Unit 8  
Plane Geometry

Lesson Outline  
*Note: This unit could stand alone and be placed anywhere in the course.

**BIG PICTURE**

Students will:
- investigate properties of geometric objects using dynamic geometry software and manipulatives;
- illustrate and explain the relationship between angles formed by parallel lines cut by a transversal and interior and exterior angles of triangles and quadrilaterals;
- determine some properties of sides and diagonals of quadrilaterals.

**Note:** Students may have a very broad range of experience with using The Geometer’s Sketchpad®4. Skills can be taught as they are needed for each lesson, or alternatively, Introduction to The Geometer’s Sketchpad® 4 file (included in Day 2) could be used at the beginning of the unit.

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<td>It’s a Parallel World</td>
<td>Describe the properties and relationships of the angles formed by parallel lines cut by a transversal.</td>
<td>MG3.01, MG3.02 CGE 5a, 5c</td>
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<td>2</td>
<td>Plane Geometry – Introduction (Part 1) GSP®4 files: Plane Geometry, Introduction to Geometer’s Sketchpad</td>
<td>Review angles, triangles, and parallel lines through exploration. Build skills required for future use of The Geometer’s Sketchpad®4 (GSP®4).</td>
<td>MG3.01, MG3.02 CGE 5a, 5e</td>
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<td>3</td>
<td>Plane Geometry – Introduction (Part 2)</td>
<td>Explore geometrical concepts (angles, triangles, parallel lines). Build skills required for future use of GSP®4.</td>
<td>MG3.01, MG3.02 CGE 4b, 5b</td>
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<td>4</td>
<td>What’s So Special? (Part 1) GSP®4 file: What’s So Special?</td>
<td>Build investigation skills by exploring geometric concepts, using GSP®4. Develop communication skills and geometric vocabulary.</td>
<td>MG3.01, MG3.02 CGE 2a, 5a</td>
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<td>5</td>
<td>What’s So Special? (Part 2)</td>
<td>Build investigation skills by exploring geometric concepts, using GSP®4. Develop communication skills and geometric vocabulary.</td>
<td>MG3.01, MG3.02 CGE 5a, 5b</td>
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<td>6</td>
<td>Interior and Exterior Angles of Triangles and Quadrilaterals GSP®4 files: Tutorial GSP, Sum of the Exterior Angles of a Triangle, Sum of the Interior Angles of a Polygon</td>
<td>Investigate the sum of the interior and exterior angles of triangles and quadrilaterals using GSP®4 and demonstration. Develop skills with GSP®4 in preparation for summative assessments.</td>
<td>MG3.01, LR1.03, LR4.04 CGE 2a, 3c</td>
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<td>7</td>
<td>Using the Properties: Connecting Algebra to Geometry</td>
<td>Practise solving problems using the geometry explored in previous lessons. Make connections to solving equations.</td>
<td>MG3.01, MG3.02, NA2.07, LR2.02, LR4.04 CGE 4b, 5a</td>
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<td>Freaky Folds</td>
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<td>Instructional Jazz</td>
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</table>
Math Learning Goals

- Describe the properties and relationships of the angles formed by parallel lines cut by a transversal.

Materials

- BLM 8.1.1, 8.1.2
- washable markers
- stickers
- butterfly fasteners
- protractors
- electronic presentation
- overheads or pictures of everyday examples of parallel lines
- scissors
- Acetate copies of BLM 8.1.1, 1/pair

Assessment Opportunities

- Have pictures, overheads, or an electronic presentation for followup to reinforce the possibilities.

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Minds On ... Small Groups → Brainstorm/Pass the Paper

In groups, students pass one piece of paper and write examples of everyday objects or situations that can be modelled using parallel lines. Examples could include fencing, ironing board top with the floor, any rectangular shape, railway tracks, etc. Record verbal responses from the groups, asking clarifying questions with respect to where the parallelism is. As a class, define parallel lines.

Action! Pairs → Guided Exploration

Provide each pair with an acetate copy of BLM 8.1.1, washable markers, a protractor, and some stickers. Cut the acetate in half, and poke a hole through each piece on the dot. Fasten the pieces of acetate with butterfly fasteners through the dots. Explain that the images on the acetates should be superimposed and rotated to create parallel lines.

Students explore, record, and justify any angle relationships they observe on BLM 8.1.2. Students can use any of the tools (e.g., stickers, markers, protractors) to determine angle relationships.

Selecting Tools and Computation Strategies/Observation/Checklist:
Circulate and observe the students as they choose their tools and strategies and as they investigate.

Consolidate Debrief Whole Class → Summarizing

Record and consolidate the relationships established by student groups. Establish that the transversal is an essential condition for angles with parallel lines. Ensure that students recognize all the possible variations of each relationship, i.e., alternate (Z), corresponding (F), co-interior (C). Students make summarizing notes.

Home Activity or Further Classroom Consolidation

Identify the types of angles found in the examples used in Minds On. Justify your answers.

Complete the practice questions.
8.1.1: Parallel Lines Exploration

Line 1

Line 2
## 8.1.2: Parallel Lines Exploration

<table>
<thead>
<tr>
<th>Explore and Reflect</th>
<th>Sketch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. How did you know when Line 1 and Line 2 were parallel?</td>
<td></td>
</tr>
</tbody>
</table>

### Angle Relationships

| 2. Find one pair of equal angles. Explain how you know they are equal. | |
| 3. Find another pair of equal angles. Explain how you know they are equal. | |
| 4. Find as many pairs of angles that are supplementary (add to 180°) as you can. Explain how you know. | |

### Summary (to be completed as a whole class)
**Math Learning Goals**
- Review angles, triangles, and parallel lines through exploration.
- Build skills required for future use of The Geometer’s Sketchpad® 4 (GSP).

<table>
<thead>
<tr>
<th>Materials</th>
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<tbody>
<tr>
<td>• BLM 8.2.1, 8.2.2, 8.2.3</td>
</tr>
<tr>
<td>• BLM 8.2.4 (Teacher)</td>
</tr>
<tr>
<td>• chart paper</td>
</tr>
<tr>
<td>• Computer with data projector</td>
</tr>
<tr>
<td>• Computer with Geometers Sketchpad 4, 1/ pair</td>
</tr>
</tbody>
</table>

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**Assessment Opportunities**

### Minds On ...

#### Small Groups ➔ Brainstorm

**Curriculum Expectations/Observation/Mental Note:** Circulate while students are working to assess prior learning diagnositcally.

For the Graffiti activity, post sheets of chart paper with the following topic titles: Triangles, Quadrilaterals, Polygons, Angles, Lines/Line Segments/Rays. Instruct groups to record prior learning on one of the concept sheets. Students might draw sketches, write definitions, state properties, etc. Groups rotate so that each group visits each topic sheet and adds information.

