GAP CLOSING

Number Sense

Junior / Intermediate Facilitator’s Guide
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INTRODUCTION

The *Gap Closing* package is designed to help Grade 6 teachers provide precisely targeted remediation for students who they identify as being significantly behind in mathematics. The goal is to close gaps in Number Sense so that the students can be successful in learning grade-appropriate mathematics. However, the materials may also be useful to Grades 4 and 5 teachers in providing alternative approaches they had not considered to some of the content or to Intermediate teachers with students who are working significantly below grade level.

For each topic, there is a diagnostic and a set of intervention materials that teachers can use to help students be more successful in their learning.

The diagnostics are designed to uncover the typical problems students have with a specific topic. Each diagnostic should be used as instructional decisions are being made for the struggling student.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with students.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:

- Questions to ask before using the approach
- Using the approach
- Consolidating and reflecting on the approach

The style and content of the consolidating questions may be useful to teachers in their regular course preparation as well.

**Getting Started**

- Identify students who are underperforming in Number Sense or in mathematics in general – students who are not where you think they should be.
- Select the Number Sense topic to focus on.
- Administer the diagnostic, allowing 20-40 minutes.
  - For students with accommodations, e.g., a scribe, supports should remain in place; they may require a longer time to complete the diagnostic.
- Refer to the Facilitator’s Guide as you administer and mark the diagnostic.
- Use the chart, Evaluating Diagnostic Results to identify which of the intervention materials aligns with the student’s learning needs.
  - This is the stage where you personalize remediation based on the precisely identified area of concern. It may be appropriate to use more than one of the intervention materials with the student.
- Select either the Open Question or the Think Sheet approach:
  - The **Open Question** provides the student with a variety of possible responses and approaches.
  - The **Think Sheet** offers a more guided and structured approach.
  - Teachers should consider student preferences and readiness when deciding which of these two approaches to use.
- Use the suggestions for the three-part structure included for each topic to provide remediation. Allow approximately 45 minutes to complete each topic, recognizing that time may vary for different students.
Considerations

Materials required for each intervention approach are indicated in the Facilitator’s Guide. There are templates included in the student book should the concrete materials not be available.

Sample answers to questions are included to give teachers language that they might use in further discussions with students. These samples sometimes may be more sophisticated than the responses that students would give. Teachers should probe further with individual students in their questioning, as needed.

Some students might need more practice with the same topic than other students. Teachers should use their professional judgment when working with individual students to decide how much practice they need. For additional practice, teachers can create similar questions to those provided using alternate numbers. There will be e-practice modules soon.

Access the Gap Closing materials at www.edugains.ca
Module 1
Representing Fractions

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REPRESENTING FRACTIONS

Relevant Learning Expectation for Grade 6

Represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire® rods, drawings, number lines, calculators) and using standard fractional notation.

Possible reasons why a student might struggle in representing fractions

- not realizing that the denominator represents the whole and the numerator a part (e.g., sees $\frac{2}{3}$ as 2 of one thing and then 3 of another)
- understanding fractions only as part of a whole, but not as part of a set or as part of a measure (like length, volume, mass)
- not understanding that the pieces representing the parts of a whole need to be equal in area, but not congruent
- not understanding that the “coloured” sections of the pieces representing the parts of a whole do not need to touch
- not understanding that a diagram showing a fraction also shows its complement (the rest of the whole) (e.g., if you see the $\frac{4}{5}$ coloured, you see the $\frac{1}{5}$ not coloured)
- not understanding that when fractions describe parts of sets, the items in the set need not be identical
- not realizing that the whole must be clearly defined in order to name a fraction of it
- an inability to create fractions even though they may be able to identify them
- an inability to create fractions when sections are not pre-divided
- not recognizing that the same part of a whole can be shown in different ways
- discomfort with improper fractions and mixed numbers

Additional consideration

Students should realize that we use the ordinal form of a number when reading the denominator, but the cardinal form of a number when reading the numerator. For example, students see $\frac{3}{10}$ and think: three (cardinal) tenths (ordinal). The exception is $\frac{1}{2}$ where we say “one half” rather than “one second.”
Administer the diagnostic

If students need help in understanding the directions for the diagnostic, clarify an item's intent. In particular, for Question 2, make sure students know that they are not necessarily looking for \(\frac{2}{3}\) being a particular colour as they did in Question 1—any form of \(\frac{2}{3}\) is acceptable. For Question 3, students can use the circles or the grid or counters.

Using diagnostic results to personalize interventions

Intervention materials are provided for dealing with part of set, part of whole, and part of measure meanings of fractions, and for half, proper, and improper fractions. You may use all or only part of these materials, based on student performance with the diagnostic.

