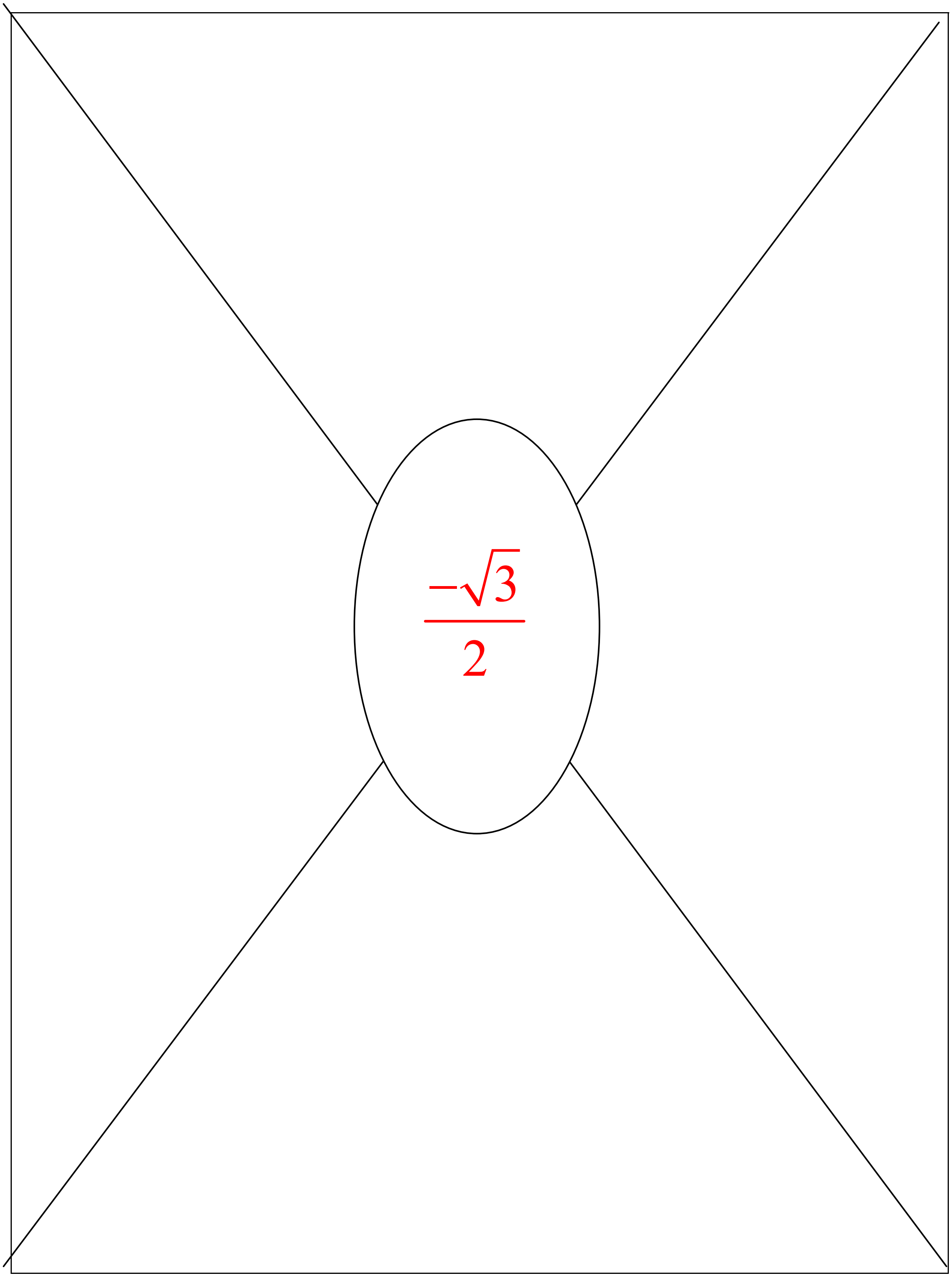

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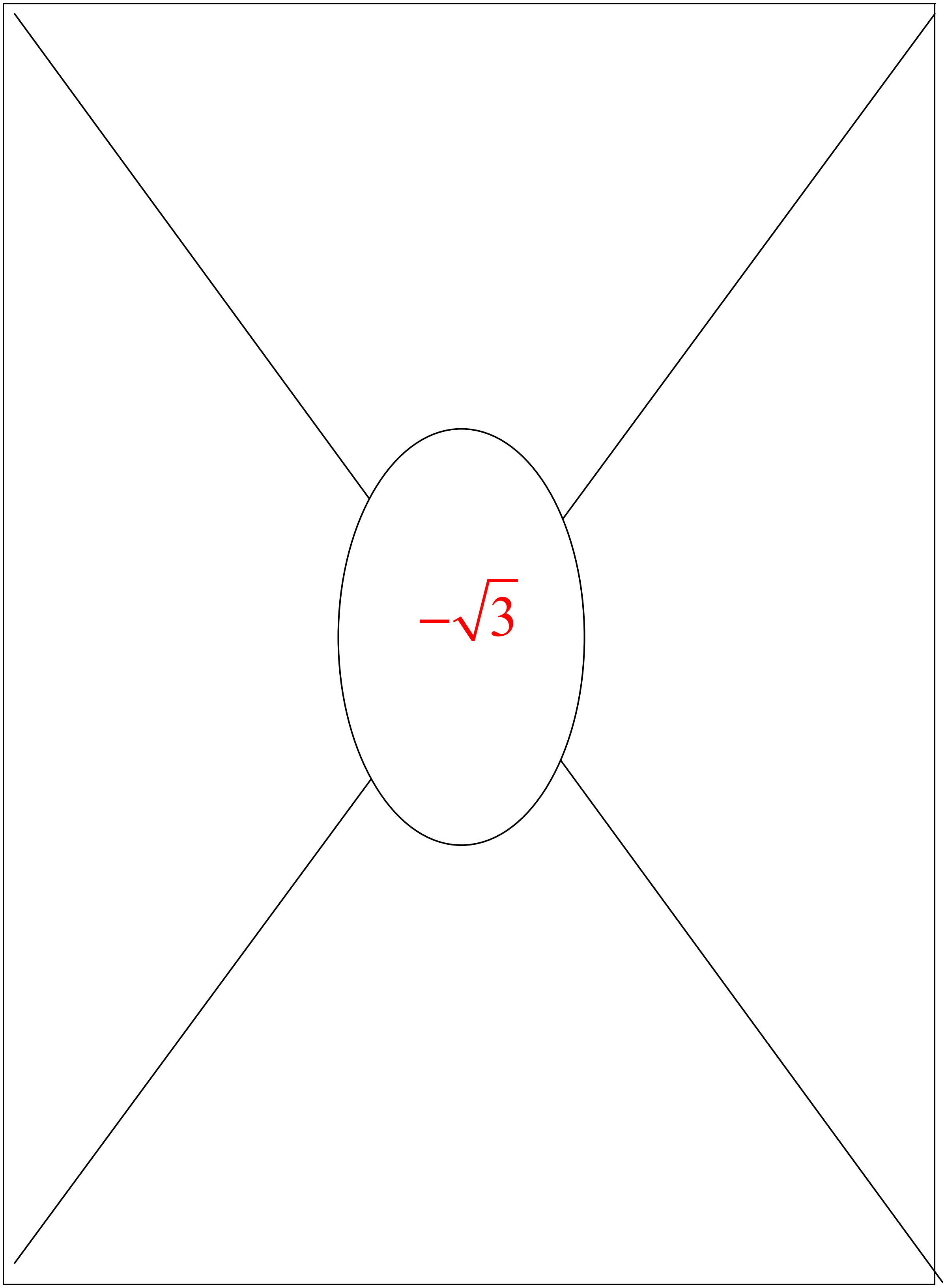

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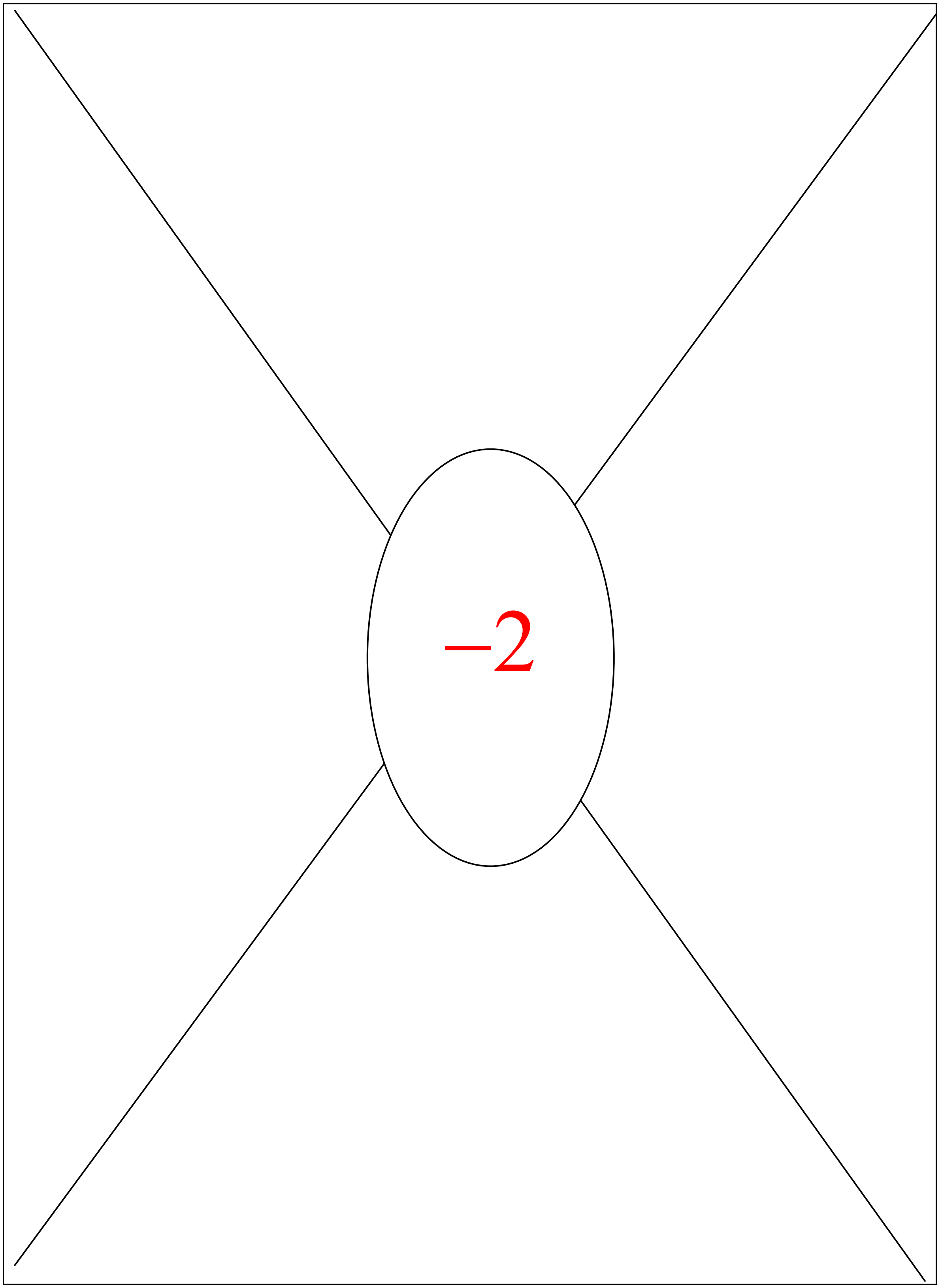