#### Whole Group ➔ Demonstration

Demonstrate how to navigate through the GSP® 4 file Plane Geometry using the How Do I...? feature and the GSP® 4 Notes Booklet (BLM 8.2.2). Students may refer to these notes throughout the unit.

### Action!

#### Pairs ➔ Guided Exploration

Students use the GSP® 4 file Plane Geometry and BLM 8.2.1 to review geometric concepts from Grades 7/8 and to build skills required for future activities with The Geometer’s Sketchpad® 4. Student roles: Driver (runs the mouse), Recorder (takes notes for the pair).

Students exchange roles part way through the activity. Provide feedback to student responses.

**Learning Skills/Work Habits/Observation/Rating Scale:** Circulate to assess how individual students stay on task and in role to complete the investigations.

### Consolidate Debrief

#### Whole Class ➔ Discussion

Discuss what has been accomplished to date.

Ask students to bring an optical illusion for Day 3.

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**Home Activity or Further Classroom Consolidation**

Make a title page for this unit.

Complete practice questions using the theorems.

Complete worksheet 8.2.3.
8.2.1: Plane Geometry Record Sheet

Use this page to record your observations and conclusions from the Plane Geometry GSP®4 file. Determine the unknown angle in the right column. Give reasons for your answer.

1) Supplementary Angles
Given: \( \triangle ABC \) is a straight angle.
Explore: Measure \( \angle ABD \) and \( \angle DBC \).
   [Measure \( \angle ABD \) and \( \angle DBC \)]
   Calculate the sum of these two angles.
   Drag point D.
Conclude: What is the sum of the two angles?

2) Complementary Angles
Given: \( \triangle AOC \) is a right angle.
Explore: Measure \( \angle AOB \) and \( \angle COB \).
   [How do I do it?]
   Calculate the sum of these two angles.
   Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.
   [Animate]
Conclude: What is the sum of the two angles?

3) Opposite Angle Theorem
Given: \( AB \) intersects \( CD \) at \( O \).
Explore: Measure \( \angle AOC \) and \( \angle DOB \).
   [How do I do it?]
   Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.
   [Animate]
Conclusion: What do you notice about \( \angle AOC \) and \( \angle DOB \)?
4) The Sum of the Interior Angles of a Triangle
Given: \( \triangle ABC \) with the interior angles identified.
Explore: Click on the action button below.

![Show the Sum of the Angles]  [Reset]

Drag each vertex of the triangle.
(Make sure the coloured sections marking interior angles are always inside the triangle.)

Conclude: What is the sum of the interior angles of a triangle?

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5) Isosceles Triangle Theorem
Given: Isosceles \( \triangle ABC \), where \( AB = AC \)
Explore: Measure the angles opposite the equal sides (\( \angle ABC \) and \( \angle ACB \))

![Show the Angles]  [Reset]

Drag each of the vertices.

Conclude: What do you notice about the angles opposite the equal sides?

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6) Equilateral Triangles
Given: \( \triangle ABC \), where \( AB = AC = BC \)
Explore: Measure each of the interior angles then drag point C.

Conclude: What do you notice about each of the interior angles?

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7) Exterior Angle Theorem
Given: \( \triangle ABC \) and exterior angle \( \angle ABD \).
Explore: Measure the non-adjacent angles \( \angle ACB \) & \( \angle CAB \)

![Measure]  [Reset]

Calculate the sum of these two angles.
Measure the exterior angle \( \angle ABC \)
Drag point A.

Conclude: What do you notice about the sum of the two non-adjacent angles, \( \angle ACB \) & \( \angle CAB \), inside the triangle and the exterior angle, \( \angle ABC \)?
8.2.1: Plane Geometry Record Sheet (continued)

8) Parallel Line Theorem - Corresponding Angles
Given: \( AB \) is parallel to \( CD \) with transversal \( PQ \).

Explore: Measure the corresponding angles \( \angle BWX \) and \( \angle DXQ \).
Drag points B and X.

Conclude: What do you notice about the corresponding angles \( \angle BWX \) and \( \angle DXQ \)?

More... Explain why this relationship is called the \( \text{F-pattern} \).
There are four "F-patterns" in this diagram. Can you find them?

9) Parallel Line Theorem - Alternate Angles
Given: \( AB \) is parallel to \( CD \) with transversal \( PQ \).

Explore: Measure the alternate angles \( \angle BWX \) and \( \angle DXC \).
Drag points B and X.

Conclude: What do you notice about the alternate angles \( \angle BWX \) and \( \angle DXC \)?

More... Explain why this relationship is called the \( \text{Z-pattern} \).
There are two "Z-patterns" in this diagram. Can you find them?

10) Parallel Line Theorem - Co-interior Angles
Given: \( AB \) is parallel to \( CD \) with transversal \( PQ \).

Explore: Measure the co-interior angles \( \angle BWX \) and \( \angle DXC \).
Calculate the sum of these two angles.
Drag points B and X.

Conclude: What do you notice about the sum of the co-interior angles \( \angle BWX \) and \( \angle DXC \)?

More... Explain why this relationship is called the \( \text{C-pattern} \).
There are two "C-patterns" in this diagram. Can you find them?
How do I …?

Geometer’s Sketchpad
Version 4

Instruction Booklet
Created by:
8.2.3: Theorems Practice Sheet
Define each principle and determine the unknown angles.

1. Supplementary Angles

\[ x^\circ = \]

\[ = \]

2. Complementary Angles

\[ m^\circ = \]

\[ = \]

3. Opposite Angle Theorem

\[ r^\circ = \]

\[ = \]

4. The Interior Angles of a Triangle

\[ q^\circ = \]

\[ = \]
5. Isosceles Triangle Theorem

\[ x = 75^\circ \]

6. Equilateral Triangles

\[ x = \]
8. Parallel Lines
   a) Corresponding Angles

   \[ \angle 68^\circ \]

   \[ \angle m \]

   \[ m^\circ = \]

   b) Alternate Angles

   \[ \angle 83^\circ \]

   \[ \angle w \]

   \[ w^\circ = \]

   c) Co-interior Angles

   \[ \angle 72^\circ \]

   \[ \angle x \]

