TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Continuum and Connections

Integers

Overview

Context Connections

- Positions integers in a larger context and shows connections to everyday situations, careers, and tasks.
- Identifies relevant manipulatives, technology, and web-based resources for addressing the mathematical theme.

Connections Across the Grades

- Outlines the scope and sequence using Grade 6, Grade 7, Grade 8, Grade 9 Applied and Academic, and Grade 10 Applied as organizers.
- Includes relevant specific expectations for each grade.
- Summarizes prior and future learning.

Instruction Connections

- Suggests instructional strategies, with examples, for each of Grade 7, Grade 8, Grade 9 Applied, and Grade 10 Applied.
- Includes advice on how to help students develop understanding.

Connections Across Strands

- Provides a sampling of connections that can be made across strands, using the theme (integers) as an organizer.

Developing Proficiency

- Provides questions related to specific expectations for a specific grade/course.
- Focuses on specific knowledge, understandings, and skills, as well as on the mathematical processes of Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, and Connecting. *Communicating is part of each question*.
- Presents short-answer questions that are procedural in nature, or identifies the question as problem solving, involving other mathematical processes, as indicated.
- Serves as a model for developing other questions relevant to the learning expected for the grade/course.

Problem Solving Across the Grades

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Models a variety of representations and strategies that students may use to solve the problem and that teachers should validate.
- Focuses on problem-solving strategies, involving multiple mathematical processes.
- Provides an opportunity for students to engage with the problem at many levels.
- Provides problems appropriate for students in Grades 7–10. The solutions illustrate that the strategies and knowledge students use may change as they mature and learn more content.

Is This Always True?

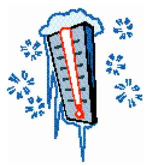
- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Focuses on the mathematical processes Reasoning and Proving, and Reflecting.

Integers

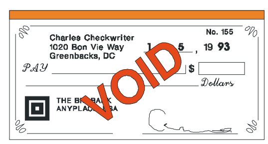
Context

- The natural numbers {1, 2, 3…} were used for counting possessions. The set of integers became necessary for counting both possessions and losses. The plus and minus signs that are used for addition and subtraction are the same signs that are used to indicate positive and negative numbers in the set of integers {…–3, –2, –1, 0, +1, +2, +3…}. The dual roles of these signs can be confusing to students.
- Each integer has two attributes: size and sign. Understanding of integers develops when students represent integers with a variety of models. Models have representations of both size and sign. When using coloured tiles, sign is represented by colour, and size is represented by number of tiles. When using number lines, sign is represented by direction, and size is represented by length (or distance). After the representations in different models are understood, contextual problems provide opportunities for students to develop conceptual understanding of operations with integers. When using coloured tiles, addition is modelled by "putting tiles together" and subtraction by "taking tiles away." The resultant colour and number of tiles represent the sign and size of the answer. Students demonstrate their depth of understanding as they use more than one model to explain their thinking about a solution to a problem.
- Procedural proficiency in operations with integers will be beneficial to students as they move to the more abstract concepts of algebra.

Context Connections

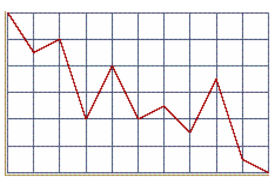

Temperature


Golf Scores

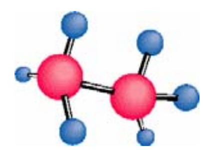


Cheque Books

Money


Graphs


Negative (Opposite)


Molecule Charge


Stock Prices


Above/Below Sea Level


Other Connections

Manipulatives

- two-coloured tiles
- number lines
- thermometers

Technology

- virtual tiles
- The Geometer's Sketchpad®4
- calculators

Other Resources

http://mathfun.com/integersworksheet.html
http://mathforum.org/dr.math/faq/faq.negxneg.html
http://www.mathgoodies.com/lessons/toc_vol5.shtm