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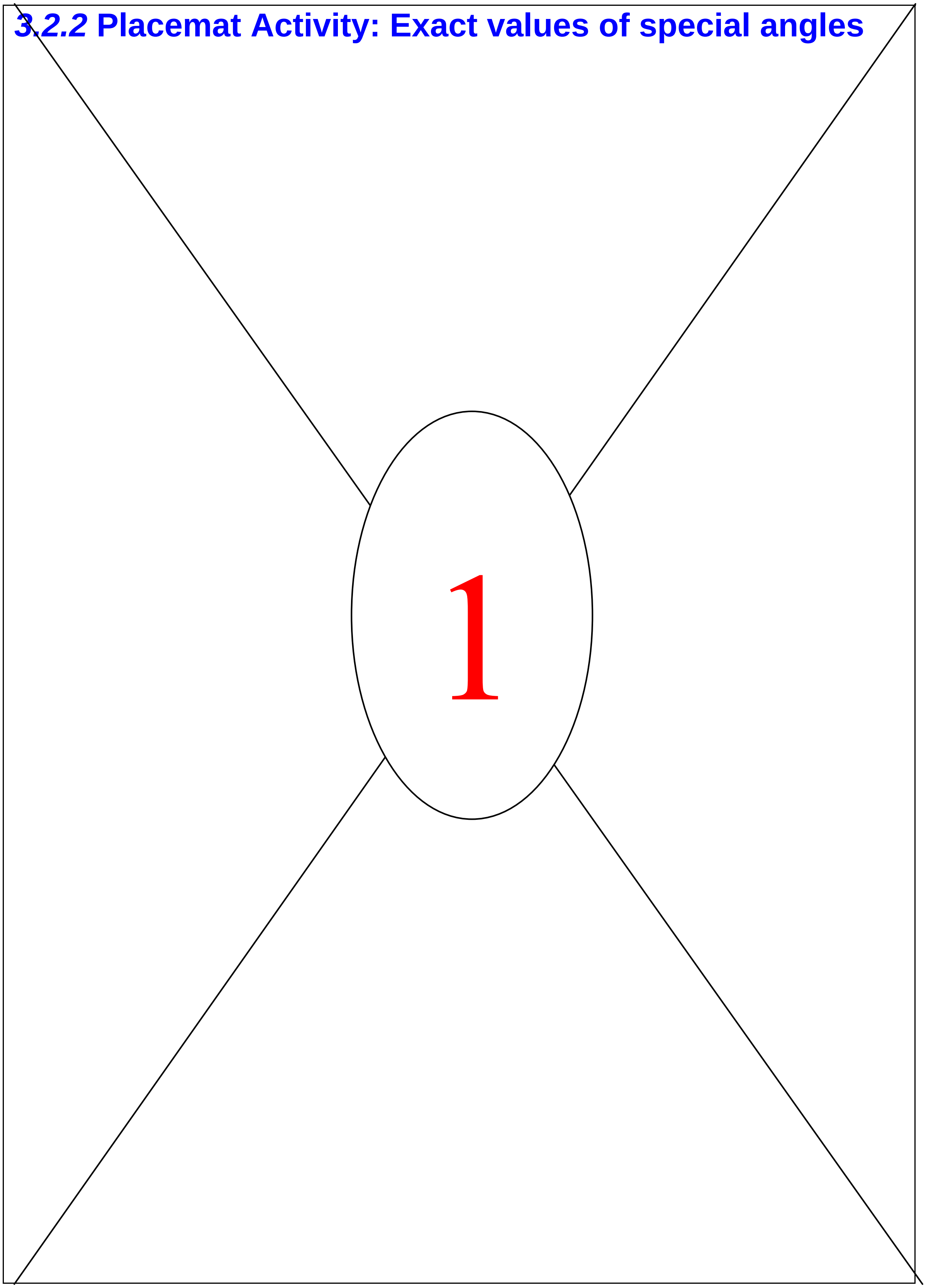

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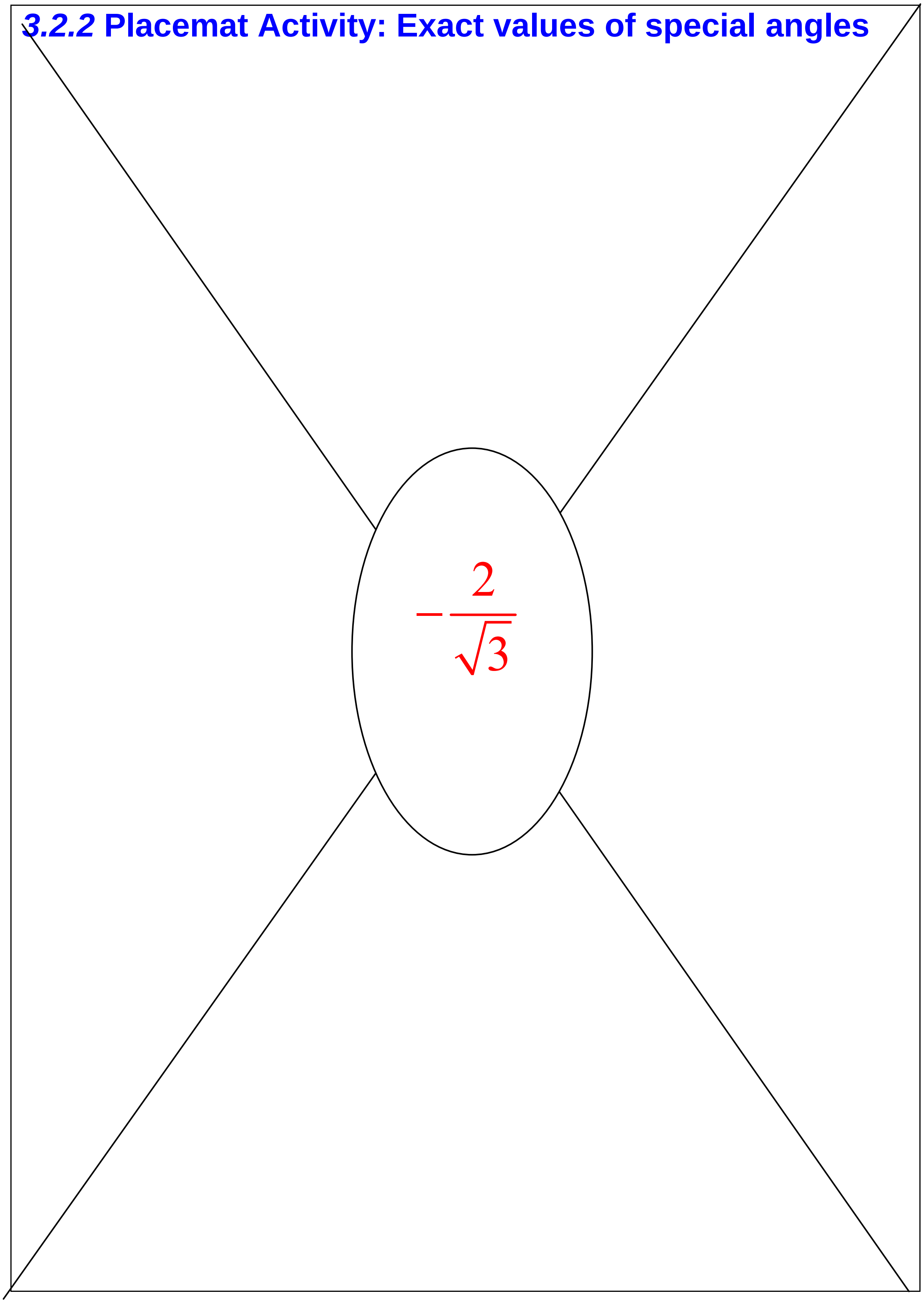

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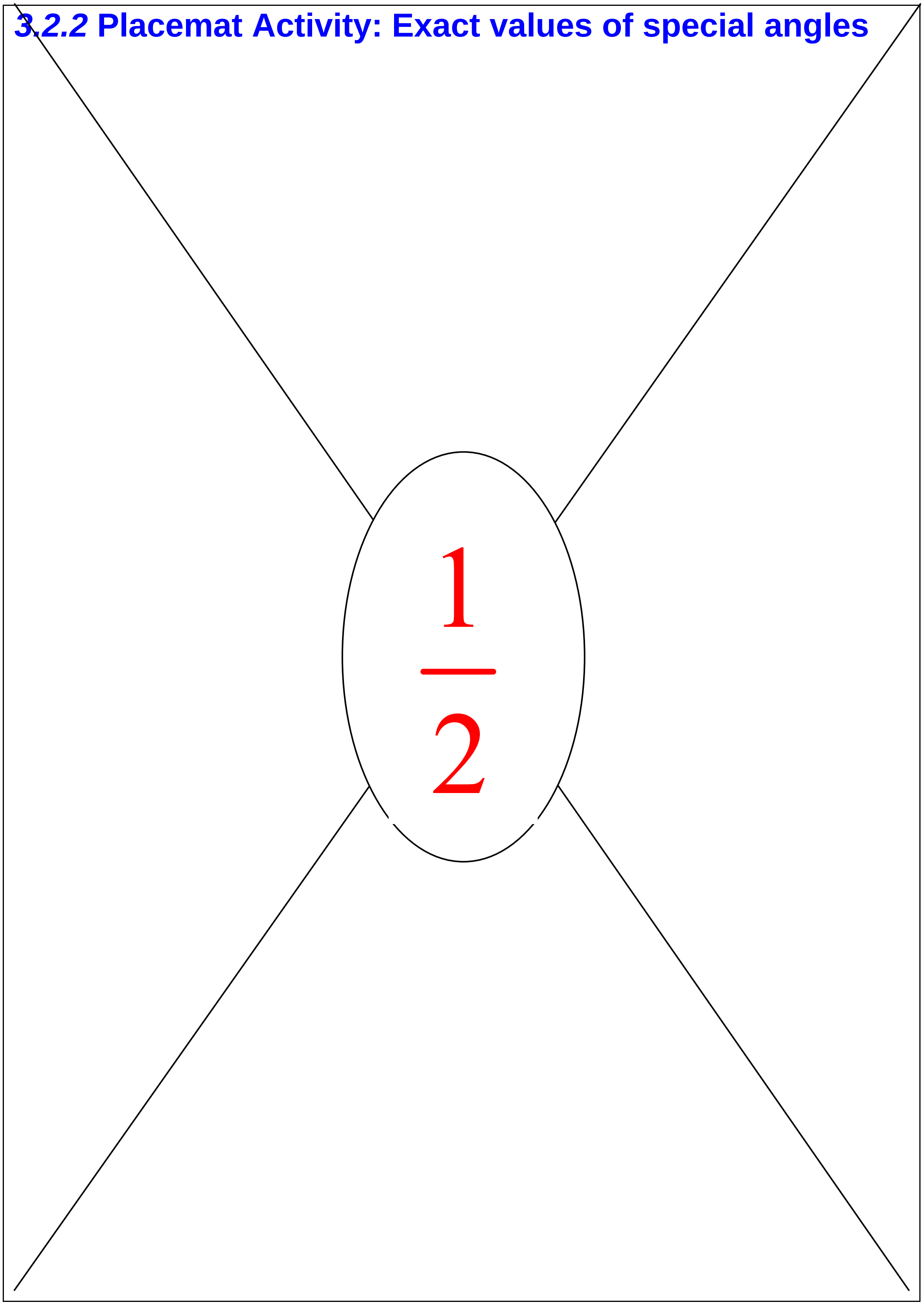


3.2.2 Placemat Activity Trigonometric Function Cards

(Teacher Notes)