<table>
<thead>
<tr>
<th>Evaluating Diagnostic Results</th>
<th>Suggested Intervention Materials</th>
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<tbody>
<tr>
<td>If students struggle with much of Question 1</td>
<td>Use the One Half materials</td>
</tr>
<tr>
<td>If students struggle with a number of: Questions 2a, 2b, 2c, 2d, 3a, 3b, 3c, 4a, 4b</td>
<td>Use Proper Fractions as Parts of a Whole materials</td>
</tr>
<tr>
<td>If students struggle with a number of Questions 2h, 2i, 2j, 3a, 3b, 3c (if represented by lengths, masses or volumes)</td>
<td>Use Fractions as Parts of a Measure materials</td>
</tr>
<tr>
<td>If students struggle with a number of Questions 2e, 2f, 2g, 3a, 3b, 3c (if represented by sets of objects), 4c, 4d</td>
<td>Use Proper Fractions as Parts of a Set materials</td>
</tr>
<tr>
<td>If students struggle with a number of Questions 3d, 3e, 3f, 4e, 4f</td>
<td>Use Improper Fractions as Parts of a Whole materials</td>
</tr>
<tr>
<td>If students struggle with several parts of Question 5</td>
<td>Use Improper Fractions as Parts of a Set materials</td>
</tr>
</tbody>
</table>

Start with the earliest pathway needed.

Recognize that success with some early interventions may or may not mitigate the need for further interventions.

Diagnostic items can be reassigned to make that decision.
### Solutions

1. **In each box, circle the pictures that show \( \frac{1}{2} \).**

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<table>
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<td>j)</td>
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2. **In each box, circle the pictures that show \( \frac{2}{3} \) (either shaded or white).**

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3. **a)**

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

- ![Fraction model](image)

**Note that students may use only part of whole, part of set, or part of measure models, or they might mix them up.**
4. a) \(\frac{1}{2}\) b) \(\frac{1}{3}\) c) \(\frac{1}{4}\) d) \(\frac{2}{3}\) e) \(\frac{2}{1}\) f) \(\frac{3}{2}\)

5. a) 

b) 

c) 

d)
USING INTERVENTION MATERIALS

The purpose of the suggested student work is to help them build a foundation with fractions so that they can use them with ratios, percents, and decimals and interpret them as division as they move into Grade 7.

Each set of intervention materials includes a single-task Open Question approach and a multiple-question Think Sheet approach. These approaches both address the same learning goals, and represent different ways of engaging and interacting with learners. You could assign just one of these approaches, or sequence the Open Question approach before, or after the Think Sheet approach.

Suggestions are provided for how best to facilitate learning before, during, and after using your choice of approaches. This three-part structure consists of:
• Questions to ask before using the approach
• Using the approach
• Consolidating and reflecting on the approach
One Half

Learning Goal

• recognizing and representing half of a whole in many ways.

Open Question

Questions to Ask Before Using the Open Question

Tell students that \( \frac{1}{2} \) is a fraction we use to tell how much one person gets if 2 people share a whole amount or a whole set of objects fairly.

◊ We read this as one half.

Ask students to think of situations where they have shared things with one other person. Ask how they check to see if the sharing is fair.

Make sure they understand what is meant by sharing fairly, perhaps by showing counter examples.

Using the Open Question

Provide suggested materials.

Ask students to show \( \frac{1}{2} \) as many ways as they can.

By viewing or listening to student responses, note if they recognize fair shares in the context of single wholes, small groups, or measurements.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe the models of fractions the students created.

Ask questions such as:

◊ I noticed this model used 4 red squares. Why did it also use 4 blue ones? (It has to be fair shares so it has to be the same amount of blue as red.)

◊ Could it show \( \frac{1}{2} \) if you used 9 squares? Explain. (e.g., No, since there is no way to have the same number of squares of each colour then.)

◊ How did you show \( \frac{1}{2} \) with the string? (It has to be two equal pieces.) Did you have to cut? (No, I could fold it end to end.)

◊ What other colour pattern blocks could you have used? (e.g., I used red and yellow but I could have used blue and green.)

◊ How could you check that you really filled \( \frac{1}{2} \) the cup? (I have to fill one whole cup and move half to another cup and see if the heights are the same.)

Solutions

e.g., □□

\[ 
\begin{array}{c}
\text{e.g., □□} \\
\text{\includegraphics[width=2cm]{example_image1}} \\
\text{\includegraphics[width=2cm]{example_image2}} \\
\text{\includegraphics[width=2cm]{example_image3}} \\
\text{\includegraphics[width=2cm]{example_image4}} \\
\end{array}
\]
**Questions to Ask Before Assigning the Think Sheet**

Tell students that \( \frac{1}{2} \) is a fraction we use to tell how much one person gets if 2 people share a whole amount or a whole set of objects fairly.

◊ *We read this as one half.*

Ask students to think of situations where they have shared things with one other person before. Ask how they check to see if the sharing is fair.

Make sure they understand what is meant by sharing fairly, perhaps by showing counter-examples.

**Using the Think Sheet**

Read through the introductory box with the students.

Make sure:

- they understand that in each picture there are 2 parts and only one is blue.
- they realize that \( \frac{1}{2} \) can apply to the star and moon, even though they are different.
- Since there is still one in each group each shape is half of the set of shapes. Assign the tasks.

By viewing or listening to student responses, note if they:

- can recognize \( \frac{1}{2} \) as either part of a set or part of a whole or part of a measure
- can recognize the role of the numerator and denominator in \( \frac{1}{2} \)
- can cut a shape into halves in more than one way

Depending on student responses, use your professional judgement to guide further follow-up.

**Consolidating and Reflecting: Questions to Ask After Using the Think Sheet**

◊ *Why does the rectangle not show \( \frac{1}{2} \)?* (The pieces are not equal in size.)