Connections Across Grades

Selected results of word search using the *Ontario Curriculum Unit Planner*

Search Words: *negative, integer, rational*

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10 Applied
Note: The expectations do not explicitly state *negative*, *integer*, or *rational*.	▪ identify and compare integers found in real-life contexts; ▪ represent and order integers, using a variety of tools; ▪ add and subtract integers, using a variety of tools.	▪ use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions, to help judge the reasonableness of a solution; ▪ represent the multiplication and division of integers, using a variety of tools; ▪ solve problems involving operations with integers, using a variety of tools; ▪ evaluate expressions that involve integers, including expressions that contain brackets and exponents, using order of operations; ▪ evaluate algebraic expressions with up to three terms, by substituting fractions, decimals and integers for the variables; ▪ solve and verify linear equations involving a one variable term and having solutions that are integers, by inspection, guess and check, and a "balance" model.	**Academic and Applied** ▪ simplify numerical expressions involving integers and rational numbers, with and without the use of technology.	**Note:** The expectations do not explicitly state *negative*, *integer*, or *rational*.

Summary of Prior Learning

In earlier years, students:
- develop proficiency in and solve problems involving whole numbers, decimals, percents, and ratios;
- use estimation to determine appropriateness of answer.

In Grade 7, students:
- identify, describe, and interpret everyday applications that involve integers, e.g., in primary and secondary data;
- order and compare integers using two-colour counters, manipulatives, and number lines;
- add or subtract two integers using concrete materials, e.g., coloured tiles, then (after doing the concrete manipulation) represent the operation using symbols;
- graph coordinate points in all four quadrants.

In Grade 8, students:
- continue to identify, describe, and interpret everyday applications that involve integers, e.g., in primary and secondary data;
- develop the rules for the multiplication and division of two integers;
- develop proficiency in multiplying and dividing integers, with and without manipulatives;
- develop proficiency in using order of operations with brackets and exponents that involve integers;
- become proficient in adding, subtracting, multiplying, and dividing integers without the use of manipulatives, i.e., using symbols;
- evaluate algebraic expressions and solve and verify simple linear equations.

In Grade 9 Applied, students:
- apply integers to simplify polynomial expressions and solve first-degree equations, in algebraic manipulation, e.g., measurement and formulas, and in solving problems;
- interpret the meaning of points in four quadrants in context;
- apply integers in calculating first differences to determine if relationship is linear, and in calculating the rate of change $\left(\frac{rise}{run}\right)$.

In Grade 10 Applied, students:
- apply integers in manipulating and solving first-degree equations and in solving systems of two linear equations;
- apply integers in determining the equation of a line, in calculating slope of a line or line segment, in determining the equation of a line, in rewriting equations of the form $y = mx + b$ and $Ax + By + C = 0$, and in describing the meaning of slope and y-intercept in the equation of a line;
- apply integers in manipulating quadratic expression, e.g., second degree polynomials, common factoring binomials and trinomials of one variable up to degree two, and factoring simple trinomials;
- apply integers in calculating the second differences to determine if a relationship is quadratic, and in identifying the key features of the graph of a parabola.

In later years

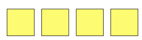
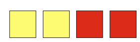
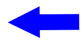
Students' choice of courses will determine the degree to which they apply their understanding of concepts related to integers.

Instruction Connections

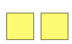
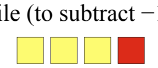
Suggested Instructional Strategies

Grade 7

- Use everyday applications to develop the concept of negative numbers, e.g., thermometers, distance above and below sea level, elevators where 0 represents the ground level.
- Use a variety of models (symbols, two-coloured tiles, directed distances, etc.) to represent integers and ensure that students understand how to represent a positive quantity, a negative quantity, and zero in each model.

	+4	−2	0
Coloured Tiles			
Directed Distances			
Kinesthetic Movement	4 steps to the right	2 steps to the left	2 steps to the right followed by 2 steps to the left

- Take integers from contexts, e.g., yards gained/lost in football, then compare, order, and represent the integers using different models.
- Present all possible combinations of addition problems, beginning with the addition of two positive integers, and ask students to find the answer using different models.
- Present all possible combinations of subtraction problems, beginning with the subtraction of familiar questions like (+5) – (+2). Progress to questions such as: (+2) – (–1) = 3. This question requires the introduction of the zero model.

(+2) − (−1) = 3 Start with 2 yellow tiles. 	Add a zero model, then remove one red tile (to subtract −1) 

Ask students to find the answer using different models.