$\tan\frac{-11\pi}{6}$	$\cot\frac{-5\pi}{3}$	$\cot\frac{\pi}{3}$	$\tan\frac{\pi}{6}$
$\cos\frac{-7\pi}{6}$	$\sin\frac{-\pi}{3}$	$\cos\frac{19\pi}{6}$	$\sin\frac{10\pi}{3}$
$\cos\frac{-\pi}{4}$	$\cos\frac{9\pi}{4}$	$\sin\frac{9\pi}{4}$	$\sin\frac{-5\pi}{4}$
$\csc\frac{4\pi}{3}$	$\csc\frac{5\pi}{3}$	$\sec\frac{7\pi}{6}$	$\sec\frac{5\pi}{6}$
$\tan\frac{5\pi}{3}$	$\tan\frac{2\pi}{3}$	$\cot\frac{5\pi}{6}$	$\cot\frac{11\pi}{6}$
$\sin\frac{13\pi}{6}$	$\sin\frac{-7\pi}{6}$	$\cos\frac{-5\pi}{3}$	$\cos\frac{5\pi}{3}$
$\sec\frac{2\pi}{3}$	$\sec\frac{4\pi}{3}$	$\csc\frac{7\pi}{6}$	$\csc\frac{11\pi}{6}$
$\cot\frac{5\pi}{4}$	$\tan\frac{-7\pi}{4}$	$\cot\frac{\pi}{4}$	$\tan\frac{-3\pi}{4}$

3.2.2 Placemat Activity Trigonometric Function Cards
(Answers)

$\frac{1}{\sqrt{3}}$		$-\frac{\sqrt{3}}{2}$	
$\tan\frac{-11\pi}{6}$	$\cot\frac{-5\pi}{3}$	$\cos\frac{-7\pi}{6}$	$\sin\frac{-\pi}{3}$
$\tan\frac{\pi}{6}$	$\cot\frac{\pi}{3}$	$\cos\frac{19\pi}{6}$	$\sin\frac{10\pi}{3}$
$\frac{1}{\sqrt{2}}$		$-\frac{2}{\sqrt{3}}$	
$\cos\frac{-\pi}{4}$	$\sin\frac{9\pi}{4}$	$\csc\frac{4\pi}{3}$	$\sec\frac{7\pi}{6}$
$\cos\frac{9\pi}{4}$	$\sin\frac{-5\pi}{4}$	$\csc\frac{5\pi}{3}$	$\sec\frac{5\pi}{6}$
1		$\frac{1}{2}$	
$\cot\frac{5\pi}{4}$	$\tan\frac{-7\pi}{4}$	$\sin\frac{13\pi}{6}$	$\cos\frac{-5\pi}{3}$
$\cot\frac{\pi}{4}$	$\tan\frac{-3\pi}{4}$	$\sin\frac{-7\pi}{6}$	$\cos\frac{5\pi}{3}$
-2		$-\sqrt{3}$	
$\csc\frac{7\pi}{6}$	$\sec\frac{2\pi}{3}$	$\tan\frac{5\pi}{3}$	$\cot\frac{5\pi}{6}$
$\csc\frac{11\pi}{6}$	$\sec\frac{4\pi}{3}$	$\tan\frac{2\pi}{3}$	$\cot\frac{11\pi}{6}$

3.2.3 HOME ACTIVITY: Radians and Special Angles

Name ____________________
Date _____________________

Knowledge

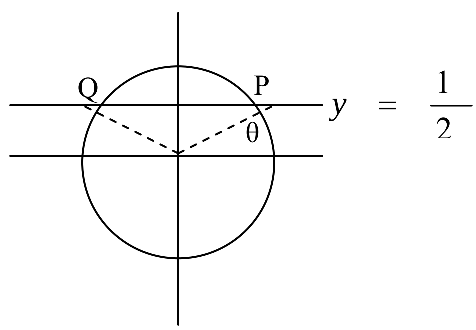
For each function, find the quadrant containing the angle, the related acute angle, and the exact value of the given function:

ANGLE	Quadrant	Related Acute	Value
1. $\sin\frac{5\pi}{4}$			
2. $\sec\frac{-7\pi}{4}$			
3. $\tan\frac{5\pi}{6}$			

Application/Communication

4. a. Find the angle θ created by the intersection of the unit circle and radius with point ***P***, as shown below.
   b. What are the coordinates of point ***P*** where the line y = ½ intersects the unit circle?
   c. Find the angle created by the intersection of the unit circle and radius with point ***Q***, as shown below.
   d. What are the coordinates of point ***Q*** where the line y = ½ intersects the unit circle?

   e. Explain how this shows that if sinθ = ½, cosθ = $\pm\frac{\sqrt{3}}{2}$



3.2.3 HOME ACTIVITY: Radians and Special Angles

(Answers)

ANGLE	Quadrant	Related Acute	Value
1. $\sin\frac{5\pi}{4}$	III	$\frac{\pi}{4}$	$-\frac{1}{\sqrt{2}}$
2. $\sec\frac{-7\pi}{4}$	I	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
3. $\tan\frac{5\pi}{6}$	II	$\frac{\pi}{6}$	$-\frac{1}{\sqrt{3}}$

4. a.

$$\sin\theta = \frac{\frac{1}{2}}{1}$$

$$\sin\theta = \frac{1}{2}$$

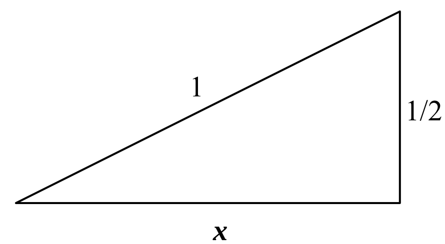
$$\theta = 30^\circ$$

b.

$$\cos\frac{\pi}{6} = \frac{x}{1}$$

$$\frac{\sqrt{3}}{2} = x$$

$$P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$



c. Related acute angle $\frac{\pi}{6}$, yielding principal angle of $\frac{5\pi}{6}$.

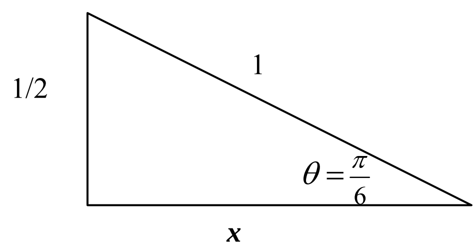
d.

$$\cos\frac{\pi}{6} = \frac{x}{1}$$

$$\frac{\sqrt{3}}{2} = x$$

In quadrant II, the value of ***x*** is negative.

$$Q\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$$



e. Sine is positive in quadrants I & II. Cosine is positive in quadrant I, and negative in quadrant II. Using the related acute angle of $\frac{\pi}{6}$ in both quadrants I & II yields a sine value of ½, and a cosine value of $\frac{\sqrt{3}}{2}$.

Unit 3: Day 3: Equivalent Trigonometric Expressions		MHF4U
Minds On: 5 Action: 50 Consolidate:20 Total=75 min	**Learning Goal**: Determine the exact values of trigonometric and reciprocal trigonometric ratios for special angles and their multiples using radian measure Recognize equivalent trigonometric expressions and verify equivalence with technology	**Materials** • Teacher Notes • BLM 3.3.1 • BLM 3.3.2

		Assessment Opportunities
Minds On…	**Individual → Calculation** Students will find the value of their trigonometric function card, using a calculator.	Cut the template from BLM 3.3.1 to give each student a card.
Action!	**Partners → Think/Pair/Share** Partner with a classmate who has the same value for their trigonometric function Conjecture why the two functions have the same value **Whole Class→Discussion** Discuss the findings of the Think/Pair/Share activity What do you notice about the answers of each pair of angles? What do you notice about each pair of angles? Learning Skills/Teamwork/Checkbric: **Teacher should circulate among the students to promote on-task behaviours and answer questions** Mathematical Process Focus: Connecting and Communicating: **Students are finding values, connecting with students having same values, and discussing questions posed by the teacher.**	
Consolidate Debrief	**Small Groups → Interview** Students will develop the cofunction identities • $\sin x = \cos\left(\frac{\pi}{2} - x\right)$ • $-\sin x = cos\left(x + \frac{\pi}{2}\right)$ or • $\cos x = \sin\left(\frac{\pi}{2} - x\right)$ Discuss as a class the advantages and disadvantages of using the cofunction identities versus the use of diagrams to illustrate equivalent trigonometric ratios.	Have the placemat activity sheets from the previous lesson posted for referencing purposes
Exploration *Application*	**Home Activity or Further Classroom Consolidation** BLM 3.3.2 Students will write a journal entry to describe how to use both a cofunction identity and a diagram to prove that two trigonometric ratios are equivalent.	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
5.3, 5.7	5.2				

3.3.1 Equivalent Trigonometric Expressions *(Teacher Notes)*

$\sin\frac{\pi}{6}$	$\sin\frac{2\pi}{9}$	$\sin\frac{\pi}{18}$	$\sin\frac{\pi}{9}$
$\cos\frac{\pi}{3}$	$\cos\frac{5\pi}{18}$	$\cos\frac{4\pi}{9}$	$\cos\frac{7\pi}{18}$
$\sin\frac{5\pi}{18}$	$\sin\frac{\pi}{3}$	$\sin\frac{7\pi}{18}$	$\sin\frac{4\pi}{9}$
$\cos\frac{2\pi}{9}$	$\cos\frac{\pi}{6}$	$\cos\frac{\pi}{9}$	$\cos\frac{\pi}{18}$
$-\sin\frac{\pi}{6}$	$-\sin\frac{\pi}{3}$	$-\sin\frac{5\pi}{18}$	$-\sin\frac{\pi}{9}$
$\cos\frac{2\pi}{3}$	$\cos\frac{5\pi}{6}$	$\cos\frac{7\pi}{9}$	$\cos\frac{11\pi}{18}$
$-\sin\frac{4\pi}{9}$	$-\sin\frac{7\pi}{18}$	$-\sin\frac{2\pi}{9}$	$-\sin\frac{\pi}{18}$
$\cos\frac{17\pi}{18}$	$\cos\frac{8\pi}{9}$	$\cos\frac{13\pi}{18}$	$\cos\frac{5\pi}{9}$

3.3.1 Equivalent Trigonometric Expressions *(Teacher Notes)*

ANSWERS

$\sin\frac{\pi}{6}=\frac{1}{2}$	$\sin\frac{2\pi}{9}\approx 0.6428$	$\sin\frac{\pi}{18}\approx 0.1736$	$\sin\frac{\pi}{9}\approx 0.3420$
$\cos\frac{\pi}{3}=\frac{1}{2}$	$\cos\frac{5\pi}{18}\approx 0.6428$	$\cos\frac{4\pi}{9}\approx 0.1736$	$\cos\frac{7\pi}{18}\approx 0.3420$
$\sin\frac{5\pi}{18}\approx 0.7660$	$\sin\frac{\pi}{3}\approx 0.8660$	$\sin\frac{7\pi}{18}\approx 0.9397$	$\sin\frac{4\pi}{9}\approx 0.9848$
$\cos\frac{2\pi}{9}\approx 0.7660$	$\cos\frac{\pi}{6}\approx 0.8660$	$\cos\frac{\pi}{9}\approx 0.9397$	$\cos\frac{\pi}{18}\approx 0.9848$
$-\sin\frac{\pi}{6}\approx -\frac{1}{2}$	$-\sin\frac{\pi}{3}\approx -0.8660$	$-\sin\frac{5\pi}{18}\approx -0.7660$	$-\sin\frac{\pi}{9}\approx -0.3420$
$\cos\frac{2\pi}{3}\approx -\frac{1}{2}$	$\cos\frac{5\pi}{6}\approx -0.8660$	$\cos\frac{7\pi}{9}\approx -0.7660$	$\cos\frac{11\pi}{18}\approx -0.3420$
$-\sin\frac{4\pi}{9}\approx -0.9848$	$-\sin\frac{7\pi}{18}\approx -0.9397$	$-\sin\frac{2\pi}{9}\approx -0.6428$	$-\sin\frac{\pi}{18}\approx -0.1736$
$\cos\frac{17\pi}{18}\approx -0.9848$	$\cos\frac{8\pi}{9}\approx -0.9397$	$\cos\frac{13\pi}{18}\approx -0.6428$	$\cos\frac{5\pi}{9}\approx -0.1736$