◊ *What would Tanya have to do with the ribbon to make it show one half?* (Fold farther down so that the two ends meet.)

◊ *Is there always more than one way to divide a shape in 2?* (yes)

◊ *How about this one?* (Show the learner a pentagon and then two lines of symmetry.)

**Solutions**

1.  

2. no, since the front part is not as long as the back part

3. a) that there are 2 equal parts  
   b) that you only want 1 of the 2 equal parts

4. e.g.,  
   a) 
   b)
Proper Fractions as Parts of a Whole

Learning Goal

- representing fractions 1 or less using an area as a whole.

Open Question

Questions to Ask Before Using the Open Question

◊ What does \( \frac{1}{2} \) mean? (The whole is divided into 2 equal parts, and you only get one of them.)
◊ What does the numerator of 1 tell? What does the denominator of 2 tell? (The 1 tells how many parts we’re using and the 2 tells how many equal parts the whole is divided into.)
◊ What do you think \( \frac{2}{3} \) means? Why? (There are 3 equal parts and you only take or colour 2 of them.)
◊ What fraction might you have if the whole is divided into 5 parts? (e.g., \( \frac{3}{5} \))
◊ What fraction might you have if the whole is divided into a lot of parts and you only want 2 of them? (e.g., \( \frac{2}{8} \))

Using the Open Question

Provide grid paper for students to use. Alternatively, provide two colours of square tiles.

By viewing or listening to student responses, note if they:
- understand the constraints of equal sections when using fraction notation and the roles of the numerator and denominator
- realize that all of the given fractions are one section short of a whole or whether they notice they are all more than half
- notice other things about the fractions

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe the pictures of fractions the students created.
Consider how they decided the fractions were alike or different.
Possibilities relate to the fact that only one section is not coloured, that each fraction is more than half, or that each involves an even number and an odd number.

If necessary, ask questions such as:
◊ How did you know you needed to shade more sections for \( \frac{4}{5} \) than for \( \frac{3}{5} \)? (You need 4 sections, not just 3.)
◊ Could the whole for the \( \frac{4}{5} \) actually be smaller than the whole for \( \frac{3}{2} \)? (e.g., yes, if you make a big rectangle to divide into fourths but a smaller one to divide into fifths)
◊ What is another fraction that would be different from all of these? (e.g., \( \frac{1}{3} \) since there would be 2 missing sections and not just one.)

Materials

- grid paper
- coloured pencils
- two colours of square tiles
**Solutions**

\[
\begin{array}{ll}
\frac{2}{3} & \\
\frac{5}{6} & \\
\frac{3}{4} & \\
\frac{4}{5} & \\
\frac{9}{10} & \\
\end{array}
\]

Fractions are alike: e.g., only 1 section is not shaded, all the parts are equal in each picture.

Fractions are different: e.g., each picture has a different number of parts shaded and a different number of total parts.

Relationship to the numerator and denominator: It is because the numerator is 1 less than the denominator that there is always one unshaded part.
Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ Why does this show \( \frac{6}{6} \)? (If the whole is divided into 6 pieces, then \( \frac{6}{6} \) is all of the whole.)
◊ What do you think \( \frac{2}{6} \) might look like? (2 triangles of the 6)

Show a pattern block hexagon (yellow), a rhombus (blue), and a triangle (green). Ask what fraction of the hexagon the triangle is. (\( \frac{2}{6} \))
Then ask what fraction of the hexagon they would use if they made a shape with 2 triangles. (\( \frac{2}{6} \))
Now ask what fraction of the rhombus a triangle is. (\( \frac{1}{2} \))
Ask what fraction of the rhombus they would use if they made a shape with 2 triangles. (\( \frac{2}{2} \))
Discuss how you need to know the whole to know what fraction to use to represent a model.

Using the Think Sheet

Read through the introductory box with the students.
Make sure they:
• understand the terms numerator and denominator and what they represent.
• notice that the coloured parts do not need to be adjacent to be counted.
• notice that the parts are equal in area
• notice that every situation shows two fractions—one for the coloured part and one for the uncoloured part

Assign the tasks.

By viewing or listening to student responses, note if they:
• can model proper fractions with pre-divided wholes
• recognize the second fraction that is always modelled when the first fraction is modelled
• recognize proper fractions when the whole is identified
• realize that the pieces must be equal in area, but not necessarily congruent
• realize that different wholes affect the look of a fraction, but not its fraction name
• understand the role of the numerator and denominator of a fraction

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why do you want a whole with 4 sections and not 3 sections to model \( \frac{3}{4} \)?
(It’s the denominator that tells how many parts there are altogether in the whole.)
◊ Does this show sixths? Why or why not?
(no, the pieces are not equal in size)

◊ Why was Question 1c a little tricky? (You have to decide if the pieces are all equal in size and it’s not that easy to tell.)
◊ Why is it easier to use grid paper than circles to show fractions like \( \frac{3}{10} \) or \( \frac{4}{5} \)?
(It’s hard to make 10 equal pieces in a circle, but you just make a rectangle with 10 squares on the grid and that’s easy.)
◊ Why do you need to know about the numerator and the denominator to model a fraction? (For any whole, there are lots of different fractions with the same denominator and different fractions with the same numerator, so I need both to be sure which fraction to show. Or, I need the denominator to tell how many equal parts I want to show in my model.)
Solutions