- To add integers, students move up and down the thermometer, starting at 0, e.g., (+4) + (–6) means the temperature goes up 4, then down 6, ending at –2. This works equally well for 4 – 6, e.g., up 4, down 6.
- Allow use of concrete materials for addition and subtraction of integers.
- Use manipulatives to develop the understanding that expressions like (−3) − (+2) and (−3) + (−2) are different representations that have the same value.
- Use patterning to develop rules for addition and subtraction of integers.

Grade 8

- Help students become proficient in subtracting integers without manipulatives by building on students' Grade 7 proficiency (in adding integers symbolically).
- Teach students to read and write shorthand notation once the concepts have been consolidated and represented using longhand notation, e.g., the shorthand notation for (+5) + (−8), which can be read as "positive 5 plus negative 8," is 5 − 8.

Helping to Develop Understanding

- Progress from concrete to visual to symbolic representations of integers.
- Make notes that incorporate pictorial (visual) representations of the two-coloured tiles.
- Help students learn to read and interpret expressions (see examples in Grade 8 strategies).
- Use Venn diagrams to illustrate the relationship between integers and other sets of numbers.
- Provide many opportunities for students to demonstrate mastery of integer skills without calculators.
- Ensure that students know how to use the +/– key on calculators.
- Avoid the expression "two negatives make a positive" – this causes confusion since the operation is not stated.
- Make practice fun through a variety of integer games and interactive Internet activities.
- Connect prior use of manipulatives and visual representations of integers to manipulatives and visual representations of algebraic terms in Grade 9.
- Connect prior knowledge of fractions and integers to develop understanding of rationals.
- Avoid the expression "to subtract integers, add the opposite," as it does not support conceptual understanding.

Suggested Instructional Strategies	Helping to Develop Understanding
• Use manipulatives and pictorial representations to introduce multiplication of a positive integer with a positive integer, and a positive integer with a negative integer. Use the commutative property of multiplication, i.e., $a \times b = b \times a$, to establish that a negative integer multiplied by a positive integer is always a negative integer. • Use patterning to explore the multiplication (and division) of a negative integer with a negative integer. • Students should be able to explain why each of the following has the value 12: $-3\ (-4)$, $(-3)(-4)$, $-3 \times (-4)$, $(-3) \times (-4)$. Students should be able to contrast these questions to $-\ 3 - 4$, which has the value of -7. • After students learn multiplication of integers, all adjacent signs can be handled with multiplication, e.g., $-\ 3 - (-2)$ means $-\ 3 + 2$, then students can move up and down a "mental" thermometer to calculate the result. • Extend understanding of order of operations to examples that involve integers. • Lead students to recognize and write divisions both horizontally and vertically, e.g., $(-14) \div 7$ and $\frac{-14}{7}$. • Lead students to recognize different ways of representing the same negative rational number, e.g., $\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}$. **Grade 9 Applied** • If some students need further practice on integer operations to be proficient with polynomials, exponents, and equations, provide materials for practice. • Review integer operations briefly using algebra tiles before combining algebraic terms. • Include exponents in order of operation questions. Emphasize that the base of a power is negative only if the negative number is contained in brackets, e.g., $(-3)^2 = (-3) \times (-3) = 9$ whereas $-3^2 = -\ (3 \times 3) = -9$, i.e., the negative of 3^2. **Grade 10 Applied** • Review integer operations briefly using algebra tiles to consolidate simplifying algebraic expressions. • Apply integer operations to determine finite differences and classify relationships as linear or non-linear. • Apply knowledge of integers when multiplying polynomials.	

Connections Across Strands

Note

Summary or synthesis of curriculum expectations is in plain font.

Verbatim curriculum expectations are in italics.