3.3.2 HOME ACTIVITY: Equivalent Trigonometric Expressions

Name ____________________ Date _____________________

Knowledge

Write each of the following in terms of the cofunction identity:

1. $\sin\frac{\pi}{12}$ 2. $\sin\frac{2\pi}{5}$ 3. $\sin\frac{5\pi}{8}$ 4. $\sin\frac{5\pi}{12}$

5. $\cos\frac{5\pi}{18}$ 6. $\cos\frac{\pi}{9}$ 7. $\cos\frac{7\pi}{36}$ 8. $\cos\frac{2\pi}{9}$

Application

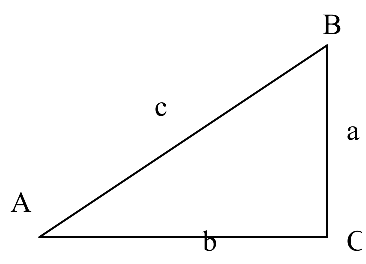
Fill in the blanks with the appropriate function name:

9. $\sin\frac{2\pi}{3} = \_\_\_\_\_\_\_\_\left(\frac{-\pi}{6}\right)$

10. $\_\_\_\_\_\_\_\_\ \frac{11\pi}{60} = \sin\frac{19\pi}{60}$

11. $\cos\frac{7\pi}{18} = \dfrac{1}{\_\_\_\_\_\_\_\_\ \frac{\pi}{9}}$

For right triangle ABC:

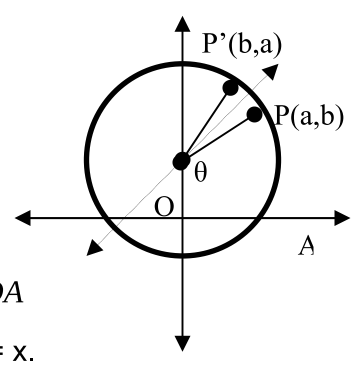


12. If $\sin A = \frac{\sqrt{3}}{3}$, what is the value of cos B?

13. If cos A=0.109, what is $\sin\left(\frac{\pi}{2} - A\right)$?

14. If $\cos\frac{11\pi}{180} = 0.9816$, what is $\sin\frac{79\pi}{180}$?

Thinking



15. The reason for the cofunction relationships can be seen from the diagram. If the sum of the measures of $\angle POA$ and $\angle P'OA$ is $\frac{\pi}{2}$, then *P* and *P'* are symmetric with respect to the line y = x. Also, if *P*=(a,b), then *P'*=(b,a) and $\sin\theta =$ y-coordinate of *P* = x-coordinate of *P'* = $\cos\left(\frac{\pi}{2} - \theta\right)$. Use this information to derive similar cofunction relationships for tangent and cotangent, as well as secant and cosecant.

3.3.2 HOME ACTIVITY: Equivalent Trigonometric Expressions (Answers)

Knowledge

Write each of the following in terms of the cofunction identity:

1. $\sin\frac{\pi}{12} = \cos\frac{5\pi}{12}$
2. $\sin\frac{2\pi}{5} = \cos\frac{\pi}{10}$
3. $\sin\frac{5\pi}{8} = \cos\frac{-\pi}{8}$
4. $\sin\frac{5\pi}{12} = \cos\frac{\pi}{12}$
5. $\cos\frac{5\pi}{18} = \sin\frac{2\pi}{9}$
6. $\cos\frac{\pi}{9} = \sin\frac{7\pi}{18}$
7. $\cos\frac{7\pi}{36} = \sin\frac{11\pi}{36}$
8. $\cos\frac{2\pi}{9} = \sin\frac{5\pi}{18}$

Application

Fill in the blanks with the appropriate function name:

9. $\sin\frac{2\pi}{3} = \cos\left(\frac{-\pi}{6}\right)$

10. $\cos\frac{11\pi}{60} = \sin\frac{19\pi}{60}$

11. $\cos\frac{7\pi}{18} = \frac{1}{\csc\frac{\pi}{9}}$

For right triangle ABC:

12. $\frac{\sqrt{3}}{3}$

13. 0.109

14. 0.9816

Thinking

15.

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

Unit 3: Day 4: Sine and Cosine in Radians

MHF4U

Minds On: 5 Action: 50 Consolidate:20 Total=75 min	**Learning Goal**: Graph f(x)=sinx and f(x)=cosx, using radian measures Make connections between the graphs of trigonometric functions generated with degrees and radians.	**Materials** • BLM 3.4.1 • BLM 3.4.2 • BLM 3.4.3 • Adhesive

Assessment Opportunities

Minds On…	**Small Groups or Class→ Puzzle** Students will Sort puzzle pieces to identify elements/characteristics of given function Compare like groups' choices and justify decisions for pieces Discuss choices for each function	Using small groups place pieces from BLM 3.4.1 into an envelope labelled as "Sine x" or "Cosine x." Students are to sort through pieces to select those which suit their function (either Sine or Cosine). Put Sine groups together (and Cosine groups together) to compare choices and discuss a united choice of pieces/characteristics or Using the entire class, provide each student with a puzzle piece from BL 3.4.1 (include additional puzzle pieces). Using the board, have each student place their puzzle piece under the title of "Sine x", "Cosine x" or "Neither". Compare and justify choices.
Action!	**Partners → Investigation** Graph Sine in degrees and radians (BLM 3.4.1) Graph Cosine in degrees and radians (BLM 3.4.1) **Groups→Discussion** Discuss characteristics of their functions Graph their functions in radians Discuss how these characteristics change when graphed in radians **Learning Skills/Teamwork/Checkbric: Teacher should circulate among groups and partners to ensure conversations are on-topic and that each student is productive** **Mathematical Process Focus: Selecting Tools & Computational Strategies, and Communicating: Students are using different strategies to graph each function and they are discussing mathematical ideas with their partners, small groups and/or class**	
Consolidate Debrief	**Whole Class → Discussion** Students will • Complete a Frayer Model of characteristics of Sine and Cosine functions in radians	
	Home Activity or Further Classroom Consolidation Journal entry: Suppose a friend missed today's lesson. Fully explain how the graphs of sine and cosine graphed in degrees are similar, yet different, from graphs in radians. Include key elements/characteristics of each graph in your explanations, and use appropriate mathematics language.	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
6.2	5.4				

3.4.1 Characteristics of Sine and Cosine (Teacher Notes)

Maximum of 1	Minimum of -1
Period 360º	Period 180°
Zeros: 0º, 180º, 360º	Zeros: 90º, 270º
Phase Shift:90º right	Phase Shift: 90º left
Maximum of -1	Minimum of 1
Amplitude 1	Amplitude 2
y-intercept: 0	y-intercept: 1
Vert. Trans.: 2 units ↑	Vert. Trans.: 2 units ↓

3.4.1 Characteristics of Sine and Cosine (Teacher Notes)

Additional Puzzle Pieces:
Use characteristics found below if the puzzle involves the entire class. Be sure to enlarge each of the characteristics so that they can be easily seen when posted on the board.

The function is not periodic	The function is periodic
Period: 360º	Amplitude: 1
*Domain: 0º - 360º see note↓	Minimum of -1
Maximum of 1	Range:-1 to 1
*Domain: 0º - 360º see note↓	Range:-1 to 1
Positive trig ratios in the 1st and 2nd quadrant	Positive trig ratios in the 1st and 4th quadrant
Positive trig ratios in the 2nd and 3rd quadrant	Positive trig ratios in the 3rd and 4th quadrant
The function is periodic	The function has asymptotes

3.4.1 Characteristics of Sine and Cosine (Answers)

Sine x	Cosine x
Maximum: 1	Maximum: 1
Minimum: -1	Minimum: -1
Period: 360º	Period: 360º
Amplitude: 1	Amplitude: 1
Zeros: 0º, 180º, 360º	Zeros: 90º, 270º
y-intercept: 0	y-intercept: 1
The function is periodic	The function is periodic
*Domain: 0º - 360º see note↓	*Domain: 0º - 360º see note↓
Range:-1 to 1	Range:-1 to 1
Positive trig ratios in the 1st and 2nd quadrant	Positive trig ratios in the 1st and 4th quadrant

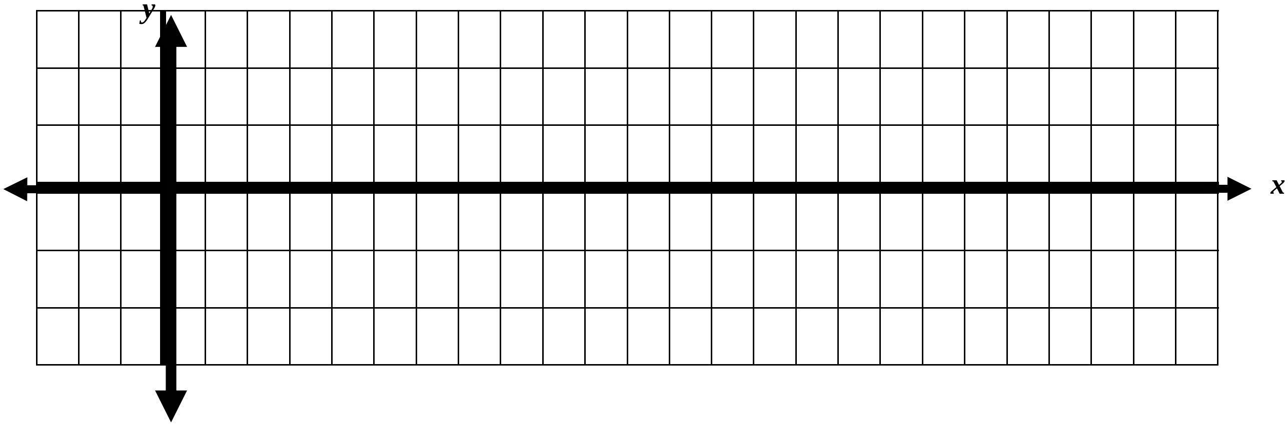
*This is not the domain of the entire sine/cosine functions but a possible domain for one period of each

Neither Sine x or Cosine x
The function is not periodic
Positive trig ratios in the 2nd and 3rd quadrant
Positive trig ratios in the 3rd and 4th quadrant
The function has asymptotes

3.4.2 Graph of Sine and Cosine in Degrees and Radians

Name ______________
Date _______________

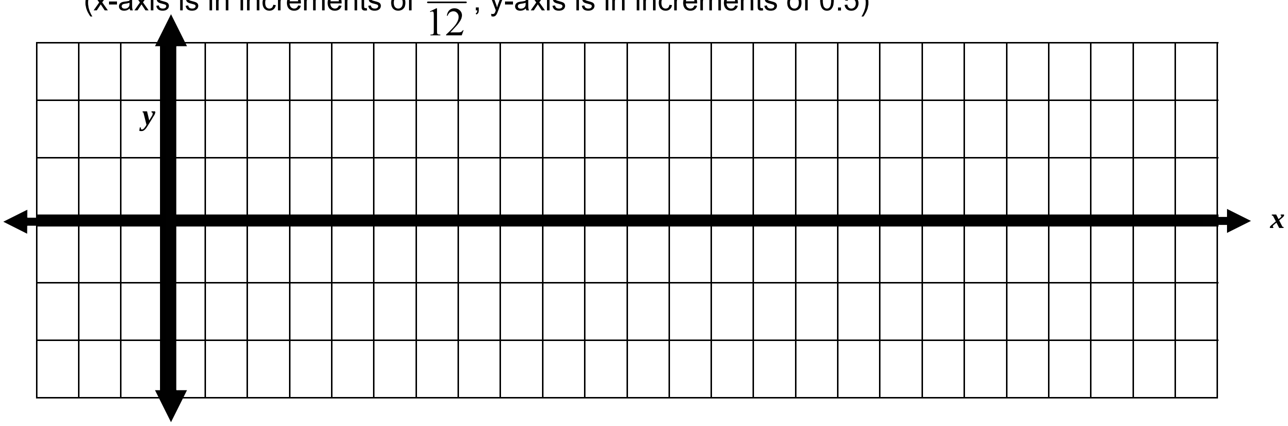
1a) Graph y=Sine (x) using degrees.
(x-axis is in increments of 15º, y-axis is in increments of 0.5)