1. $\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{2}{10}, \frac{5}{6}, \frac{2}{9}$

2. a) \[
\begin{array}{c}
\text{\framebox{1}} \\
\text{\framebox{3}} \\
\end{array}
\]
   \[
\frac{1}{3}
\]

   b) \[
\begin{array}{c}
\text{\framebox{3}} \\
\text{\framebox{2}} \\
\text{\framebox{1}} \\
\end{array}
\]
   \[
\frac{3}{8}
\]

   c) \[
\begin{array}{c}
\text{\framebox{2}} \\
\text{\framebox{5}} \\
\end{array}
\]
   \[
\frac{2}{5}
\]

   d) \[
\begin{array}{c}
\text{\framebox{1}} \\
\text{\framebox{4}} \\
\end{array}
\]
   \[
\frac{1}{4}
\]

3. I think it is. You can cut up the top triangle and turn the two pieces to fit on the right one. The top and bottom are the same and the left and right are the same, so they're all the same. That means all 4 parts are equal.

4. a) e.g., \[
\begin{array}{c}
\text{\framebox{2}} \\
\text{\framebox{3}} \\
\text{\framebox{4}} \\
\text{\framebox{5}} \\
\end{array}
\]
   \[
\text{\framebox{6}}
\]

   b) e.g., \[
\begin{array}{c}
\text{\framebox{7}} \\
\text{\framebox{8}} \\
\text{\framebox{9}} \\
\text{\framebox{10}} \\
\end{array}
\]

   c) e.g., \[
\begin{array}{c}
\text{\framebox{1}} \\
\text{\framebox{2}} \\
\text{\framebox{3}} \\
\text{\framebox{4}} \\
\end{array}
\]

   d) e.g., \[
\begin{array}{c}
\text{\framebox{5}} \\
\text{\framebox{6}} \\
\text{\framebox{7}} \\
\text{\framebox{8}} \\
\end{array}
\]

   e) e.g., \[
\begin{array}{c}
\text{\framebox{9}} \\
\text{\framebox{10}} \\
\text{\framebox{11}} \\
\text{\framebox{12}} \\
\end{array}
\]

5. a) e.g., $\frac{2}{6}$

   b) e.g., $\frac{2}{5}$ and $\frac{3}{5}$

   c) e.g., $\frac{2}{10}$

   d) e.g., $\frac{3}{4}$
Fractions as Parts of Measures

Learning Goal

• representing fractions 1 or less using a volume, length or capacity as a whole.

Open Question

Questions to Ask Before Using the Open Question

◊ Fill a straight sided container \(\frac{3}{4}\) way up. What fraction tells how full this is? Why? (\(\frac{3}{4}\) since it’s just as empty as it is full.)
◊ What does the numerator of 1 tell? What does the denominator of 2 tell? (The 1 tells you that there is one full section and the 2 tells you that there are 2 equal sections.)
◊ Make a linking cube tower that is 2 red and 3 green cubes. Ask: What fraction is green? How do you know? (\(\frac{3}{5}\) since it’s 3 out of 5.)
◊ Put together 2 yellow Cuisenaire® rods and 1 green one. Ask: Is \(\frac{2}{3}\) of the length yellow? Explain. (No—there are 3 parts and 2 are yellow, but the parts aren’t equal.)

Using the Open Question

By viewing or listening to student responses, note if they:

• understand the roles of the numerator and denominator
• realize that all of the given fractions are less than half of the whole.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe student models.

Consider how they decided the fractions were alike or different.

Possibilities relate to the fact that all of the fractions are not as much as half of the whole, that all involve only even or only odd numerators and denominators, or that all involve much smaller numerators than denominators.

If necessary, ask questions such as:

◊ How did you know how many cubes to use to make your tower? (I used the denominator to tell me.)
◊ How did you make sure that you only showed \(\frac{3}{5}\)? (I only made 2 sections the colour I had chosen.)
◊ How could you have used your tower of \(\frac{2}{3}\) to fill a container \(\frac{2}{5}\) of the way? (e.g., If I had a container the same height as the tower, I would only fill it to the height of 2 cubes.)

Materials

• rulers
• a measuring cup and filler material
• coloured pencils
• linking cubes
• Cuisenaire® rods
Fractions are alike: all look like not much of the whole

Fractions are different: e.g., each whole has a different number of equal parts

Relationship to the numerator and denominator: The numerator is always a lot less than the denominator; the denominator is always more than double the numerator; the numerators are all very small
Questions to Ask Before Assigning the Think Sheet

◊ Have students watch you fill a container $\frac{3}{5}$ way up. What fraction tells how full this is? Why? ($\frac{3}{5}$ since it’s just as empty as it is full.)
◊ What does the numerator of 1 tell? What does the denominator of 2 tell? (The 1 tells you that there is one full section and the 2 tells you that there are 2 equal sections.)
◊ Make a linking cube tower that is 2 red and 3 green cubes. Ask: What fraction is green? How do you know? ($\frac{3}{5}$ since it’s 3 out of 5.)
◊ Put together 2 yellow Cuisenaire® rods and 1 green one. Ask: Is $\frac{2}{5}$ of the length yellow? Explain. (No—there are 3 parts and 2 are yellow, but the parts aren’t equal.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they:
• understand the terms numerator and denominator and what they represent.
• notice that the tower parts or the line parts are equal in size

Assign the tasks. Encourage students to use concrete manipulatives as well as drawings (e.g., Question 4).