Grade 7

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• See Connections Across the Grades, p. 3.	N/A	• *plot points using all four quadrants of the Cartesian coordinate plane* • *create and analyse designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes, using a variety of tools* (rotation of 90° can be associated with a rotation of positive/negative 90°)	• recognize patterns and use them to make predictions • establish or verify strategies for operating with integers	• primary and secondary data may involve the need to read, interpret, and draw conclusions from charts, tables, and graphs that involve integers • *determine, through investigation, the effect on a measure of central tendency of adding or removing a value or values* • *identify and describe trends, based on the distribution of the data presented in tables and graphs, using informal language*

Grade 8

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• *represent, compare, and order rational numbers* • See Connections Across the Grades, p. 3.	N/A	• *graph the image of a point, or set of points, on the Cartesian coordinate plane after applying a transformation to the original point(s) (i.e., translation; reflection in the x-axis, the y-axis, or the angle bisector of the axes that passes through the first and third quadrants; rotation of 90°, 180°, or 270° about the origin)*	• *evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables* • *solve and verify linear equations involving a one-variable term and having solutions that are integers, by using inspection, guess and check, and a "balance" model*	• primary and secondary data (categorical, discrete, or continuous) may involve the need to read, interpret, and draw conclusions from charts, tables, and graphs that involve integers • *identify and describe trends, based on the rate of change of data from tables and graphs, using informal language*

Grade 9 Academic

Number Sense and Algebra	Measurement and Geometry	Analytic Geometry	Linear Relations
• *determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one variable* • *add and subtract polynomials with up to two variables, using a variety of tools* • *expand and simplify polynomial expressions involving one variable, using a variety of tools* • *solve first-degree equations, including equations with fractional coefficients, using a variety of tools and strategies* • algebraic manipulation (including formulas)	N/A	• *express the equation of a line in the form* $y = mx + b$*, given the form* $ax + by + c = 0$ • determine the slope of a line or line segment using the various formulas of slope • determine the equation of a line given slope and y-intercept, slope and a point, or two points on the line • explain the geometric significance of m and b in the equation $y = mx + b$ • *describe the meaning of the slope and y-intercept for a linear relation arising from a realistic situation* • *determine graphically the point of intersection of two linear relations, and interpret the intersection point in the context of an application*	• *interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant* • determine if a relationship is linear by calculating first differences • *determine other representations of a linear relation, given one representation* • *describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied*

Grade 9 Applied

Number Sense and Algebra	Measurement and Geometry	Linear Relations
• *add and subtract polynomials involving the same variable up to degree three, using a variety of tools* • *solve first-degree equations with non-fractional coefficients, using a variety of tools and strategies* • manipulate algebraic expressions (including formulas) • *multiply a polynomial by a monomial involving the same variable to give results up to degree three*	N/A	• *interpret the meanings of points on scatter plots or graphs that represent linear relations, including scatter plots or graphs in more than one quadrant* • describe trends, relationships, and make inferences from linear and non-linear data • determine if a relationship is linear by calculating first differences • determine rate of change of a linear relation using any two points on the line of a relation • *solve problems that can be modelled with first-degree relations, and compare the algebraic method to other solution methods* • *describe the effects on a linear graph and make the corresponding changes to the linear equation when the conditions of the situation they represent are varied*

Grade 10 Applied

Measurement and Trigonometry	Modelling Linear Relations	Quadratic Relations of the Form $y = ax^2 + bx + c$
N/A	• *solve first-degree equations involving one variable, including equations with fractional coefficients* • *determine the value of a variable in the first degree, using a formula* • *express the equation of a line in the form $y = mx + b$, given the form $ax + by + c = 0$* • explain the geometric significance of m and b in the equation $y = mx + b$ • determine properties of the slopes of lines and line segments (e.g., negative rate of change) • *determine the equation of a line, given its graph, the slope and y-intercept, the slope and a point on the line, or two points on the line* • *solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination*	• *expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials, using a variety of tools* • *factor binomials (e.g., $4x^2 + 8x$) and trinomials (e.g., $3x^2 + 9x – 15$) involving one variable up to degree two, by determining a common factor using a variety of tools* • *factor simple trinomials of the form $x^2 + bx + c$, using a variety of tools* • determine if a relationship is quadratic by calculating second differences • identify the key features of a graph of a parabola

Developing Proficiency

Grade 7

Name:
Date:

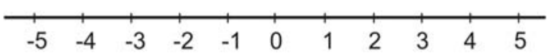
Expectation – Number Sense and Numeration, 7m26:
Add and subtract integers, using a variety of tools (e.g., two-colour counters, virtual manipulatives, number lines).