   \[ x^\circ = \]
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<td>acute angle</td>
<td>obtuse triangle</td>
</tr>
<tr>
<td>acute triangle</td>
<td>octagon</td>
</tr>
<tr>
<td>adjacent angles</td>
<td>opposite angle</td>
</tr>
<tr>
<td>alternate angles</td>
<td>parallelogram</td>
</tr>
<tr>
<td>bisector</td>
<td>pentagon</td>
</tr>
<tr>
<td>circle</td>
<td>perpendicular</td>
</tr>
<tr>
<td>co-interior angles</td>
<td>perpendicular bisector</td>
</tr>
<tr>
<td>complementary angles</td>
<td>point</td>
</tr>
<tr>
<td>congruent</td>
<td>polygon</td>
</tr>
<tr>
<td>corresponding angles</td>
<td>quadrilateral</td>
</tr>
<tr>
<td>diagonal</td>
<td>radius</td>
</tr>
<tr>
<td>diameter</td>
<td>ray</td>
</tr>
<tr>
<td>equilateral triangle</td>
<td>rectangle</td>
</tr>
<tr>
<td>exterior angle</td>
<td>reflex angle</td>
</tr>
<tr>
<td>hexagon</td>
<td>rhombus</td>
</tr>
<tr>
<td>interior angle</td>
<td>right angle</td>
</tr>
<tr>
<td>isosceles triangle</td>
<td>right angle triangle</td>
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<tr>
<td>kite</td>
<td>scalene triangle</td>
</tr>
<tr>
<td>line</td>
<td>side</td>
</tr>
<tr>
<td>line segment</td>
<td>similar figures</td>
</tr>
<tr>
<td>midpoint</td>
<td>supplementary angles</td>
</tr>
<tr>
<td>obtuse angle</td>
<td>transversal</td>
</tr>
</tbody>
</table>
Introduction to the Geometer’s Sketchpad  

**Intro GSP.gsp**

### Introduction to Geometer’s Sketchpad.

Geometry’s Sketchpad allows us to construct, measure, and animate and move geometric objects easily.

The four tools for constructing objects are at the left of the screen:
1. **Point Tool** (dot)
2. **Compass Tool** (circle)
3. **Segment Tool** (line segment)
4. **Text Tool** (letter A)

Try experimenting with these tools by constructing various objects on the right of the screen.

When you are finished, click on the Link button to go to Page 2.

### Constructions.

**To construct the midpoint of CD**, click on the line segment then, from the **Construct menu**, choose **Midpoint**.

**To construct a line parallel to FG**, highlight point E and segment FG. Then, from the **Construct** menu, choose **Parallel Line**.

**To construct a line perpendicular to FG**, highlight point E and segment FG. Then, from the **Construct** menu, choose **Perpendicular Line**.

Click on the Link button to go to Page 4.

### Constructions 2

**To construct the angle bisector of ABC**, click on the points ABC (order is important) then, from the **Construct Menu**, choose **Angle Bisector**.

**To construct the interior of ABC**, click on the points DEF. Then, from the **Construct Menu**, choose **Triangle Interior**.

You can change the color of the interior by highlighting it then, from the **Display Menu**, choose **Color**.

Click on the Link button to go to Page 5.

### Measuring Objects (segments and angles).

A line segment is drawn at right. **To measure the length of the line segment**, click on it then go to the **Measure menu** above and choose **Length**.

Be sure that only the line segment is highlighted. **To de-highlight an object** click anywhere on the screen.

**∠CDE** is drawn at right. **To measure the angle**, click on points CDE (order is important) then go to the **Measure menu** and choose **Angle**.

To change the precision of the measurements, go to the **Edit tab** and choose Preferences.

Click on the Link button to go to Page 6.

### Measuring Objects (perimeter and area)

Construct the interior of ΔABC (click on points ABC then choose Triangle Interior from the **Construct Menu**).

Highlight the interior of the triangle then, from the **Measure Menu**, choose **Area**.

**Practice**: Determine the area and perimeter of parallelogram DEFG.

Click on the Link button to go to Page 7.
**Introduction to the Geometer’s Sketchpad (GSP®4 file)**

(continued)

**Measuring Angles (tabulating)**

△DEF is constructed at right. All of the angles have been measured as well as the sum of the angles.
Move point D to change the measures of the angles, then double-click on the tabulation table.

What do you notice? ______

Move point F then double-click on the tabulation table again.

On the next page you will learn how to make a tabulation table.

**Measuring (sum of angles)**

Determine the measures of the three angles of the triangle (see page 5).

When you have determined the three measures, click on the Measure menu and choose Calculate.

When you see the calculator, click on the first measure followed by the plus sign (+) then click on the second measure followed by the plus sign (+) then click on the third measure followed by OK.

Highlight all three measures as well as the sum.
Then, from the Graph menu, choose Tabulate.

Change the size of the triangle then click the tabulate box. What do you notice? Do this a few times.

Click on the Link button to go to Page 9

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**Practice**

1- Construct any triangle and determine the midpoints of the three sides.

2- Join the midpoints of the sides.

3- Determine the area of the small triangle in the middle formed by joining the midpoints.

4- Determine the area of the large triangle.

5- Use the Calculator to divide the area of the large triangle by the area of the smaller triangle.

6- What do you notice? ______

Click on the Link button to go to Page 10

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**Practice**

Construct line segments and triangles and practice constructing the following:

1- parallel lines
2- perpendicular lines
3- angle bisectors
4- lengths of line segments
5- area and perimeters
8.2.1 Plane Geometry

Explore each of the ten activities. (Start by clicking on number 1.)

Each activity has a button which will bring you back to this page.

If you need help on how to use Geometer's Sketchpad go to the “How do I...?” page by pressing the button:

1) Supplementary Angles

Given: \( \triangle ABC \) is a straight angle.

Explore:

Measure \( \angle ABD \) and \( \angle DBC \).

Calculate the sum of these two angles.

Drag point D.

Conclude: What is the sum of the two angles?

2) Complementary Angles

Given: \( \angle AOC \) is a right angle.

Explore:

Measure \( \angle AOB \) and \( \angle COB \).

Calculate the sum of these two angles.

Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.

Conclude: What is the sum of the two angles?

3) Opposite Angle Theorem

Given: \( AB \) intersects \( CD \) at \( O \).

Explore:

Measure \( \angle AOC \) and \( \angle DOB \).

Animate the diagram by pressing the button below. You can pause the animation by pressing the button again.

Conclusion: What do you notice about \( \angle AOC \) and \( \angle DOB \)?

4) The Sum of the Interior Angles of a Triangle

Given: \( \triangle ABC \) with the interior angles identified

Explore:

Click on the action button below.

Drag each vertex of the triangle.

(Make sure the colored sections marking interior angles are always inside the triangle.)

Conclusion: What is the sum of the interior angles of a triangle?

5) Isosceles Triangle Theorem

Given: Isosceles \( \triangle ABC \), where \( AB = AC \)

Explore:

Measure the angles opposite the equal sides \( \angle A \) and \( \angle C \).

Drag each of the vertices.

Conclusion: What do you notice about the angles opposite the equal sides?
6) Equilateral Triangles
Given: \( \triangle ABC \), where \( AB = AC = BC \)
Explore: Measure each of the interior angles then drag point C.
Conclude: What do you notice about each of the interior angles?

7) Exterior Angle Theorem
Given: \( \triangle ABC \) and exterior angle \( \angle ABD \).
Explore: Measure the non-adjacent angles \( \angle ACB \) and \( \angle CAB \).
Calculate the sum of these two angles.
Measure the exterior angle \( \angle ABD \).
Drag point A.
Conclude: What do you notice about the sum of the two non-adjacent angles, \( \angle ACB \) and \( \angle CAB \), inside the triangle and the exterior angle, \( \angle ABD \)?