By viewing or listening to student responses, note if they:
• relate the numerator and denominator of the fraction to the parts of the model
• can identify proper fractions as parts of measures
• can model proper fractions as parts of measures
• recognize that whenever a fraction is shown, its complement (the rest of the whole) is also shown
• relate the denominator to the total number of equal sections in the set and the numerator to the number under consideration
• recognize that the parts of the measure need to be identical

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ How did you use the picture to decide on the numerator? (I looked for shaded or coloured parts.) How did you use the picture to decide on the denominator? (I looked for the total number of parts.)
◊ Why do you always see $\frac{2}{5}$ whenever you see $\frac{3}{5}$? (The $\frac{2}{5}$ is the part that is not coloured or filled when the $\frac{3}{5}$ is coloured or filled.)
◊ Suppose a tower had 3 red pieces and 2 blue ones. Why might it not be $\frac{3}{5}$ red? (If the pieces are not the same size.)
◊ How is showing $\frac{3}{5}$ as a part of a cube tower like showing it as part of a rectangle? (Both times there are 5 equal parts and 3 of them are shaded or coloured.)

Material
• rulers
• linking cubes
• measuring cup and filler material
• coloured pencils
• Cuisenaire® rods
Solutions

1. a) \(\frac{2}{3}\)  
   b) \(\frac{7}{8}\)  
   c) \(\frac{1}{10}\)  
   d) \(\frac{6}{6}\)  
   e) \(\frac{3}{8}\)  
   f) \(\frac{8}{9}\)

2. a) ![](image1)  
   b) ![](image2)  
   c) ![](image3)  
   d) ![](image4)

3. a) The blue section is too long, so it isn’t 2 out of 3 equal parts.  
   b) The pieces aren’t all the same size—the bottom red one is too big  
   c) The container is wider at the top so less than \(\frac{1}{2}\) of it is full

4. a) e.g., ![](image5)  
   b) e.g., ![](image6)  
   c) e.g., ![](image7)  
   d) e.g., ![](image8)  
   e) e.g., ![](image9)

5. Fractions are alike: They all have 1 part missing and the rest full.  
Fractions are different: There are different numbers of parts in the whole.
**Proper Fractions as Parts of a Set**

**Learning Goal**

- representing fractions 1 or less using a set of objects of a given size as a whole.

**Open Question**

**Questions to Ask Before Using the Open Question**

◊ Here are 2 counters. What would $\frac{1}{2}$ of them be? (1 of the counters)
  Why? ($\frac{1}{2}$ means that you share them fairly.)

◊ What does the numerator of 1 tell? What does the denominator of 2 tell?
  (The 1 tells you only want 1 and the 2 tells there are 2 altogether.)

◊ Here are some counters and linking cubes. What are some different ways to show $\frac{3}{5}$?
  (e.g., 2 counters and 1 linking cube or 2 linking cubes and 1 counter)

◊ How would $\frac{2}{3}$ of a set of counters and $\frac{2}{4}$ of a set of counters look different and how would they look the same?
  (Both times you would only get 2 counters but one time there would be 4 to pick from and the other time there would be 8.)

**Using the Open Question**

Provide counters or linking cubes for students to use as their “cookies.”

By viewing or listening to student responses, note if they:

- understand the roles of the numerator and denominator
- realize that all of the given fractions are three items short of the whole or whether they focus on their relative sizes

Depending on student responses, use your professional judgement to guide specific follow-up.

**Consolidating and Reflecting on the Open Question**

Observe the cookie pictures the students make.

Consider how they decided the fractions were alike or different.

Possibilities relate to the fact that there are 3 parts of the whole missing each time, the relative size of the fractions compared to $\frac{1}{2}$, the fact that numerators and denominators are opposite in terms of being even or odd or the fact that the denominator is always 3 more than the numerator.

If necessary, ask questions such as:

◊ How did you know you needed more counters to show $\frac{7}{10}$ than $\frac{3}{6}$? (You need 10 counters if there are 10 cookies in a package but only 6 if there are only 6 cookies in a package.)

◊ What is another fraction that would be like all of these? (e.g., $\frac{5}{8}$ since the numerator is 3 less than the denominator.)

◊ What is another fraction not like these? (e.g., $\frac{5}{8}$ since you wouldn’t have any cookies.)

**Materials**

- counters
- linking cubes
Fractions are alike: 3 items in a set are different each time

Fractions are different: each whole has a different number of cookies in it

Relationship to the numerator and denominator: The numerator is always 3 less than the denominator; there is always one even and one odd; the numerator and denominator add up to different values
Think Sheet

Questions to Ask Before Assigning the Think Sheet

◊ Here are 2 counters. What would \( \frac{1}{2} \) of them be? (1 of the counters)
   Why? (\( \frac{1}{2} \) means that you share them fairly.)
◊ What does the numerator of 1 tell? What does the denominator of 2 tell?
   (The 1 tells you only want 1 and the 2 tells there are 2 parts altogether.)
◊ What are some different ways to show \( \frac{3}{2} \) using counters and linking cubes?
   (e.g., 2 counters and 1 linking cube or 2 linking cubes and 1 counter)
◊ How would \( \frac{2}{3} \) of a set of counters and \( \frac{3}{4} \) of a set of counters look different
   and how would they look the same? (Both times you would only get 2 counters but
   one time there would be 4 to pick from and the other time there would be 8.)