Knowledge and Understanding
(Facts and Procedures)

Place the following on the number line, using the letters provided:

A –2 **C** –5 + 5 = _____

B 5 **D** –10 + 6 =_____



Knowledge and Understanding
(Conceptual Understanding)

You are given several counters.

 represents –1

 represents +1

Make a diagram to show a representation for the following expression: (–5) + (+3)

Diagram:

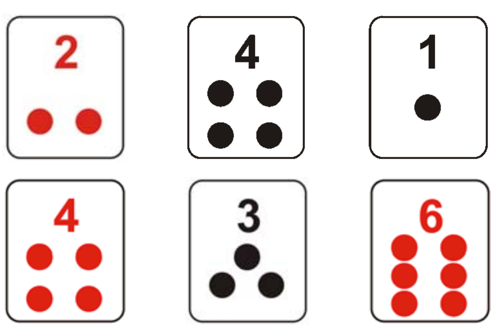
Answer:

Problem Solving
(Reasoning and Proving)

Cards are made using the digits from 1 to 9.

Red represents a negative number.	**Black** represents a positive number.
 represents –2	 represents +3

Which cards can be taken out of the following collection to give a total value of +2?



Give reasons for your answer.

Problem Solving
(Connecting)

Bob recently opened a bank account. The bank allows him an overdraft of $5, i.e., he is allowed to withdraw $5 more than what he has in the bank.

Bob decided to use integers to keep track of the money. A positive integer represents how much he deposits into his account and a negative integer represents the amount he withdraws.

Bob recorded the following integers to represent his recent transactions.

(+20) , (+6) , (– 4) , (+2) , (–25)

Write a story to describe Bob's transactions. What is his account balance after the five transactions? What does the balance represent?

Developing Proficiency

Grade 8

Name:
Date:

Expectation – Number Sense and Numeration, 8m23:
Evaluate expressions that involve integers, including expressions that contain brackets and exponents, using order of operations.

Knowledge and Understanding
(Facts and Procedures)

Evaluate the following.

a) $\frac{6}{-2}$

b) $-8 \times (4)$

c) $10 - (-2)(+3)$

Knowledge and Understanding
(Conceptual Understanding)

You are given several counters.

 represents –1

 represents +1

Make a diagram to show a representation for the following expression: 3(–2)

Diagram:

Answer:

Problem Solving
(Connecting)

Bob recently opened a bank account.

He decided to use integers to keep track of the money. A positive integer represents how much he deposits into his account and a negative integer represents the amount he withdraws.

Bob wrote the following expression to represent a series of transactions.

$20 + 4(-2) + 3(+4)$

Write a story to describe the expression that Bob recorded. What is his account balance after these transactions?

Problem Solving
(Representing, Reflecting)

Cards are made using the digits from 1 to 9.

Red represents a negative number.	**Black** represents a positive number.
	
represents –2	represents +3

Group	Number of Cards	Card	Value of Card(s)
A	1	**Black 7**	
B	2	**Black 4**	
C	2	**Red 8**	
D	3	**Red 3**	

Remove either Group A, B, C, or D so that the average of all the remaining cards is +1.

Give reasons for your answer.

Developing Proficiency

Grade 9 Applied

Name:
Date:

Expectation – Number Sense and Algebra, NA2.04:
Substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases.
** The knowledge and skills described in this expectation are to be introduced as needed and applied and consolidated throughout the course.

Knowledge and Understanding
(Facts and Procedures)

Evaluate the following expressions for $x = 3$ and $y = –2$

a) $–4x^2y^3$

b) $x^3(2x – 3y)$

Knowledge and Understanding
(Conceptual Understanding)

For $x = –3$, determine each of the following. Explain your thinking.

x^2 (x^2) $–x^2$

Problem Solving
(Reasoning and Proving, Connecting)

A movie theatre wants to compare the volumes of popcorn in two containers, a cube with edge length 8.1 cm and a cylinder with radius 4.5 cm and height 8 cm. Determine which container holds more popcorn.

Explain your thinking.

Problem Solving
(Selecting Tools, and Computational Strategies, Connecting)

A scientist checks a petri dish that has 120 bacteria in it. If the number of bacteria doubles every half hour, how many bacteria will there be in $1\frac{1}{2}$ hours, and in 4 hours?