8) Parallel Line Theorem - Corresponding Angles
Given: \( \overline{AB} \parallel \overline{CD} \) with transversal \( \overline{PQ} \).
Explore: Measure the corresponding angles \( \angle BWX \) and \( \angle DXP \).
Drag points B and X.
Conclude: What do you notice about the corresponding angles \( \angle BWX \) and \( \angle DXP \)?

9) Parallel Line Theorem - Alternate Angles
Given: \( \overline{AB} \parallel \overline{CD} \) with transversal \( \overline{PQ} \).
Explore: Measure the alternate angles \( \angle BWX \) and \( \angle WXC \).
Drag points B and X.
Conclude: What do you notice about the alternate angles \( \angle BWX \) and \( \angle WXC \)?

10) Parallel Line Theorem - Co-interior Angles
Given: \( \overline{AB} \parallel \overline{CD} \) with transversal \( \overline{PQ} \).
Explore: Measure the co-interior angles \( \angle BWX \) and \( \angle WX D \).
Calculate the sum of these two angles.
Drag points B and X.
Conclude: What do you notice about the sum of the co-interior angles \( \angle BWX \) and \( \angle WX D \)?

How do I ...?

...measure the length of a side?
To measure \( \overline{CB} \):
1. Deselect.
2. Select the two endpoints (C and B).
3. Under Measure choose Distance.

...measure an angle?
To measure \( \angle ABC \):
1. Deselect.
2. Select point A, then B, then C. Order is important!
3. Under Measure choose Angle.

...measure an area or perimeter?
...construct a polygon interior?
...measure an angle or side?
TIPS4RM: Grade 9 Applied – Unit 8: Plane Geometry

**Unit 8: Day 3: Plane Geometry – Introduction (Part 2)**

**Math Learning Goals**
- Explore geometrical concepts (angles, triangles, parallel lines).
- Build skills required for future use of The Geometer’s Sketchpad®4 (GSP).

**Materials**
- computer/pair
- BLM 8.2.1

**Assessment Opportunities**
- Acronyms are good time savers when students are asked to give reasons for their answers.
- Students should use full terminology in work handed in. They could write acronyms on their practice sheet (BLM 8.2.3).
- Examining the theorem with GSP®4 provides convincing evidence that the theorem always works.

**Minds On ...**

**Pairs → Discussion**
Students post their optical illusions and find concrete examples in the classroom of the geometric concepts that they explored yesterday. (For example, swing the door and ask what type of angles does the door create with the wall?)

**Whole Group → Discussion**
Review some of the concepts from the last lesson and ask students to share their concrete examples. Discuss some of the problems you observed with the students’ use of GSP®4 and BLM 8.2.2 GSP®4 Notes Booklet (see Day 2).
As a class make up acronyms for some of the theorems e.g., students may suggest OAT for the Opposite Angles Theorem. In later lessons students may use the acronyms when asked to give reasons for their answers.
Discuss the idea of a theorem and the need to confirm your hypothesis with measurement.

**Action!**

**Pairs → Guided Exploration**

**Learning Skills/Work Habits/Checklist/Rating Scale:** Circulate to check homework and see unit title page while students are working.

Students use the GSP®4 file Plane Geometry and BLM 8.2.1 to complete the review of geometric concepts from Grades 7/8 and to build skills required for future activities with Geometer’s Sketchpad. As well, they should work on completing BLM 8.2.2 GSP®4 Notes Booklet.

Student roles: Driver, Recorder. Students exchange roles part way through the activity. Provide feedback to student responses during circulation. (Students can check their answers by using the GSP®4 sketch.)

**Consolidate Debrief**

**Whole Class → Discussion**
Using the Graffiti sheets from Day 2, students determine if anything needs to be added/changed/deleted on the topic sheets, based on what they learned during the computer exploration.
Students provide more concrete illustrations of the concepts, using objects in the classroom, e.g., pages in a book, ceiling/floor tiles.
As a class, continue to make up acronyms for the theorems, e.g., students may suggest OAT for the Opposite Angles Theorem. Students record them on the Word Wall and/or in their notes. In later lessons students may use the acronyms when asked to give reasons for their answers.

**Home Activity or Further Classroom Consolidation**
Ask students to bring an optical illusion to the next day’s class if they haven’t already.
Continue working on the title page for this unit, if necessary.
Complete practise questions using the theorems.

**Reflection Concept Practice**
- Provide appropriate practice questions.
Unit 8: Day 4: What’s So Special? (Part 1)

Math Learning Goals
- Build investigation skills by exploring geometric concepts, using GSP®4.
- Develop communication skills and geometric vocabulary.

Materials
- BLM 8.4.1, 8.4.2
- data projector

Assessment Opportunities
- Special.gsp

Minds On ...

Whole Class ➔ Demonstration
Students post their optical illusions. Display the GSP®4 file What’s So Special? using a data projector. After each demonstration, invite students to share their observations.
- Discuss the two optical illusions on the first sketch. Students will see different things when they look at geometric diagrams.
- Click the first two demonstration buttons. Emphasize the importance of discussing and recording observations so students can learn from each other. Use the posted optical illusions to reinforce messages.
- Click the Demonstration 3 button. Discuss the hidden geometry in the constructed equilateral triangle. (Students may respond that some sketches are special because some things always remain true in any drag test.)
- Click the Demonstration 4 button and drag the points to demonstrate how to collect evidence that shows that two quantities are proportional. Show students how to measure the areas of both circles and how to determine the ratio of the areas by using the Calculate command under the Measure menu.
- Use the first activity (Special Triangles) to demonstrate how to look for something special in a sketch. Explore the first two triangles. Demonstrate how to use “tabulate” to collect evidence that supports an hypothesis, e.g., for the second triangle collect at least three table entries that show that angle LKM has a measure of 90 degrees.
- Review the process by using BLM 8.4.1 and 8.4.2, and suggest that students use them as guides for GSP®4 explorations.

Action!

Pairs ➔ Investigation
Students work on triangle ABC and KLM of Investigation 1 in the GSP®4 file What’s So Special? Circulate and assist groups who are experiencing difficulties.

Consolidate Debrief

Whole Class ➔ Discussion
Using the demonstration, discuss what is so special about the triangle. Discuss what must be measured to show the special feature.

Curriculum Expectations/Quiz/Marking Key: Create a quiz that is based on material from Days 1–3.

Home Activity or Further Classroom Consolidation
Find or create a logo that has a geometric design. Describe the design so someone else has a clear picture of it.
8.4.1: “What’s So Special?” Guide Sheet

Explore!
Drag each vertex in the figure.