Using the Think Sheet

Read through the introductory box with the students.

Make sure they:
• understand the terms numerator and denominator and what they represent.
• notice that the items in the whole set need not be identical
• notice that sometimes they might focus on colour to describe the fraction but not
   always

Assign the tasks.

By viewing or listening to student responses, note if they:
• can identify proper fractions as parts of sets
• can model fractions as parts of sets
• relate the denominator to the total number in the set and the numerator to the
   number under consideration
• recognize that the set items need not be identical

Depending on student responses, use your professional judgement to guide further
follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ If there are 6 things in the set, what do you know about all the fractions describing
   parts of the set? (Denominator is always 6.)
◊ If there are 2 hearts in a set and some other shapes, what do you know about the
   fractions describing the hearts? (Numerator is always 2.)
◊ In Question 1a, why would you use different fractions to tell about the hearts than to
   tell what fraction of the set is red? (There are 2 out of 4 hearts but 3 out of 4 red.)
◊ What’s special about the fraction in 1f? (The numerator and denominator are
   equal; it’s 1 whole.)
◊ Why doesn’t it matter in 1e what size the shapes are if you want to know the
   fraction of hearts? (They are hearts no matter what size they are.)
◊ Why can you always see a \( \frac{3}{2} \) if you can see a \( \frac{1}{2} \)? (You must be able to see the
   whole to know it’s \( \frac{1}{2} \) and then the other 3 parts have to make \( \frac{3}{2} \).)
◊ If you were making a set to show a fraction, what would the denominator tell you
   and what would the numerator tell you? (The denominator tells how many things in
   the set and the numerator tells how many I want to use.)
◊ Why are there lots of answers to 5c? (You could make the whole set have any
   amount more than 3 shapes, so the denominator could keep changing.)

Materials
• coloured counters
• linking cubes
Solutions

1. a) \(\frac{2}{4}\)  
b) \(\frac{2}{4}\)  
c) \(\frac{4}{8}\)  
d) \(\frac{4}{6}\)  
e) \(\frac{7}{8}\)  
f) \(\frac{6}{6}\)

2. a) 5; e.g.,  
   b) 10; e.g.,  
   c) 8; e.g.,  
   d) 5; e.g.,

3. They both show \(\frac{2}{5}\). In the first one the \(\frac{2}{5}\) tells about the blue squares and in the second one the \(\frac{2}{5}\) also tells about the blue shapes—I know the shapes don’t all have to look the same.

4. a) e.g.,  
   b) e.g.,  
   c) e.g.,  
   d) e.g.,  
   e) e.g.,

5. a) e.g., \(\frac{1}{6}\)  
   b) e.g., \(\frac{2}{6}\)  
   c) e.g., \(\frac{2}{10}\)  
   d) e.g., \(\frac{4}{9}\)
Improper Fractions as Parts of Wholes

Learning Goal

• representing fractions greater than 1 using an area as a whole.

Open Question

Questions to Ask Before Using the Open Question

◊ What does \( \frac{2}{3} \) mean? (The whole is divided into 3 equal parts, and 2 are the number of parts we’re talking about.)

◊ Would this be \( \frac{2}{3} \)?

◊ Why not? (The parts are not equal.)

◊ What does the numerator of 2 tell? What does the denominator of 3 tell? (The 2 tells how many parts we’re using and the 3 tells how many equal parts the whole is divided into.)

◊ What would \( \frac{2}{3} \) mean? Why? (The entire whole, since there are 3 equal pieces and all are being used.)

◊ What do you think \( \frac{3}{3} \) might mean? Why? (Eight pieces that are the size of \( \frac{1}{3} \) of a whole.)

◊ Would you still call it a fraction? (I guess so since it has a numerator and denominator, but I’m not sure.)

At this point, intervene to suggest that the answer is reasonable, and that, in fact, it is called a fraction.

Using the Open Question

Provide a number of blank circles pre-divided into thirds, fourths, fifths, and eighths as well as some blank circles so students can divide them as they wish.

By viewing or listening to student responses, note if they:
• can make sense of improper fractions and whether, without prompting, they think of them in terms of mixed numbers or not
• realize that all of the given fractions are more than 2 wholes, but less than 3 wholes
• notice other things about the fractions

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe the pictures of fractions the students created.

Consider how they decided the fractions were alike or different.

Possibilities relate to the size of the numbers, the fact that all the fractions are improper or that the numerator is greater than the denominator, whether the denominators and numerators are even or odd, etc.

If necessary, ask questions such as:

◊ How did you know you needed to draw more than 2 wholes? (\( \frac{6}{3} \) would be two wholes and I had \( \frac{3}{3} \))

◊ What is another fraction that would be like all of these? (e.g., \( \frac{10}{4} \) since it’s also more than 2 wholes but less than 3 wholes)

◊ What is another fraction that would be different from all of these? (e.g., \( \frac{2}{3} \) since it’s not even one whole)

◊ I noticed that the shaded pieces in the partially shaded circle are touching. Do they have to be? (no)
Solutions

$\frac{8}{3}$  

$\frac{10}{4}$  

$\frac{14}{5}$  

$\frac{22}{8}$

Fractions are alike: They can all be represented with 2 whole circles and part of a third circle; they are all fractions, they all have even numerators, they all have numerators greater than the denominators, they all have only 1 or 2 sections left unshaded.