Characteristics:
Max. value:___________ Min. value: ____________

y intercept: ___________ x intercept (zeros): _________

1b) Graph y=Sine (x) using radians.
(x-axis is in increments of $\frac{\pi}{12}$, y-axis is in increments of 0.5)

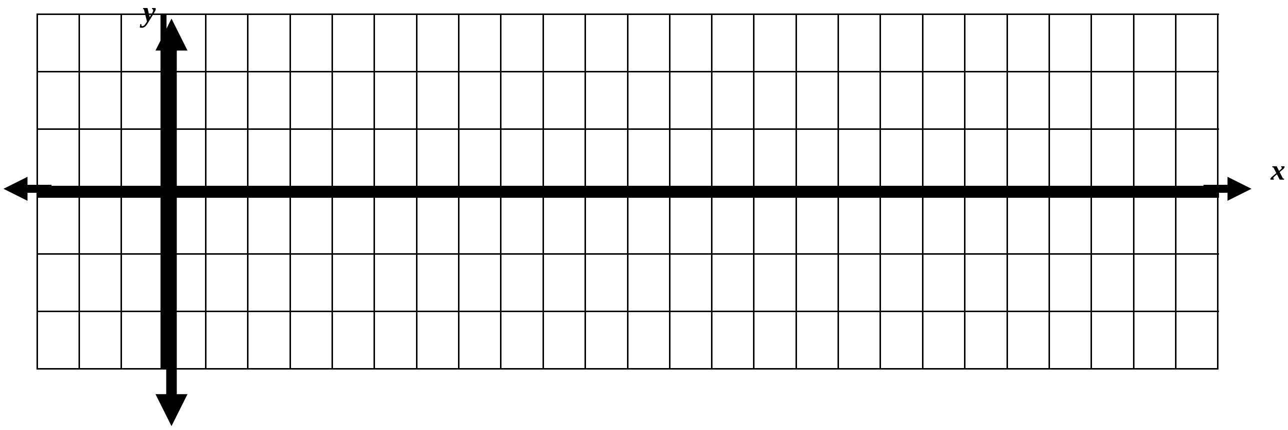


Characteristics:
Max. value:___________ Min. value: ____________

y intercept: ___________ x intercept (zeros): _________

3.4.2 Graph of Sine and Cosine in Degrees and Radians
(Continued)

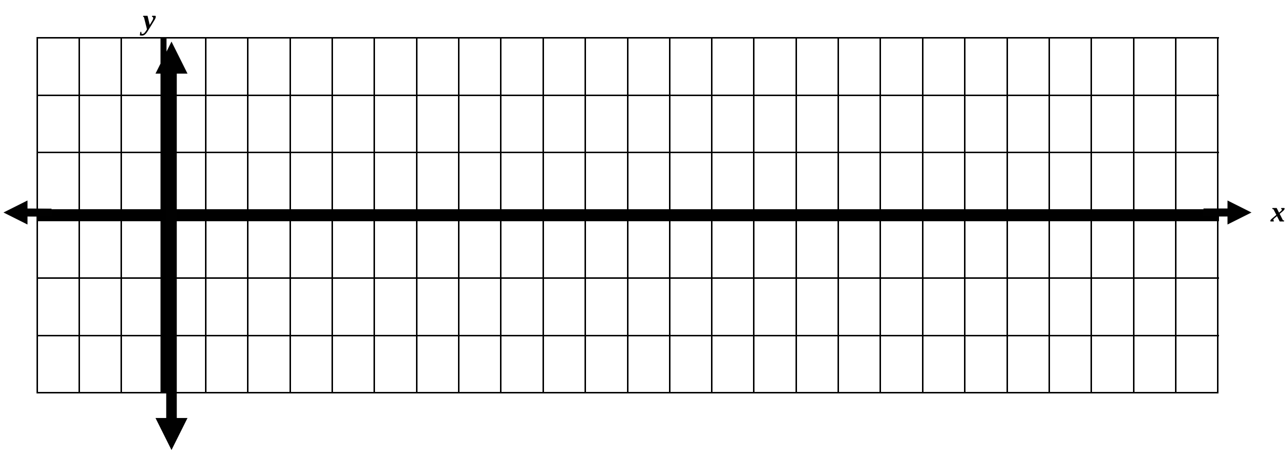
2a) Graph y=Cosine x using degrees.
(x-axis is in increments of 15º, y-axis is in increments of 0.5)



Characteristics:
Max. value:___________ Min. value: ____________

y intercept: ___________ x intercepts (zeros): _________

2b) Graph y=Cosine x using radians.
(x-axis is in increments of $\frac{\pi}{12}$, y-axis is in increments of 0.5)



Characteristics:
Max. value:___________ Min. value: ____________

y intercept: ___________ x intercepts (zeros): _________

3.4.2 Graph of Sine and Cosine in Degrees and Radians

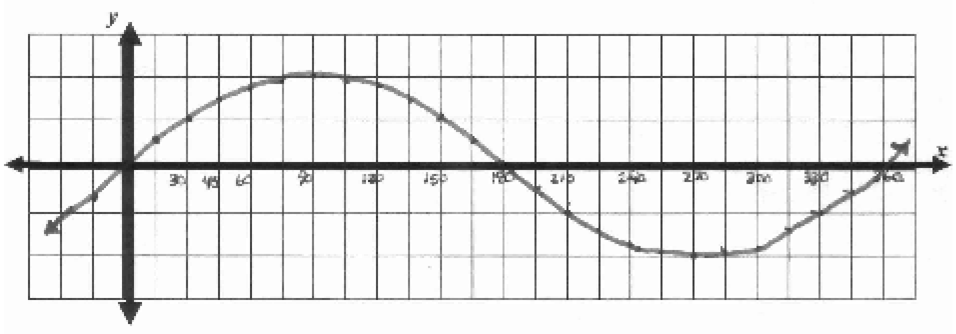
ANSWERS

BLM 3.4.1

<u>Graphs of Sine and Cosine: Radians</u>

On the given set of axes, graph Sine θ and Cosine θ.
(x-axis is in increments of 15°)
(y-axis is in increments of 0.5)

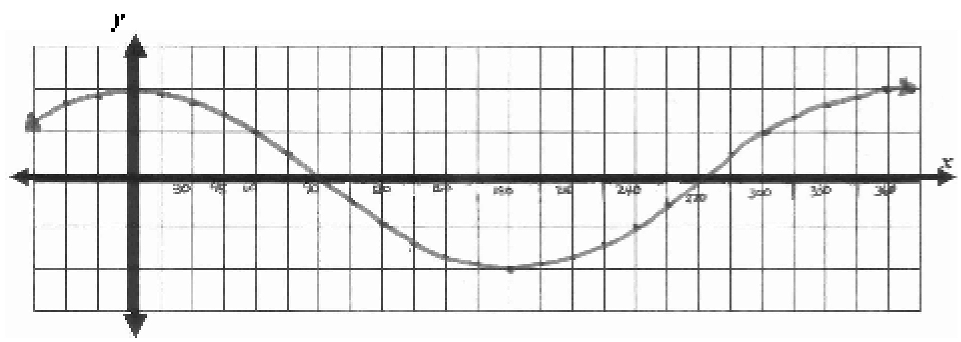
y=Sine θ



Characteristics:

maximum: 1
minimum: -1
y intercept: 0
x-int./zeros: 0, 180°, 360°

Cosine θ



Characteristics:

maximum: 1
minimum: -1
y intercept: 1
x-int. /zeros 90°, 270°

3.4.3 Frayer Model for Sine and Cosine Functions Using Radians

Name ____________________ Date ________________

Complete each Frayer Model with information on each function IN RADIANS.

Sine θ

Period

Zeros

Y-intercept

Characteristics

- Maximum:
- Minimum:
- Amplitude:

Cosine θ

Period

Zeros

Y-intercept

Characteristics

- Maximum:
- Minimum:
- Amplitude:

3.4.3 Frayer Model for Sine and Cosine Functions Using Radians (Answers)

Complete each Frayer Model with information on each function IN RADIANS.

Sine θ	
Period 2π	Zeros Zeros: $0, \pi, 2\pi, k\pi$
Y-intercept 0	Characteristics • Maximum: 1 • Minimum: -1 • Amplitude: 1

Cosine θ	
Period 2π	Zeros $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}+k\pi$
Y-intercept 1	Characteristics • Maximum: 1 • Minimum: -1 • Amplitude: 1

Unit 3: Day 5: Graphs of Sine & Cosine Reciprocals in Radians

Minds On: 10 Action: 50 Consolidate:15 Total=75 min	**Learning Goal**: Graph the reciprocals, using radian measure and properties of rational functions	**Materials** BLM 3.5.1 BLM 3.5.2 BLM 3.5.3 Graphing Calculators

Assessment Opportunities

Minds On…	**Whole Class → Quiz** Using BLM 3.5.1 complete a matching quiz on functions and their reciprocals **Whole Class → Discussion** Correct quizzes Discuss any errors to clarify understanding	
Action!	**Partners → Investigation** Using BLM 3.5.2 and graphing calculators complete investigation on graphs of trigonometric functions.. Use knowledge of restrictions of rational functions to identify asymptotes Identify key elements of primary trig. functions and how they relate to the graphs of the reciprocal functions **Learning Skills/Teamwork/Checkbric: Teacher should circulate among students to listen and determine if students have successfully identified key elements of graphs.** **Mathematical Process Focus: Reasoning & Proving, Communicating: Students will discover that asymptotes of reciprocal functions occur at zeros of original functions, and communicate this with their partners.**	
Consolidate Debrief	**Whole Class → Discussion** Compare and discuss their findings from BLM 3.5.2	
	Home Activity or Further Classroom Consolidation Complete BLM 3.5.3	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
6.2	5.4				

3.5.1 Reciprocal Trigonometric Functions

Name ____________________

Match the functions on the left with their reciprocals on the right.

1. $\sin\theta$	a. $\dfrac{1}{\cos\theta}$
2. $\cos\theta$	b. $\dfrac{1}{\cot\theta}$
3. $\tan\theta$	c. $\dfrac{1}{\tan\theta}$
4. $\sec\theta$	d. $\dfrac{1}{\csc\theta}$
5. $\csc\theta$	e. $\dfrac{1}{\sin\theta}$
6. $\cot\theta$	f. $\dfrac{1}{\sec\theta}$

State restrictions on each function:

7. $\dfrac{(2x+3)(x-7)}{(x-4)(x+2)}$
8. $\dfrac{x(2x+1)}{(3x-2)(x+2)}$
9. $\dfrac{(x-4)(x+4)}{x(x-3)(x-2)}$
10. $\dfrac{(x-7)(2x+5)}{x(x-9)(3x+4)}$

3.5.1 Reciprocal Trigonometric Functions (Answers)

Name ____________________

Match the functions on the left with their reciprocals on the right.