The petri dish can only hold 40 000 bacteria, how long will it take before it is full?

Show your work and explain your reasoning.

Developing Proficiency

Grade 10 Applied

Name:
Date:

Expectation – Manipulating Quadratic Expressions, QR1.01:
Expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials or the square of a binomial using a variety of tools and strategies.

Knowledge and Understanding
(Facts and Procedures)

a) Simplify and collect terms like terms:
$(x^2 + 3x – 5) – (2x^2 + 7x – 6)$

b) Square the binomial:
$(2a – 3)^2$

Knowledge and Understanding
(Conceptual Understanding)

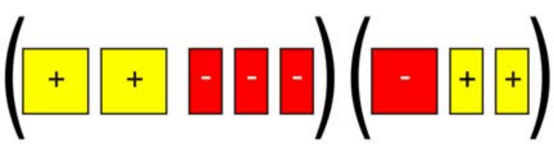
When multiplying these binomials, explain why there is no *x* term.

$(7x + 2)\ (7x – 2)$

Problem Solving
(Representing, Reasoning and Proving)

Express the following algebra tile representation using symbols.

Calculate the answer and verify using tiles.



Problem Solving
(Connecting)

Compare and contrast these three quadratic expressions:

a) $(x – 4)(x + 6)$

b) $x^2 + 2x – 24$

c) $(x –1)^2 – 25$

Problem Solving Across the Grades   Sample 1

Name:
Date:

Your math teacher has asked you to help a classmate determine when the sum of two integers is a negative integer.

Explore the possible types of combinations for adding two integers.

Take into account the size and the sign of each integer.

Summarize your conclusions in a format that will be easy for your classmates to understand.

Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have available a variety of tools from which students can choose to assist them with their thinking and communication.

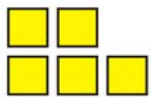
Problem-Solving Strategies:
- Use manipulative representation
- Draw a diagram
- Act it out

1. Students most comfortable with concrete materials might use integer tiles, counters, or linking blocks.
(Yellow represents positive and red represents negative.)

Add two positive integers

Example: 2 + 3

2 + 3 = 5

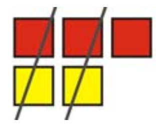


All of the tiles will always be yellow so the sum will always be positive. This is not what we want since we are checking for negative answers.

Add a negative to a positive integer

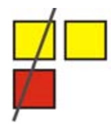
Example: 2 + (–3)

2 + (–3) = –1



Example: 2 + (–1)

2 + (–1) = 1

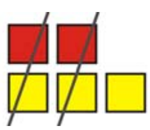


If the size of the negative integer is greater than that of the positive, the sum will be negative. This is what we want.

Add a positive to a negative integer

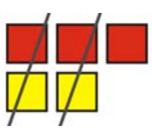
Example: –3 + 2

–3 + 2 = –1



Example: –2 + 3

–2 + 3 = 1

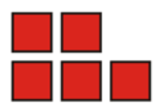


If the size of the positive integer is less than that of the negative, the sum will be negative. This is what we want.

Add two negative integers

Example: –2 + (–3)

–2 + (–3) = – 5



All of the tiles will always be red (negative) so the sum will always be negative. This is what we want.

When adding two integers, the sum will be negative if the integer with the larger size is negative or if both integers are negative.

2.

Problem-Solving Strategies:
- Use patterning
- Solve an easier problem

$5 + 2 = 7$		$5 + (-2) = 3$	
$4 + 2 = 6$		$4 + (-2) = 2$	
$3 + 2 = 5$		$3 + (-2) = 1$	
$2 + 2 = 4$		$2 + (-2) = 0$	
$1 + 2 = 3$		$1 + (-2) = -1$	
$0 + 2 = 2$		$0 + (-2) = -2$	Pattern implies results of –1, –2, –3, –4…
$-1 + 2 = 1$		$-1 + (-2) = -3$	
$-2 + 2 = 0$		$-2 + (-2) = -4$	
$-3 + 2 =$	Pattern implies results of –1, –2, –3	$-3 + (-2) =$	
$-4 + 2 =$		$-4 + (-2) =$	
$-5 + 2 =$		$-5 + (-2) =$	

Extending the pattern from what we know to what we don't know, gives negative results when the size of the negative number is greater that the size of the positive number being added.