Fractions are different: Some of the denominators are odd and some are even; some of the numerators are less than 10 and some are more than 10, some leave 1 section unshaded but some leave two sections unshaded.
Questions to Ask Before Assigning the Think Sheet

◊ Why does this show $\frac{5}{6}$? (If the whole is divided into 6 equal pieces, then $\frac{5}{6}$ is all of the whole.)
◊ What do you think $\frac{7}{9}$ might look like?

Show a pattern block hexagon (yellow), a rhombus (blue), and a triangle (green). Ask what fraction of the hexagon the triangle is. ($\frac{1}{6}$)

Then ask what fraction they’d use if they made a shape with 9 triangles ($\frac{9}{6}$).

Now ask what fraction of the rhombus a triangle is. ($\frac{1}{3}$)

Ask what fraction they’d use now if they made a shape with 9 triangles ($\frac{9}{3}$).

Discuss how you need to know the whole to know what fraction to use to represent a model.

Using the Think Sheet

Read through the introductory box with the students. Make sure they understand how the whole is identified and how the fractions are named. In particular, make sure they know why $\frac{1}{3}$ was broken into $\frac{1}{3}$ and $\frac{1}{3}$ into $\frac{1}{3}$ and $\frac{1}{3}$. (to make as many wholes as possible first)

Assign the tasks. Provide grid paper and paper circles.

By viewing or listening to student responses, note if they:
• can model improper fractions with pre-divided wholes
• can recognize improper fractions when the whole is identified
• relate improper fractions to mixed numbers (not using an algorithm)
• recognize the role of the whole in giving a fraction a name
• understand the role of the numerator and denominator of a fraction

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why did the wholes have to be labeled in Question 1? (Without the label, there is no way to know how many equal pieces make up the whole.)
◊ Suppose an improper fraction had a denominator of 6. If the mixed number were 2 and a bit, what might the numerator of the improper fraction have been? (e.g., 13 or 14)
◊ Is it possible for one person to think that a picture shows $\frac{3}{2}$ and another $\frac{3}{4}$ and both answers make sense? (If there were 2 circles cut in half and 3 parts were coloured; it’s $\frac{3}{2}$ if one circle is a whole, but $\frac{3}{4}$ if both circles together is what the whole is.)
◊ Suppose a numerator is 2 more than the denominator. What might be the whole number parts of the mixed numbers? (Usually it will be 1, e.g., $\frac{3}{2} = 1\frac{1}{2}$ or $\frac{11}{8} = 1\frac{3}{8}$, but if it’s $\frac{1}{3}$, the whole number is 3 and if it’s $\frac{4}{3}$, the whole number is 2.)
◊ Suppose a numerator is 4 times a denominator. How do you know that the mixed number is actually a whole number? (It is always 4.)
Solutions

1. a) 
   \[\boxed{\text{1 whole}}\]

   b) 
   \[\boxed{\text{1 whole}}\]

   c) 
   \[\boxed{\text{1 whole}}\]

   d) 
   \[\boxed{\text{1 whole}}\]

2. b) \(1\frac{2}{5}\)
   c) \(2\frac{3}{5}\)
   d) \(2\frac{1}{3}\)
   e) \(3\)
   f) \(1\frac{3}{4}\)

3. a) \(3\)
   b) \(3\)
   c) \(2\)
   d) \(5\)
   e) \(3\)
   f) \(3\)

4. It depends on whether the top rectangle or the full picture is the whole. You can’t decide until someone tells you what the whole is.

5. a) e.g., \(\frac{22}{5} = 4\frac{2}{5}\)
   b) e.g., \(\frac{7}{3} = 2\frac{1}{3}\)
   c) e.g., \(\frac{8}{2} = 4\)
   d) e.g., \(\frac{14}{10} = 1\frac{4}{10}\)
   e) e.g., \(\frac{6}{4} = 1\frac{1}{4}\)
Improper Fractions as Parts of Sets

Learning Goal

• representing fractions greater than 1 using a set of objects of a given size as a whole.

Open Question

Questions to Ask Before Using the Open Question

◊ Show 2 red and 1 blue counters equal in size. Why does this show \( \frac{2}{3} \)? (There are 3 things altogether and 2 are red.)

◊ Show 1 red and 1 blue counter and 1 red linking cube, all separated. Does this still show \( \frac{2}{3} \)? (If the student says no, indicate that it does.) Why? (There are still 3 things and 2 are red. It doesn’t matter what the things look like.)

◊ What does the numerator of 2 tell? What does the denominator of 3 tell? (The 2 tells how many parts we’re using and the 3 tells how many things there are altogether.)

◊ What would \( \frac{5}{4} \) mean? (the whole group)

◊ Put out 4 counters in a small plastic bag and another counter sitting by itself. Why might you call this \( \frac{5}{4} \)? (e.g., There are 5 counters but it only takes 4 to fill one whole bag.)