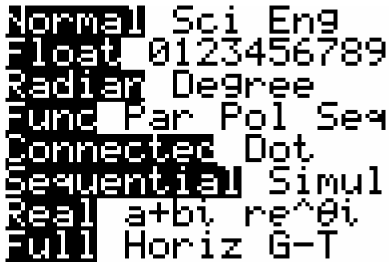
1. $\sin\theta$ D	a. $\dfrac{1}{\cos\theta}$
2. $\cos\theta$ F	b. $\dfrac{1}{\cot\theta}$
3. $\tan\theta$ B	c. $\dfrac{1}{\tan\theta}$
4. $\sec\theta$ A	d. $\dfrac{1}{\csc\theta}$
5. $\csc\theta$ E	e. $\dfrac{1}{\sin\theta}$
6. $\cot\theta$ C	f. $\dfrac{1}{\sec\theta}$

State restrictions on each function:

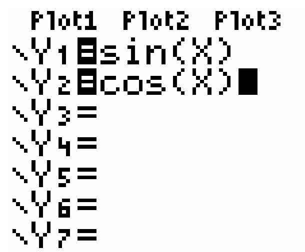
7. $\dfrac{(2x+3)(x-7)}{(x-4)(x+2)}$	$x \neq 4, -2$
8. $\dfrac{x(2x+1)}{(3x-2)(x+2)}$	$x \neq \frac{2}{3}, -2$
9. $\dfrac{(x-4)(x+4)}{x(x-3)(x-2)}$	$x \neq 0, 3, 2$
10. $\dfrac{(x-7)(2x+5)}{x(x-9)(3x+4)}$	$x \neq 0, 9, -\frac{3}{4}$

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians

Ensure that the calculator is set to RADIAN mode (MODE)



Graph ***sin x*** and ***cos x***



Use the TRACE function to identify key characteristics of the functions:

Sine x	**Cosine x**
Period:	Period:
Maximum Point:	Maximum Points:
Minimum Point:	Minimum Point:
Y-intercept:	Y-intercept:
Zeros:	Zeros:

To view the table of values in radians, it is important to set the table restrictions.

Press 2nd and WINDOW .

For TblStart=, enter $-\pi \div 3$

For Δ Tbl=, enter $\pi \div 12$

(the calculator will change these values to decimal equivalents)

To view the table of values, press 2nd and GRAPH

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Continued)

Complete the table as shown:

x	Sin (x)		Cos (x)	
$\frac{-\pi}{3}$				
$\frac{-\pi}{4}$				
$\frac{-\pi}{6}$				
$\frac{-\pi}{12}$				
0				
$\frac{\pi}{12}$				
$\frac{\pi}{6}$				
$\frac{\pi}{4}$				
$\frac{\pi}{3}$				
$\frac{5\pi}{12}$				
$\frac{\pi}{2}$				
$\frac{7\pi}{12}$				
$\frac{2\pi}{3}$				
$\frac{3\pi}{4}$				
$\frac{5\pi}{6}$				
$\frac{11\pi}{12}$				
π				

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Continued)

x	Sin (x)		Cos (x)	
$\frac{13\pi}{12}$				
$\frac{7\pi}{6}$				
$\frac{5\pi}{4}$				
$\frac{4\pi}{3}$				
$\frac{17\pi}{12}$				
$\frac{3\pi}{2}$				
$\frac{19\pi}{12}$				
$\frac{5\pi}{3}$				
$\frac{7\pi}{4}$				
$\frac{11\pi}{6}$				
$\frac{23\pi}{12}$				
2π				

The remaining columns of the table are for the RECIPROCAL trigonometric functions.

You know that $\csc x = \frac{1}{\sin x}$ and $\sec x = \frac{1}{\cos x}$.

To find the values to graph these functions, simply divide "1" by each of the values from sin x or cos x.

For instance, since $\sin\frac{4\pi}{3} = -0.8660$, $\csc\frac{4\pi}{3} = \frac{1}{-0.8660} = -1.1547$

Label the top of the extra columns with **csc (x)** and **sec (x)** , then fill in their corresponding values.

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Continued)

What do you notice about $\csc 0$, $\csc \pi$, $\csc 2\pi$, $\sec \frac{\pi}{2}$, $\sec \frac{3\pi}{2}$?

Why does this happen?

What occurs on the graphs of the reciprocals at those points?

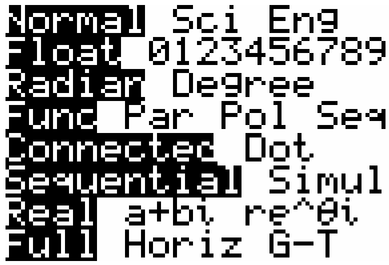
State the restrictions of the secant and cosecant functions:

Secant:

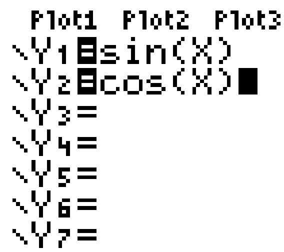
Cosecant:

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Answers)

Ensure that the calculator is set to RADIAN mode (MODE)



Graph ***sin (x)*** and ***cos (x)***



Use the TRACE function to identify key characteristics of the functions:

Sine x	**Cosine x**
Period: 2π	Period: 2π
Maximum Point: $\left(\frac{\pi}{2},1\right)$	Maximum Points: $(0,1)$ $(2\pi,1)$
Minimum Point: $\left(\frac{3\pi}{2},-1\right)$	Minimum Point: $(\pi,-1)$
Y-intercept: 0	Y-intercept: 1
Zeros: $0, 2\pi$	Zeros: $\frac{\pi}{2}, \frac{3\pi}{2}$

To view the table of values in radians, it is important to set the table restrictions.

Press 2nd and WINDOW .

For TblStart=, enter $-\pi \div 3$

For Δ Tbl=, enter $\pi \div 12$

(the calculator will change these values to decimal equivalents)

To view the table of values, press 2nd and GRAPH

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Answers continued)

Complete the table as shown:

x	Sin (x)	Csc (x)	Cos (x)	Sec (x)
$\frac{-\pi}{3}$	-0.8660	-1.155	0.5	2
$\frac{-\pi}{4}$	-0.7071	-1.414	0.7071	1.4142
$\frac{-\pi}{6}$	-0.5	-2	0.8660	1.1547
$\frac{-\pi}{12}$	-0.2588	-3.864	0.9659	1.0353
0	0	ERROR	1	1
$\frac{\pi}{12}$	0.2588	3.8637	0.9659	1.0353
$\frac{\pi}{6}$	0.5	2	0.8660	1.1547
$\frac{\pi}{4}$	0.7071	1.4142	0.7071	1.4142
$\frac{\pi}{3}$	0.8660	1.1547	0.5	2
$\frac{5\pi}{12}$	0.9659	1.0353	0.2588	3.8637
$\frac{\pi}{2}$	1	1	0	ERROR
$\frac{7\pi}{12}$	0.9659	1.0353	-0.2588	-3.864
$\frac{2\pi}{3}$	0.8660	1.1547	-0.5	-2
$\frac{3\pi}{4}$	0.7071	1.4142	-0.7071	-1.414
$\frac{5\pi}{6}$	0.5	2	-0.8660	-1.155
$\frac{11\pi}{12}$	0.2588	3.8637	-0.9659	-1.035
π	0	ERROR	-1	-1

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Answers continued)

x	Sin (x)	Csc (x)	Cos (x)	Sec (x)
$\frac{13\pi}{12}$	-0.2588	-3.864	-0.9659	-1.035
$\frac{7\pi}{6}$	-0.5	-2	-0.8660	-1.155
$\frac{5\pi}{4}$	-0.7071	-1.414	-0.7071	-1.414
$\frac{4\pi}{3}$	-0.8660	-1.155	-0.5	-2
$\frac{17\pi}{12}$	-0.9659	-1.035	-0.2588	-3.864
$\frac{3\pi}{2}$	-1	-1	0	ERROR
$\frac{19\pi}{12}$	-0.9659	-1.035	0.2588	3.8637
$\frac{5\pi}{3}$	-0.8660	-1.155	0.5	2
$\frac{7\pi}{4}$	-0.7071	-1.414	0.7071	1.4142
$\frac{11\pi}{6}$	-0.5	-2	0.8660	1.1547
$\frac{23\pi}{12}$	-0.2588	-3.864	0.9659	1.0353
2π	0	ERROR	1	1

The remaining columns of the table are for the RECIPROCAL trigonometric functions.

You know that $\csc x = \frac{1}{\sin x}$ and $\sec x = \frac{1}{\cos x}$.

To find the values to graph these functions, simply divide "1" by each of the values from sin x or cos x.

For instance, since $\sin\frac{4\pi}{3} = -0.8660$, $\csc\frac{4\pi}{3} = \frac{1}{-0.8660} = -1.1547$

Label the top of the extra columns with **csc (x)** and **sec (x)** , then fill in their corresponding values.

3.5.2 Investigation: Graphing Secondary Trig. Functions in Radians (Answers continued)

What do you notice about $\csc 0$, $\csc \pi$, $\csc 2\pi$, $\sec \frac{\pi}{2}$, $\sec \frac{3\pi}{2}$?

ERROR

Why does this happen?

Because you are dividing by zero, which is undefined

What occurs on the graphs of the reciprocals at those points?

Vertical lines

State the restrictions of the secant and cosecant functions:

Secant: $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$ nor any decrease or increase by π

Cosecant: $x \neq 0, \pi, 2\pi$ nor any of their multiples

3.5.3 Reciprocal Trigonometric Functions Practice

Knowledge

Find each function value:

1. $\csc\theta$, if $\sin\theta = \frac{\sqrt{2}}{4}$
2. $\cos\theta$, if $\sec\theta = -2.5$
3. $\sin\theta$, if $\csc\theta = 3$
4. $\sin\theta$, if $\csc\theta = \sqrt{15}$
5. $\sec\theta$, if $\cos\theta = \frac{-1}{\sqrt{7}}$
6. $\sec\theta$, if $\cos\theta = \frac{5}{\sqrt{26}}$
7. $\csc\theta$, if $\sin\theta = \frac{\sqrt{11}}{6}$
8. $\cos\theta$, if $\sec\theta = \frac{-\sqrt{14}}{3}$
9. $\sin\theta$, if $\csc\theta = \frac{\sqrt{3}}{3}$
10. $\sec\theta$, if $\cos\theta = \frac{\sqrt{6}}{12}$

Application

Find each function value (keep answers in radical form):

11. $\csc\theta$, if $\tan\theta = \frac{\sqrt{6}}{12}$
12. $\sec\theta$, if $\sin\theta = \frac{\sqrt{3}}{3}$
13. $\cos\theta$, if $\cot\theta = \frac{-\sqrt{3}}{3}$
14. $\sin\theta$, if $\cos\theta = \frac{\sqrt{3}}{2}$
15. $\sec\theta$, if $\csc\theta = \sqrt{15}$
16. $\cos\theta$, if $\csc\theta = \sqrt{15}$
17. $\sec\theta$, if $\tan\theta = \sqrt{3}$
18. $\csc\theta$, if $\sin\theta = \frac{-2}{\sqrt{12}}$
19. $\cos\theta$, if $\sin\theta = \frac{5}{13}$
20. $\sin\theta$, if $\tan\theta = \frac{-2}{\sqrt{5}}$

ANSWERS:

1. $\frac{4}{\sqrt{2}}$
2. -0.4
3. $\frac{1}{3}$
4. $\frac{1}{\sqrt{15}}$
5. $-\sqrt{7}$
6. $\frac{\sqrt{26}}{5}$
7. $\frac{6}{\sqrt{11}}$
8. $-\frac{3}{\sqrt{14}}$
9. $\frac{3}{\sqrt{3}}$
10. $\frac{12}{\sqrt{6}}$
11. 5
12. $\frac{\sqrt{6}}{3}$
13. $-\frac{1}{2}$
14. $\frac{1}{2}$
15. $\frac{\sqrt{15}}{\sqrt{14}}$
16. $\frac{\sqrt{14}}{\sqrt{15}}$
17. 2
18. $-\frac{\sqrt{12}}{2}$
19. $\frac{12}{13}$
20. $-\frac{2}{3}$