As you drag vertices, look for some of the following:
• measurements that always seem to be equal to each other
• measurements that never seem to change
• measurements that might have a constant ratio (proportional)
• lines that always seem to be parallel or perpendicular
• line segments that always seem to be bisected
• figures that always seem to be congruent
• objects that don’t seem to be connected, yet they move together when something is dragged

Make an Hypothesis
Decide which measurements you need to test your hypothesis.
Drag each vertex again while you pay close attention to the way the object moves and to the way the measurements change.

Test Your Hypothesis
Collect and record evidence to test your hypothesis.

<table>
<thead>
<tr>
<th>What can you measure?</th>
<th>What can you calculate?</th>
</tr>
</thead>
<tbody>
<tr>
<td>angles</td>
<td>sums</td>
</tr>
<tr>
<td>lengths</td>
<td>ratios</td>
</tr>
<tr>
<td>areas</td>
<td>formulas</td>
</tr>
<tr>
<td>perimeters</td>
<td></td>
</tr>
<tr>
<td>slopes</td>
<td></td>
</tr>
</tbody>
</table>
8.4.2: Guiding Questions

**Examine the angles...**

- Are any of the angles equal?
- Do any of the angle measures always add to give the same total?
- Does the measure of any angle always stay the same?
- Are any of the angles cut in half (bisected)?

**Examine the line segments...**

- Are any of the lengths equal?
- Is any length proportional with any other length?
- Are any of the line segments cut in half (bisected)?

**Examine the lines...**

- Are any of the lines parallel?
- Are any of the lines perpendicular?
- Are any of the slopes of the lines equal?

**Examine areas and perimeters...**

- Are any of the areas equal?
- Are any of the perimeters equal?
- Are any of the shapes congruent?
- Are any of the shapes similar?
- Are any of the areas proportional?
- Are any of the perimeters proportional?
What’s So Special? (GSP®4 file)

**Special.gsp**

### Investigation 1: Special Triangles?

**Exploring:**
- Drag each vertex of every triangle.
- Which edge was drawn at a dot angle for an equilateral triangle?
- How does the area of the green circle compare to the area of the yellow circle?

**Hypothesizing:**
- Make a hypothesis about each triangle for why one of the triangle is special.

**Collecting Evidence:**
- What evidence can you find to support your hypothesis? How are you going to measure? Calculate? Tabulate?

**Making a Conclusion:**
- What is special about the intersecting line segments? Were your hypotheses correct?
- Justify your answer using the evidence you collected.

### Investigation 2: Parallel or Perpendicular?

**Exploring:**
- Explore all three figures.
- Do you notice anything special? Record your observations.

**Hypothesizing:**
- Make a hypothesis based on your exploration.

**Collecting Evidence:**
- What evidence can you find to support your hypothesis? What are you going to measure? Calculate? Tabulate?

**Making a Conclusion:**
- What is special about some of the line segments? Were your hypotheses correct? Justify your answer using the evidence you collected.

### Investigation 3: Bisected?

**Exploring:**
- Explore all three figures. Do you notice anything special? Record your observations.

**Hypothesizing:**
- Make a hypothesis.

**Collecting Evidence:**
- What evidence can you find to support your hypothesis? What are you going to measure? Calculate? Tabulate?

**Making a Conclusion:**
- What is special about the intersecting line segments? Were your hypotheses correct? Justify your answer using the evidence you collected.
What’s So Special? (GSP®4 file) (continued)

**Investigation 4: Proportional?**

<table>
<thead>
<tr>
<th>Exploring:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using math software. Do you notice anything special? Measure the area and the perimeter of triangles ADF and AEI.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hypothesizing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is there anything special in this diagram?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Collecting Evidence:</th>
</tr>
</thead>
<tbody>
<tr>
<td>What evidence did you use to support your hypothesis? What are you going to measure?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Making a Calculation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are you going to measure or calculate?</td>
</tr>
</tbody>
</table>

**How do I ...?**

1. [Measure an angle or segment]
2. [Measure another angle or segment]
3. [Measure two or more segments]
4. [Measure two or more angles]
5. [Measure two or more segments and angles]

...make a calculation?

To calculate the sum of the interior angles:
1. Deselect.
2. Under **Measure** choose **Calculate**.
3. Select the measurement m\(\angle\text{CAB}\) (in the upper right corner) then the + sign (on the calculator) then the measurement \(m\angle\text{ABC}\), then the + sign, then the measurement \(m\angle\text{BCA}\), then OK.

**Investigation 5: Special Quadrilaterals?**

<table>
<thead>
<tr>
<th>Exploring:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does each quadrilateral have any special features?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hypothesizing:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make a hypothesis about each quadrilateral. Are any of the quadrilaterals special?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Collecting Evidence:</th>
</tr>
</thead>
<tbody>
<tr>
<td>What evidence do you need to support your hypothesis? What are you going to measure or calculate?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Making a Conclusion:</th>
</tr>
</thead>
</table>
| What is your conclusion? 
Is it correct? If incorrect, what do you need to do to make it correct? |

**Definitions**

- **Acute** \(\triangle\): A triangle in which each of the three interior angles is acute.
- **Obtuse** \(\triangle\): A triangle containing one obtuse angle.
- **Right** \(\triangle\): A triangle containing a 90° angle.
- **Equilateral** \(\triangle\): A triangle with all sides equal.
- **Isosceles** \(\triangle\): A triangle with two equal sides.
- **Scalene** \(\triangle\): A triangle with no equal sides.
- **Parallel Lines**: Lines in the same plane that do not intersect. They are always the same distance apart.
- **Bisect**: Means to cut in half. Angles or lengths can be bisected.
- **Angle Bisector**: A line that divides an angle into two equal parts.
- **Right Bisector of a line segment**: A line that is perpendicular to line segment and divides the line segment into two equal parts.
- **Proportional**: A statement that two ratios are equal. (Two objects have exactly the same shape but are different sizes).
Math Learning Goals
- Build investigation skills by exploring geometric concepts, using GSP®4.
- Develop communication skills and geometric vocabulary.

Materials
- BLM 8.5.1
- Examples of logos with geometric designs
- Computer with Geometers Sketchpad 4, 1/pair

Assessment Opportunities
- Have available some examples of logos with geometric designs.

Minds On ...
**Small Groups → Brainstorm**
Students form small groups and challenge each other to sketch the logo that they describe from the Home Activity assignment. Students brainstorm how to create an effective description.

**Whole Class → Discussion**
Demonstrate the two triangles from Day 4.
Emphasize that there is very little that is always special about triangle ABC. (It will always have three sides and three angles).
Emphasize that triangle KLM always has a right angle at K.
Explore the other triangles in the sketch to determine what is special.

Action!
**Pairs → Investigation**
Students investigate geometric relationships using sketches in the GSP®4 file, What’s So Special? (See Day 4.)
Assign one investigation to each pair for reporting purposes during Consolidate/Debrief.
Pairs may have time to do more than one of the investigations.