◊ Would you still call it a fraction? (I guess so since it has a numerator and denominator, but I’m not sure.) [At this point, intervene to suggest that the answer is reasonable and that, in fact, it is called a fraction.]

Using the Open Question

Provide counters and/or linking cubes of several colors students can use.

By viewing or listening to student responses, note if they:

• make sense of improper fractions and whether, without prompting, they think of them in terms of mixed numbers or not
• realize that all of the given fractions are of the form \( \frac{3}{4} \) if written as mixed numbers
• notice other things about the fractions.

Depending on student responses, use your professional judgement to guide specific follow-up.

Consolidating and Reflecting on the Open Question

Observe the pictures of fractions the students created.

Consider how they decided the fractions were alike or different.

Possibilities relate to the size of the numbers, the fact that all the fractions are improper or that the numerator is greater than the denominator, whether the denominators and numerators are even or odd, etc.

If necessary, ask questions such as:

◊ How did you know you needed to draw more than 3 whole boxes? (e.g., \( \frac{11}{3} \) would be three wholes and I had \( \frac{2}{3} \).)

◊ What is another fraction that would be like all of these? (e.g., \( \frac{40}{12} \) since it’s also 3 wholes and 4 parts of another one.)

◊ What is another fraction that would be different from all of these? (e.g., \( \frac{16}{9} \) since it’s only 2 wholes and another 4 parts of a whole.)

◊ Why does it make sense to call these fractions even though they are more than a whole? (e.g., because they are written like fractions and because they are just counting parts of wholes.)

Materials

• red and blue counters
• linking cubes
• counters and clear plastic bags
Fractions are alike: e.g., There were always 3 boxes and 4 extra cookies. The whole number part is always 3 and the fraction part always has a numerator of 4. Either both numbers were even or both were odd. They were all improper fractions.

Fractions are different: e.g., The numbers of cookies in a whole box were different. The denominators were different. Some numerators and denominators were more than 10 and some were less.
Think Sheet

Questions to Ask Before Assigning the Think Sheet

Show 2 red and 1 blue counter; the counters are equal in size.
◊ Why does this show \( \frac{2}{3} \)? (There are 3 things altogether and 2 are red.)

Show 1 red and 1 blue counter and 1 red linking cube, all separated.
◊ Is this still \( \frac{2}{3} \)? (If the student says no, indicate that it does.) Why? (There are still 3 things and 2 are red. It doesn’t matter what the things look like.)
◊ What does the numerator of 2 tell? What does the denominator of 3 tell? (The 2 tells how many parts we’re using and the 3 tells how many things there are altogether.)
◊ What would \( \frac{3}{3} \) mean? (the whole group)
◊ Put 4 counters in a small plastic bag and another counter sitting alone. Why might you call this \( \frac{4}{3} \)? (e.g., There are 5 counters but it only takes 4 to fill one whole bag.)
◊ Would you still call it a fraction? (I guess so since it has a numerator and denominator, but I’m not sure.)

Suggest that the answer is reasonable, and that, in fact, it is called a fraction.

Using the Think Sheet

Read through the introductory box with the students. Make sure they understand how the denominator tells the number in one whole set (or the number of counters they would put in one bag) and the numerator tells the total number of items. In particular, make sure they know why \( \frac{9}{6} \) was broken into \( \frac{3}{2} \) and \( \frac{22}{10} \) into \( \frac{10}{5} \) and \( \frac{10}{5} \) (to make as many wholes as possible first).

Assign the tasks. Provide plastic bags and several colours of counters or linking cubes students can use to help them.

By viewing or listening to student responses, note if they:
- recognize that the denominator tells how many items are in one whole set and the numerator the number of items altogether
- relate the improper fraction to the number of whole sets and part of a set left over
- recognize that identification of a fraction is not possible without first identifying the whole

Depending on student responses, use your professional judgement to guide further follow-up.

Consolidating and Reflecting: Questions to Ask After Using the Think Sheet

◊ Why did you need to know the number of objects in a set to answer Question 1? (Without knowing that, I wouldn’t know how many wholes there are.)
◊ How did you know there would be more than one whole? (The denominator is less than the numerator.)
◊ How could you predict that the fraction \( \frac{22}{3} \) would be more than 4 wholes? (e.g., \( \frac{10}{3} \) is 2 wholes, so there would be \( \frac{10}{3} \) and \( \frac{10}{3} \) and more.)
◊ How does skip counting help you figure out how much \( \frac{30}{2} \) is? (I could say \( \frac{1}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4} \) up to \( \frac{30}{2} \) and that would be 30 numbers, but I group them in 4s since it takes 4 numbers to make another whole. That shows it is about 7 and some more.)
Solutions

1. a)

b)

c)

d)

2. b) $\frac{3}{3}$
   c) $\frac{4}{2}$
   d) $\frac{5}{2}$
   e) $3$
   f) $1 \frac{9}{10}$

3. a) 2, 4
   b) 3, 5
   c) 4, 4
   d) 10, 2
   e) 10, 1
   f) 3, 8

4. It depends on whether the full set is 8 hearts or 4 hearts.

5. a) $\frac{7}{2}$
   b) e.g., $\frac{20}{8}; \frac{32}{8}$
   c) e.g., $\frac{10}{3}; \frac{31}{3}$
   d) e.g., $\frac{20}{8}; \frac{24}{8}$