Unit 3: Day 6: Graphs of Tangent and Cotangent

MHF4U

Minds On: 10 Action: 45 Consolidate:20 Total=75 min	**Learning Goal**: Make connections between the tangent ratio and the tangent function using technology Graph the reciprocal trig functions for angles in radians with technology, and determine and describe the key properties Understand notation used to represent the reciprocal functions	**Materials** Graphing calculators BLM 3.6.1 BLM 3.6.2 BLM 3.6.3

Assessment Opportunities

Minds On…	**Small Groups → Puzzle** Sort puzzle pieces to identify elements/characteristics of given function Compare like groups' choices and justify decisions for pieces Discuss choices for each function	Place pieces from BLM 3.6.1 into an envelope labelled as "Tangent x" or "Cotangent x." Students are to sort through pieces to select those which suit their function (either Tangent or Cotangent). Put Tangent groups together (and Cotangent groups together) to compare choices and discuss a united choice of pieces/characteristics
Action!	**Partners → Investigation** Graph Tangent and Cotangent in degrees Graph Tangent and Cotangent in radians **Groups→Discussion** Discuss characteristics of their functions Graph their functions in radians Discuss how these characteristics change when graphed in radians **Learning Skills/Teamwork/Checkbric: Teacher should circulate among groups and partners to ensure conversations are on-topic and students' work is productive** **Mathematical Process Focus: Selecting Tools & Computational Strategies, and Communicating: Students are using strategies to graph, and discussing with their partners or small groups**	
Consolidate Debrief	**Whole Class → Discussion** Complete a Frayer Model of characteristics of Tangent and Cotangent functions in radians	
	Home Activity or Further Classroom Consolidation Journal entry: Suppose a friend missed today's lesson. Fully explain how the graphs of tangent and cotangent graphed in degrees are similar, yet different, from graphs in radians. Include key elements/characteristics of each graph in your explanations, and use appropriate mathematics language.	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
6.6	5.4				

3.6.1 Characteristics of Tangent and Cotangent Functions *(Teacher Notes)*

Maximum of 1	Minimum of -1
Period 360º	Period 180°
Zeros: 0º, 180º, 360º	Zeros: 90º, 270º
No maximum	No minimum
Undefined at 90º, 270º	Undefined at 180º, 360º
Undefined at 45º, 225º	Undefined at 135º, 315º
y-intercept: 0	y-intercept: 90º

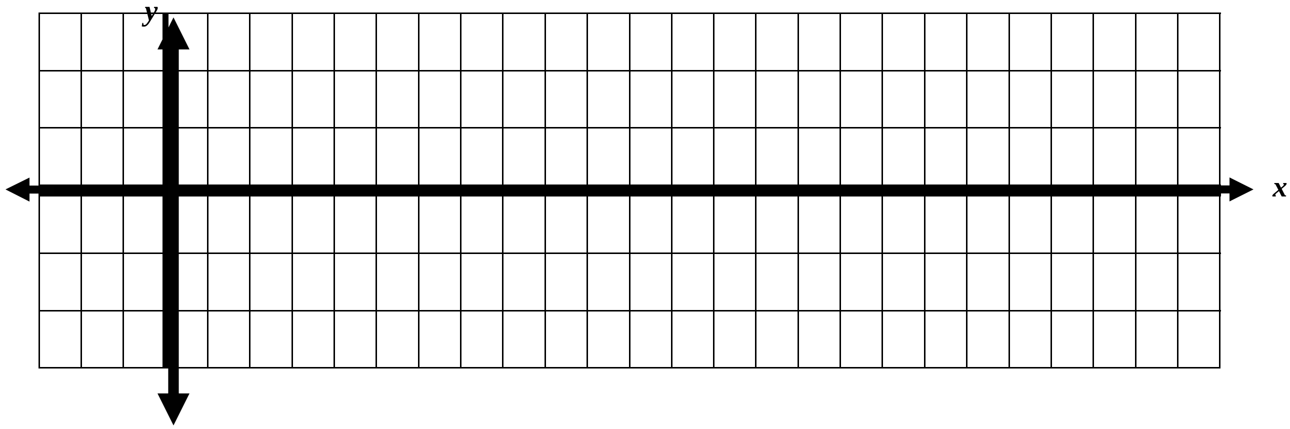
3.6.1 Characteristics of Tangent and Cotangent Functions *(Answers)*

Tangent x	Cotangent x
No maximum	No maximum
No minimum	No minimum
Period: 180º	Period: 180º
Zeros: 0º, 180º, 360º	Zeros: 90º, 270º
y-intercept: 0	y-intercept: 1

3.6.1 Graphs of Tangent and Cotangent in Degrees

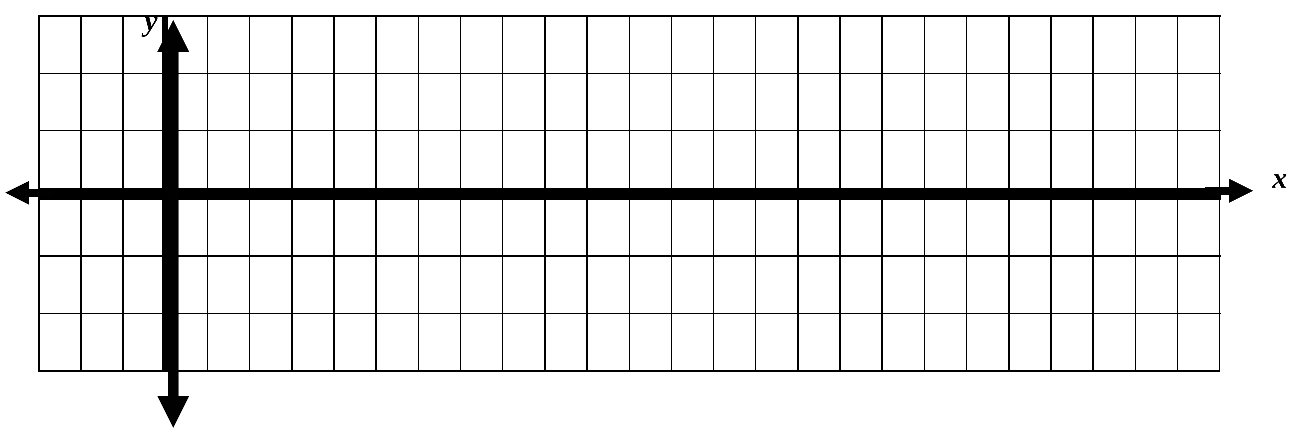
On the given set of axes, graph Tangent x and Cotangent x.
(x-axis is in increments of 15º)
(y-axis is in increments of 0.5)

y = Tangent (x)



Characteristics:

y = Cotangent (x)



Characteristics:

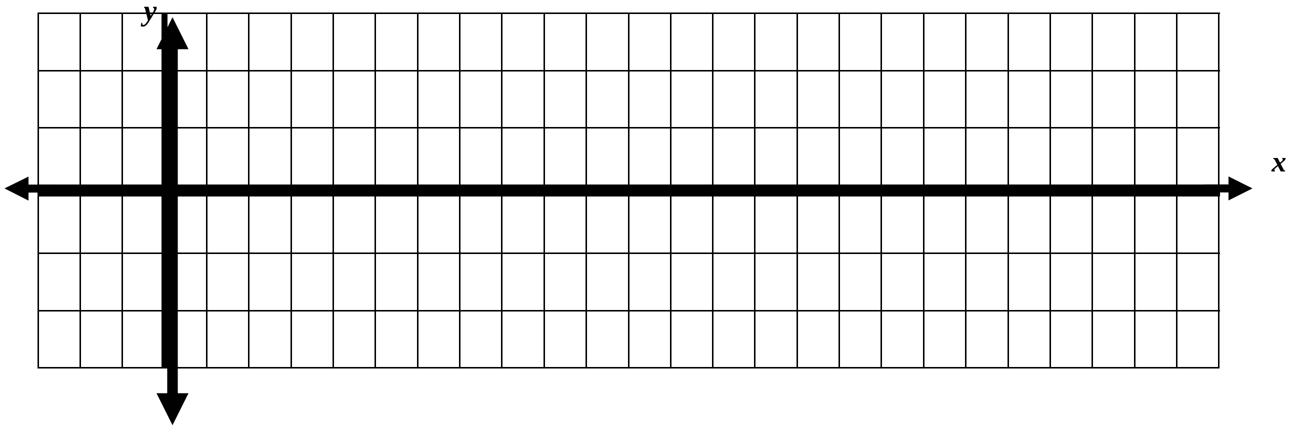
3.6.1 Graphs of Tangent and Cotangent in Radians

On the given set of axes, graph Tangent x and Cotangent x.

(x-axis is in increments of $\frac{\pi}{12}$)

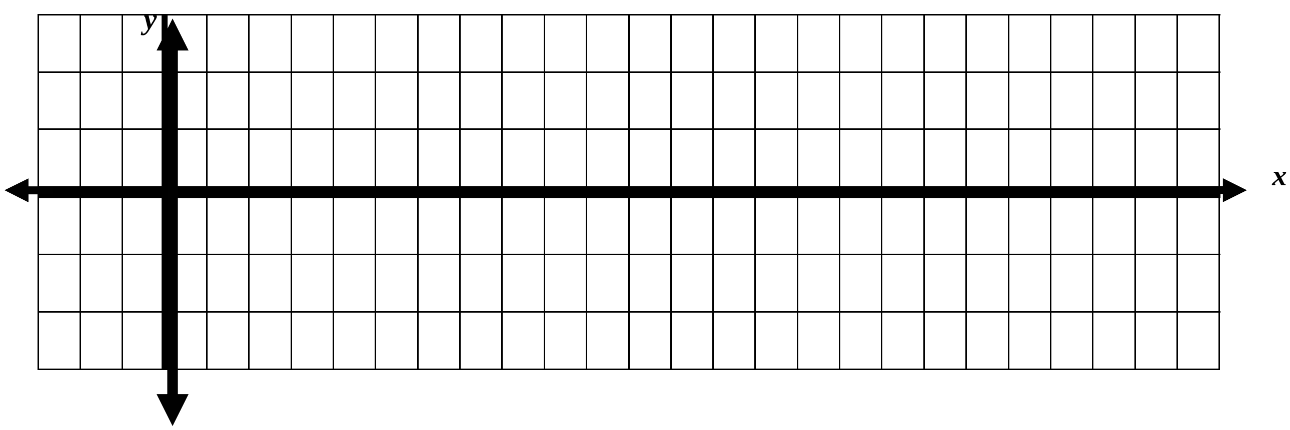
(y-axis is in increments of 0.5)

y = Tangent (x)



Characteristics:

y = Cotangent (x)



Characteristics:

3.6.1 Graphs of Tangent and Cotangent in Radians (Answers)

*I disagree with the solution given for cotx=- the graph does not have any holes, only asymptotes

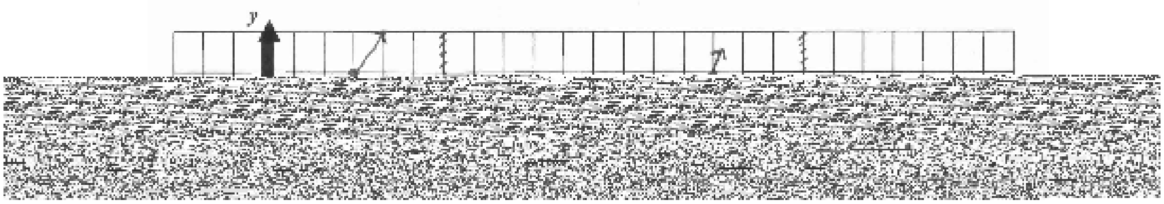
ANSWERS

BLM 3.4.1

Graphs of Tangent and Cotangent: Radians

On the given set of axes, graph Tangent θ and Cotangent θ.
(x-axis is in increments of 15°)
(y-axis is in increments of 0.5)

y=Tangent θ



3.6.2 Frayer Model for Tangent and Cotangent

Complete each Frayer Model with information on each function IN RADIANS.