Learning Skills (Teamwork)/Observation/Rating Scale: Circulate to assess how students contribute to the group to complete the activity.

Consolidate
**Debrief**
For each of the four activities choose pairs of students to present their work to the class.

**Whole Class → Discussion**
Debrief the presentations to help students understand how GSP®4 shows geometric principles.
Ask such questions as:
- How does The Geometer’s Sketchpad®4 help to collect evidence?
- How much evidence is needed to convince you that something is always true?

Home Activity or Further Classroom Consolidation
Complete worksheet 8.5.1 Learn the Lingo.
### 8.5.1: Learn the Lingo

1. Part a) shows an example of how to complete a word chart.
   Complete the remaining word charts.

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral Triangle</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
</tr>
<tr>
<td>An <em>equilateral triangle</em> is a triangle for which all sides have the same length.</td>
<td>A Yield sign</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td></td>
</tr>
<tr>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior Angle</td>
<td><img src="image" alt="Exterior Angle" /></td>
</tr>
<tr>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Visual Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior Angle</td>
<td></td>
</tr>
<tr>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### 8.5.1: Learn the Lingo (continued)

<table>
<thead>
<tr>
<th>e) Term: Parallel Lines</th>
<th>Visual Representation:</th>
<th>f) Term: Transversal</th>
<th>Visual Representation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>g) Term: Perpendicular Bisector</th>
<th>Visual Representation:</th>
<th>h) Term: Diagonal</th>
<th>Visual Representation:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
<td><strong>Definition:</strong></td>
<td><strong>Association:</strong></td>
</tr>
</tbody>
</table>
### 8.5.1: Learn the Lingo (continued)

2. Determine the unknown angle. Give reasons for your answer.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>b)</td>
<td>c)</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
<tr>
<td>$\triangle ABC = \triangle AC = \triangle BC$</td>
<td>$\angle DEG = ___$</td>
<td>$\angle TZ \parallel UY$ $\angle TXW = 75^\circ$ $\angle UVW = ___$</td>
</tr>
<tr>
<td>$\angle ACB = ___$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d)</th>
<th>e)</th>
<th>f)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Diagram" /></td>
<td><img src="image5.png" alt="Diagram" /></td>
<td><img src="image6.png" alt="Diagram" /></td>
</tr>
<tr>
<td>$\angle COD = 64^\circ$</td>
<td>$\angle NQR = 115^\circ$ $\angle MRQ = ___$</td>
<td>$\angle BOC = 43^\circ$ $\angle COE = ___$ $\angle EOD = ___$</td>
</tr>
<tr>
<td>$\angle FOE = ___$</td>
<td>$\angle COF = ___$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>g)</th>
<th>h)</th>
<th>i)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Diagram" /></td>
<td><img src="image8.png" alt="Diagram" /></td>
<td>Create your own question!</td>
</tr>
<tr>
<td>$\angle VRW = 42^\circ$ $\angle RT = RU$ $\angle SRT = 19^\circ$ $\angle RSZ = ___$</td>
<td>$\angle YWX = 118^\circ$ $\angle WXZ = ___$</td>
<td></td>
</tr>
</tbody>
</table>

---

TIPS4RM: Grade 9 Applied – Unit 8: Plane Geometry
Math Learning Goals

- Investigate the sum of the interior and exterior angles of triangles and quadrilaterals using The Geometer’s Sketchpad®4 and demonstration.
- Develop skills with The Geometer’s Sketchpad®4 in preparation for summative assessments.

Materials

- data projector
- BLM 8.6.1, 8.6.2
- Computer with Geometers Sketchpad 4, 1 / pair

Assessment Opportunities

Tutorial GSP.gsp
Angles Triangles.gsp
Angles Polygon.gsp

Minds On ...

Pairs ➔ Tutorial

In pairs, students review how to use The Geometer’s Sketchpad®4 tools for today’s activity using the GSP®4 file, Quick Tutorial and BLM 8.2.2 GSP®4 Notes Booklet.

Whole Class ➔ Demonstration

Demonstrate how to construct a triangle using rays.
Demonstrate how to measure the exterior angles. Have students make a prediction.
Demonstrate how to calculate the sum of the exterior angles and tabulate the sum. Drag each of the vertices of the triangle and tabulate the results.
Students make a prediction about the exterior angles of a quadrilateral.

Action!

Pairs ➔ Exploration

Students title a blank sketch “Exterior Angles of a Quadrilateral.” They write their hypothesis about the sum of the exterior angles of the quadrilateral. They construct a quadrilateral, measure the angles, calculate the sum of the angles and tabulate their results, then write a conclusion.
Students create another sketch titled “Exterior Angles of a Polygon.” They may choose a polygon with any number of sides and repeat the same process.
Students can use BLM 8.6.1 to help guide their activity.

Reasoning and/or Proving/Observation/Mental Note: Observe students’ facility with the inquiry process to determine if they need differentiated instruction to prepare for the assessment.

Whole Class ➔ Demonstration

Consolidate what students should have learned from their investigation using the GSP®4 files, Sum of the Exterior Angles of a Triangle and Sum of the Interior Angles of a Polygon.

Consolidate Debrief

Pairs ➔ Application

Students complete BLM 8.6.2.

Home Activity or Further Classroom Consolidation

Write a letter to Abe, who missed Math class, explaining how he can determine the sum of the interior and exterior angles in a decagon (10-sided polygon).

Sample response:

\[ S = 10 \times 180° - 360° = 1440° \]
8.6.1: Exterior and Interior Angles of a Polygon

Part A – Exterior Angles of a Triangle
Follow the instructions carefully to make the diagram shown.

- Plot point A and draw a ray.
- Drag point B along the ray AB and then draw a ray from B.
- Select points C and A (order is important) then use the construct menu to construct ray CA.
- Construct points D, E, and F on rays AB, BC, and CA respectively.
- Drag test your construction.

Hypothesis
I think that the sum of the exterior angles of a triangle is ________ because

Conclusion
Form a conclusion based on your evidence. Refer to your hypothesis.

Part B – Exterior Angles of a Quadrilateral
Use the steps from Part A to construct and explore the sum of the exterior angles of a quadrilateral.

Part C – Exterior Angles of Any Polygon
Compare the conclusions you reached in Part A and Part B. Test your hypothesis about the sum of the exterior angles of a polygon by constructing and measuring the angles of another polygon (pentagon, hexagon, etc.). Record your information and write your final conclusion about the sum of the exterior angles of any polygon.
8.6.2: Interior Angle Sums

1. Complete the chart.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Number of sides</th>
<th>Sum of interior angles</th>
<th>Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Triangle" /></td>
<td>3</td>
<td>180°</td>
<td>The sum of the angles in any triangle is 180°.</td>
</tr>
<tr>
<td><img src="image" alt="Quadrilateral" /></td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Pentagon" /></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Hexagon" /></td>
<td>n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. a) Determine the sum of the interior angles in a polygon with 15 sides. Show your work.

b) Determine the number of sides in a polygon if the sum of the interior angles is 5400°. Show your work.
3. Derek is building a deck for his summer job in the shape of a regular octagon.
   a) Define: regular octagon
   b) Determine the measure of the interior angles of the deck. Show your work.

4. A Canadian $1 coin, known as a loonie, is a regular polygon with 11 sides, called an undecagon.
   a) Define a regular polygon with 11 sides.
   b) Determine the sum of the interior angles of the loonie.
   c) What is the size of one of the interior angles?
### Quick Tutorial - Geometer's Sketchpad

Geometer's Sketchpad allows us to construct, measure, animate, and move geometric objects easily.

#### Name of Tools

<table>
<thead>
<tr>
<th>Tool</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection Arrow</td>
<td>Allows you to select and drag objects.</td>
</tr>
<tr>
<td>Point Tool</td>
<td>Used to construct points. It looks like a dot.</td>
</tr>
<tr>
<td>Compass Tool</td>
<td>Used to construct circles. If you hold the selection arrow over it, it will change to allow you to select a segment, ray or line tool.</td>
</tr>
<tr>
<td>Straightedge Tool</td>
<td>Used to construct straight objects. It looks like a circle with cross hairs.</td>
</tr>
<tr>
<td>Text Tool</td>
<td>Used to create and edit text. It can be used to label objects. It looks like the letter A.</td>
</tr>
</tbody>
</table>

#### 1) Labeling Objects.

A point and a line segment are drawn at right.

To label them, click on the Text Tool (letter A) then click on the point or the line segment.

To hide a label, click on the object, then click the Display tab then choose Hide Label.

Another way to show or hide the label is to highlight the object, then from the Display menu, choose Show / Hide Label.

By highlighting an object you can also hide the object from the Display tab. This does not delete it, but hides it from view.

#### 2) Measuring Segments and Angles

**Line segment AB is drawn at right.**

One way to measure the length of the line segment, click on the line segment then go to the Measure menu above and choose Length.

Another way to measure the length of the line is to measure the distance between points. Select Point A then Point B. Go to the Measure menu above and choose Distance.

**Which way do you like better?**

\[ \angle CDE \] is drawn at right.

To measure the angle, click on points CDE (order is important) then go to the Measure menu and choose Angle.

**HINT:** To change the precision of the measurements, go to the Edit tab and choose Preferences.

#### 3) Constructing Midpoints and Angle Bisectors

To construct the angle bisector of ABC, click on the points ABC (order is important) then, from the Construct Menu, choose Angle Bisector.

**Did it really cut the angle in half?**

Measure it!

To construct the midpoint of DE, click on the line segment then, from the Construct menu, choose Midpoint.

Label the Midpoint M.

**Is the distance between D and M the same as M and E?**

Construct a ray through point P. Hold down the straightedge tool button and choose the middle option - ray tool. Click in the blank space then on point P.

#### 4) Constructing Interiors

To construct the interior of \( \triangle DEF \), click on the points D, E, and F. Then, from the Construct Menu, choose Triangle Interior.

You can change the color of the interior by highlighting it then, from the Display Menu, choose Color.

**TRY IT**

Make a quadrilateral and shade the interior blue.

#### 5) Measuring Perimeter and Area of Objects

**Construct the interior of \( \triangle ABC \):**

Highlight the interior of the triangle then, from the Measure Menu, choose Area.

**Highlight the interior of the triangle then, from the Measure Menu, choose Perimeter**

**TRY IT**

Determine the area and perimeter of parallelogram DEFG.
6) Measuring the sum of angles

Determine the measures of the three angles of the triangle ∠ABC, ∠ACB and ∠CBA.

When you have determined the three measures, click on the Measure menu and choose Calculate.

When you see the calculator, click on the first measure (m∠ABC) followed by the plus sign (+) then click on the second measure followed by the plus sign (+) then click on the third measure followed by OK.

Highlight all three measures as well as the sum. Then, from the Graph menu, choose Tabulate.

Change the size of the triangle then double-click the tabulate box. What do you notice? Do this a few times.

7) Constructing Parallel and Perpendicular Lines

To construct a line parallel to BC through A. Highlight point A and segment BC.

Then, from the Construct menu, choose Parallel Line.

Test to see if they are parallel by dragging point C.

To construct a line perpendicular to FG through E. Highlight point E and segment FG then, from the Construct menu, choose Perpendicular Line.

Test to see if they are perpendicular.

8) Practice

1- Construct any triangle. Label it ABC.

2- Determine the midpoints of the three sides. Label them P, Q and R.

3- Join the midpoints of the sides.

4- Determine the area of the small triangle in the middle formed by joining the midpoints.

5- Determine the area of the large triangle.

6- Use the Calculator to divide the area of the large triangle by the area of the smaller triangle.

7- Tabulate your measurements.

8- What do you notice? ______

9) Picture Practice

Make a picture that includes construction of the following:

1- parallel lines

2- perpendicular lines

3- angle bisectors

4- lengths of the line segments

5- area and perimeters
Sum of the Exterior Angles of a Triangle (GSP®4 file)

Angles Triangle.gsp
Sum of the Exterior Angles of a Triangle (GSP®4 file) (continued)

**Sum of the Interior Angles of a Polygon**

**Tearing Corners**
- Triangle
- Quadrilateral

**Diagonal Divisions**
- Summary
- Quadrilateral
- Pentagon
- Hexagon vs. Regular Hexagon

**Tiny Triangles**
- Quadrilateral

These two investigations simulate tearing off the corners of the polygon and joining them.

These investigations use the fact that any polygon can be divided into triangles by drawing diagonals. It can also be used to explore the pattern of the sum of the angles and number of sides.

This investigation uses the fact that any polygon can be divided into triangles by connecting a center point to two of the vertices. It also uses the idea of subtracting one complete rotation.

**Quadrilateral - Sum of the Interior Angles**

Hypothesis: What is the sum of the interior angles of a quadrilateral?

If you move the vertices of the quadrilateral does the sum remain the same?

**Interior Angles of Polygons - Looking for Patterns**

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Number of Sides</th>
<th>Sum of Interior Angles</th>
<th>Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td>3</td>
<td>180°</td>
<td>FACT: The sum of the angles in any triangle is 180°</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
<td>4</td>
<td><img src="image3" alt="Show me" /></td>
<td>The interior of a quadrilateral has two triangles, and sum of the angles in each triangle is 180°, so 2 x 180° = 360°</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
<td>5</td>
<td><img src="image5" alt="Show me" /></td>
<td></td>
</tr>
</tbody>
</table>

TIPS4RM: Grade 9 Applied – Unit 8: Plane Geometry
Sum of the Interior Angles of a Polygon (GSP®4 file)

Angles Polygon.gsp
## Math Learning Goals
- Practise solving problems using the geometry explored in previous lessons.
- Make connections to solving equations.