Tangent θ

Period

Zeros

Y-intercept

Characteristics

- Maximum:
- Minimum:
- Asymptotes:

Cotangent θ

Period

Zeros

Y-intercept

Characteristics

- Maximum:
- Minimum:
- Asymptotes:

3.6.2 Frayer Model for Tangent and Cotangent (Answers)

Complete each Frayer Model with information on each function IN RADIANS.

Tangent θ

Period

π

Zeros

$0, \pi, 2\pi$

Y-intercept

0

Characteristics

- Maximum: None
- Minimum: None
- Asymptotes: $\frac{\pi}{2}, \frac{3\pi}{2}$

Cotangent θ

Period

π

Zeros

Y-intercept

None

'Holes' at $\frac{\pi}{2}, \frac{3\pi}{2}$

Characteristics

- Maximum: None
- Minimum: None
- Asymptotes: $0, \pi, 2\pi$

Unit 3: Day 8: Trigonometric Rates of Change		MHF4U
Minds On: 5 Action: 50 Consolidate:20 Total=75 min	**Learning Goal**: Solve problems involving average and instantaneous rates of change at a point using numerical and graphical methods	**Materials** BLM3.8.1 BLM 3.8.2
		Assessment Opportunities
Minds On…	**Individual → Quiz** Complete a quiz on finding the average and instantaneous rate of change of a simple trigonometric function (BLM 3.8.1)	
Action!	**Small Groups → Assignment** Choose/Be assigned one of the questions to solve in small groups (BLM 3.8.2) Present their solutions to the class Discuss problems, solutions, methods	
Consolidate Debrief	**Whole Class → Summarize** Consolidate their understanding from the presentations of the groups and the small group assignment	
Exploration *Application*	**Home Activity or Further Classroom Consolidation** .Students will complete the remaining exercises from the Small Groups Assignment.	

3.8.1 Rate of Change for Trigonometric Functions

Given the function: $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$

1. Sketch $f(\theta)$ on an interval $\left[\frac{\pi}{6},\frac{7\pi}{6}\right]$

2. Is the function increasing or decreasing on the interval $\frac{\pi}{3}$ to $\frac{2\pi}{3}$.

3. Draw the line through the points $f\left(\frac{\pi}{3}\right)$ and $f\left(\frac{2\pi}{3}\right)$

4. Find the average rate of change of the function $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$ from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$.

5. What does this mean?

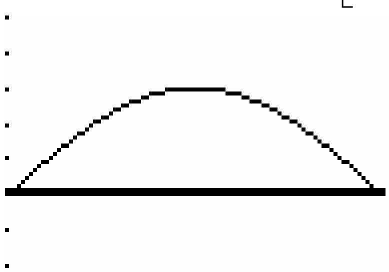
6. Describe how to find the instantaneous rate of change of $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$ at $\frac{\pi}{3}$. What does this mean?

3.8.1 Rate of Change for Trigonometric Functions (Answers)

Given the function: $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$

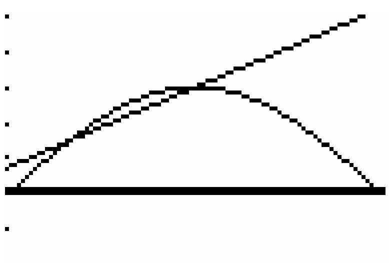
*And the points: $\frac{\pi}{3}$ $\frac{2\pi}{3}$

1. Sketch on an interval $\left[\frac{\pi}{6},\frac{7\pi}{6}\right]$



2. Is the function increasing or decreasing on the interval $\frac{\pi}{3}$ to $\frac{2\pi}{3}$. Increasing

3. Draw the line through the points $f\left(\frac{\pi}{3}\right)$ and $f\left(\frac{2\pi}{3}\right)$



4. Find the average rate of change of the function $f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$ from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$.

$$\frac{f\left(\frac{\pi}{3}\right)-f\left(\frac{2\pi}{3}\right)}{\frac{\pi}{3}-\frac{2\pi}{3}}=\frac{1.5-3}{\frac{-\pi}{3}}=\frac{-1.5}{\frac{-\pi}{3}}=0.025$$

3.8.1 Rate of Change for Trigonometric Functions
(Answers continued)

5. What does this mean?

This is the slope of the line through the points $\left(\frac{\pi}{3}, 1.5\right)$ and $\left(\frac{2\pi}{3}, 3\right)$

6. Find the instantaneous rate of change at $\frac{\pi}{3}$.

To find instantaneous rate of change at $\frac{\pi}{3}$, choose values for θ which move closer to $\frac{\pi}{3}$ from $\frac{2\pi}{3}$.

At $\frac{\pi}{2}$ $$\frac{f\left(\frac{\pi}{2}\right)-f\left(\frac{\pi}{3}\right)}{\frac{\pi}{2}-\frac{\pi}{3}}=\frac{2.5981-1.5}{\frac{\pi}{6}}=\frac{1.0981}{\frac{\pi}{6}}=0.0366$$

At $\frac{5\pi}{12}$ $$\frac{f\left(\frac{5\pi}{12}\right)-f\left(\frac{\pi}{3}\right)}{\frac{5\pi}{12}-\frac{\pi}{3}}=\frac{2.1213-1.5}{\frac{\pi}{12}}=\frac{0.6213}{\frac{\pi}{12}}=0.0414$$

At $\frac{7\pi}{18}$ $$\frac{f\left(\frac{7\pi}{18}\right)-f\left(\frac{\pi}{3}\right)}{\frac{7\pi}{18}-\frac{\pi}{3}}=\frac{1.9284-1.5}{\frac{\pi}{18}}=\frac{0.4284}{\frac{\pi}{18}}=0.0428$$

At $\frac{13\pi}{36}$ $$\frac{f\left(\frac{13\pi}{36}\right)-f\left(\frac{\pi}{3}\right)}{\frac{13\pi}{36}-\frac{\pi}{3}}=\frac{1.7207-1.5}{\frac{\pi}{36}}=\frac{0.2207}{\frac{\pi}{36}}=0.0441$$

At $\frac{61\pi}{180}$ $$\frac{f\left(\frac{61\pi}{180}\right)-f\left(\frac{\pi}{3}\right)}{\frac{61\pi}{180}-\frac{\pi}{3}}=\frac{1.5451-1.5}{1}=\frac{0.0451}{1}=0.0451$$

Approaches 0.05. This means that the slope of the line tangent to

$f(\theta)=3\sin\left(\theta-\frac{\pi}{6}\right)$ at $\frac{\pi}{3}$ is 0.05

3.8.2 Rate of Change for Trigonometric Functions: Problems

For each of the following functions, sketch the graph on the indicated interval. Find the average rate of change using the identified points, then find the instantaneous rate of change at the indicated point.

1. In a simple arc for an alternating current circuit, the current at any instant ***t*** is given by the function $f(t)=15\sin(60t)$. Graph the function on the interval $0 \le t \le 5$. Find the average rate of change as ***t*** goes from 2 to 3. Find the instantaneous rate of change at ***t*** = 2.

2. The weight at the end of a spring is observed to be undergoing simple harmonic motion which can be modeled by the function $D(t)=12\sin(60\pi t)$. Graph the function on the interval $0 \le t \le 1$. Find the average rate of change as ***t*** goes from 0.05 to 0.40. Find the instantaneous rate of change at ***t*** = 0.40.

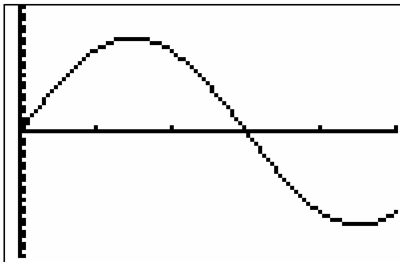
3. In a predator-prey system, the number of predators and the number of prey tend to vary in a periodic manner. In a certain region with cats as predators and mice as prey, the mice population ***M*** varied according to the equation $M=110250\sin(1/2)\pi t$, where ***t*** is the time in years since January 1996. Graph the function on the interval $0 \le t \le 2$. Find the average rate of change as ***t*** goes from 0.75 to 0.85. Find the instantaneous rate of change at ***t*** = 0.85.

4. A Ferris Wheel with a diameter of 50 ft rotates every 30 seconds. The vertical position of a person on the Ferris Wheel, above and below an imaginary horizontal plane through the center of the wheel can be modeled by the equation $h(t)=25\sin 12t$. Graph the function on the interval $15 \le t \le 30$. Find the average rate of change as ***t*** goes from 24 to 24.5. Find the instantaneous rate of change at ***t*** = 24.

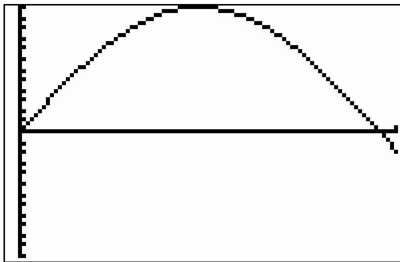
5. The depth of water at the end of a pier in Vacation Village varies with the tides throughout the day and can be modeled by the equation $D=1.5\cos[0.575(t-3.5)]+3.8$. Graph the function on the interval $0 \le t \le 10$. Find the average rate of change as ***t*** goes from 4.0 to 6.5. Find the instantaneous rate of change at $t=6.5$.

3.8.2 Rate of Change for Trigonometric Functions: Problems (Answers)

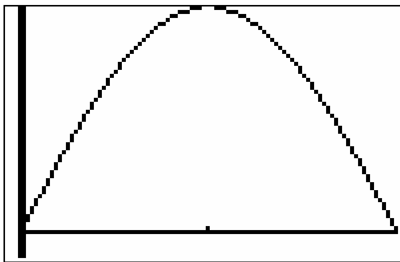
1.

	AVERAGE RATE OF CHANGE = -12.99	INSTANTANEOUS RATE OF CHANGE = -8

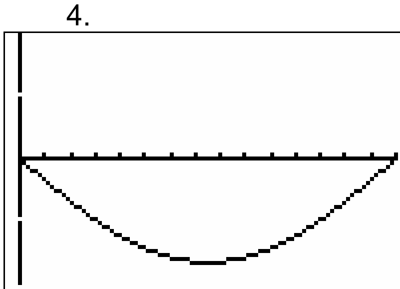
2.

	AVERAGE RATE OF CHANGE = 27.5629	INSTANTANEOUS RATE OF CHANGE = 10

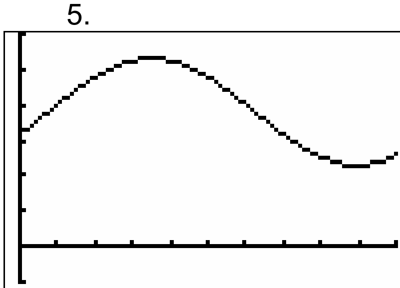
3.

	AVERAGE RATE OF CHANGE = 53460	INSTANTANEOUS RATE OF CHANGE = 40,000

4.

	AVERAGE RATE OF CHANGE = 1.88	INSTANTANEOUS RATE OF CHANGE = 1.620

5.

	AVERAGE RATE OF CHANGE = -0.66756	INSTANTANEOUS RATE OF CHANGE = -0.9