### Assessment Opportunities

**Minds On ...**

**Pairs \rightarrow Equation Game**
Use BLM 8.7.1 Equation Strips. Cut each solution into strips and place in an envelope.
Pairs of students solve the equations by correctly ordering the strips.

**Whole Class \rightarrow Discussion**
Review the concept that solving an equation means finding the value of the variable that makes the statement true, i.e., so that the expression on the left side of the equal sign balances the expression on the right side of the equal sign.

**Action!**

**Small Groups \rightarrow Discussion**
Students scan BLM 8.7.2 Connecting Algebra to Geometry and determine three clarifying questions to help them understand the task.
As a class, discuss the clarifying questions.

**Pairs \rightarrow Practice**
Students complete BLM 8.7.2.

**Learning Skills (Teamwork)/Observation/Mental Note:** Continuously check as pairs work. Check one pair’s work (this first pair becomes the expert pair on a question) and refer other pairs to the first pair for a correct solution.

**Consolidate Debrief**

**Whole Class \rightarrow Concept Map**
Use Smart Ideas software to create a concept map with various titles, for example, Angle Theorems.
Brainstorm a list of theorems.
Review the list and categorize the theorems, e.g. triangle angle theorems.
Discuss the categories and display the concept map.
Home Activity or Further Classroom Consolidation
Create five quiz questions, write solutions, and check them for accuracy.

These quiz questions could be exchanged with a partner for review or form part of a review package for the class.
### 8.7.1: Equation Strips

1. 

\[ 2a + a = 90 \]
\[ 3a = 90 \]
\[ \frac{3a}{3} = \frac{90}{3} \]
\[ a = 30 \]

2. 

\[ 2b = 40 + 70 \]
\[ 2b = 110 \]
\[ \frac{2b}{2} = \frac{110}{2} \]
\[ b = 55 \]
3. \[ 2b + 3x = 100 \]

\[ 5x = 100 \]

\[ \frac{5x}{5} = \frac{100}{5} \]

\[ x = 20 \]

4. \[ 4c + 50 = 170 \]

\[ 4c + 50 - 50 = 170 - 50 \]

\[ 4c = 120 \]

\[ \frac{4c}{4} = \frac{120}{4} \]

\[ c = 30 \]
5. \[
5g - 32 = 108
\]
\[
5g - 32 + 32 = 108 + 32
\]
\[
5g = 140
\]
\[
\frac{5g}{5} = \frac{140}{5}
\]
\[
g = 28
\]
8.7.2: Connecting Algebra to Geometry

1. a) The sum of the interior angles in a triangle is:

b) An equation that models the sum of the interior angles in this triangle is:

c) Solve the equation to determine the value of $x$.

d) Use the value of $x$ to calculate the size of:

   $\angle W$: $\angle Y$: $\angle Z$: 

2. a) The sum of the angles in a right angle is:

b) Write 2 equations to model the sums of the 2 sets of angles that add to 90°:

   (i) 

   (ii) 

c) Solve these equations to determine the values.

   (i) solve for $x^\circ$  
   (ii) solve for $y^\circ$

d) Use the values of $x$ and $y$ to calculate the size of:

   $\angle CBP$: $\angle ABQ$: 

3. Write an equation and solve for the unknown. State the theorem used to make the equation.

a) \[ 60^\circ = 2a^\circ \]

b) \[ 130^\circ = b^\circ + 20^\circ \]

c) \[ 120^\circ = 2c^\circ + 40^\circ \]

d) \[ 100^\circ = 3d^\circ + 10^\circ \]

e) \[ 75^\circ = e^\circ - 15^\circ \]

f) \[ 78^\circ = f^\circ - 32^\circ \]

g) \[ 3g^\circ + 44^\circ = 5g^\circ - 12^\circ \]

h) \[ 4h^\circ - 56^\circ = 2h^\circ + 44^\circ \]
8.7.2: Connecting Algebra to Geometry (continued)

i) \[
\begin{align*}
\angle f &= 2r^\circ \\
\angle 75^\circ &= k^\circ
\end{align*}
\]

j) \[
\begin{align*}
\angle f &= 75^\circ \\
\angle 2l^\circ + 20^\circ &= 2l^\circ
\end{align*}
\]

k) \[
\begin{align*}
\angle 2k^\circ &= k^\circ \\
\angle 30^\circ &= 80^\circ
\end{align*}
\]

l) \[
\begin{align*}
\angle f &= 2l^\circ + 20^\circ \\
\angle 2n^\circ &= 120^\circ
\end{align*}
\]

m) \[
\begin{align*}
\angle 30^\circ &= 2k^\circ \\
\angle 80^\circ &= 2l^\circ
\end{align*}
\]

n) \[
\begin{align*}
\angle 2n^\circ &= 120^\circ \\
\angle 3n^\circ &= 2n^\circ
\end{align*}
\]
**Unit 8: Day 8: Freaky Folds**

**Math Learning Goals**
- Use paper folding to illustrate geometric properties.

**Materials**
- origami paper
- scissors
- BLM 8.8.1
  (Teacher)

**Assessment Opportunities**

**Minds On ... Whole Class ➔ Demonstration**
Introduce origami to the class as the art of paper folding. Give each student an origami or square piece of paper. Demonstrate how to fold the paper to form a paper drinking cup (use an overhead of BLM 8.8.1). Students perform the same folds.

Students then unfold their paper cup. Discuss the kinds of geometry they see in the folds.

For example:
- triangle EFB, triangle BFG, triangle DHI, triangle DIJ are congruent and isosceles
- triangle HCG and triangle JAE are right isosceles triangles
- EF is parallel to IH
- JI is parallel to FG
- JE || DB || GH

This website is designed for young children but provides a good selection of appropriate origami patterns.
For example:
- fish
- iris (a bit tricky)
- cat (easy!)
- dragon fortune teller

**Action! Pairs ➔ Investigation**
Students make two of the same item – leave one folded and the second one unfolded. They tape their unfolded one onto another piece of paper and label all the geometry in preparation for their presentation. Ask students to make measurements to provide evidence that they have correctly identified the geometry.

**Consolidate Debrief**
**Pairs ➔ Presentation**
Students present to the class their origami figure and explain the geometry they found in the unfolded shape.

**Communicating/Presentation/Rubric:** Assess students on their use of appropriate terminology and the clarity of their justifications as they make their presentations.

**Differentiated Reflection Reasoning and Proving**

**Home Activity or Further Classroom Consolidation**
Look in newspapers, magazines, or on the Internet for a logo that has some geometric properties. Paste a picture of your logo on a piece of paper and identify and describe the geometry in the logo.

Submit your work for assessment.
1. Cut out the pattern and fold the square paper in half (lines facing out).

2. Fold right on the dotted line towards you, fold left outwards from you.

3. Tuck the top triangle inside the paper, one on each side paper.