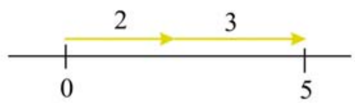
Extending the pattern from what we know to what we don't know, gives a negative result when the size of the negative number being added is greater than the original positive number, or when both numbers are negative.

3a. Students comfortable with drawings might choose to use integer line arrows.

Problem-Solving Strategies:
- Draw a diagram

Add Two Positives

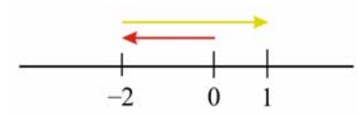
2 + 3



2 + 3 = 5
Since the arrows start at zero and can only move to the right, the sum will always be positive.
This is not what we want.

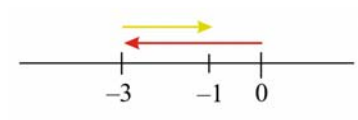
Add a Negative and a Positive

–2 + 3



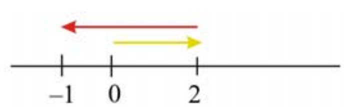
–2 + 3 = 1
The positive arrow goes past the negative arrow, so the sum is positive.
This is not what we want.

–3 + 2



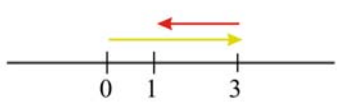
–3 + 2 = –1
The positive arrow is not large enough to take the sum into the positive part of the line. This is what we want.

2 + (–3)



2 + (–3) = –1
The negative arrow goes past the positive arrow, so the sum is negative.
This is what we want.

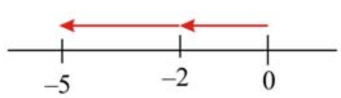
3 + (–2)



3 + (–2) = 1
The negative arrow is not large enough to take the sum into the negative part of the line. This is not what we want.

Add Two Negatives

–2 + (–3)



–2 + (–3) = –5
Since the arrows start at zero and can only move to the left, the sum will always be negative.
This is what we want.

Summary

Add	+	–
+	always +	could be – or +
–	could be + or –	always –

b. Some kinesthetic learners might choose to walk a number line on the floor. Walking to the right indicates a positive integer and walking to the left represents a negative integer. The diagrams above show how students would walk.

Problem-Solving Strategies:
- Act it out

Students' solutions could include any of the Grades 7 and 8 answers.

1a. Some students may use the Samples folder in The Geometer's Sketchpad®4. Select → Teaching Mathematics → Add Integers.

Problem-Solving Strategies:
- Use technology (GSP®4, scientific calculator)

b. Students could experiment with a scientific calculator. They could track their investigation on a chart:

1st integer	2nd integer	Result
–2	–5	–
3	21	+
1	–8	–
–1	8	+

Note: Different brands of calculators have different conventions and keys for entering negative numbers.

Some students could quote memorized rules and apply them without understanding. This is not recommended.

positive + **positive** = **positive** and add the sizes

positive + **negative** = sign of the integer with larger size and subtract the sizes

negative + **negative** = **negative** and add the sizes

When I apply the rules, I find that the sum of two integers is a negative when I add two negative integers or add a positive integer and negative integer where the size of the negative integer is larger.

Name:
Date:

Janet says that two negatives make a positive.

Investigate her claim with respect to the four basic operations with integers, and summarize your conclusions.

Adding Integers	Subtracting Integers
Multiplying Integers	**Dividing Integers**

Although the teacher may expect a student to apply specific mathematical knowledge in a problem-solving context, the student may find some unexpected way to solve the problem.

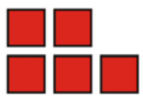
Have available a variety of tools from which students can choose to assist them with their thinking and communication.

Problem-Solving Strategies:
- Use concrete material (integer tiles)
- Draw a diagram (number line)
- Apply a rule
- Use technology (GSP®4, scientific calculator)

1. Students may not have made the connection that the sign rules for multiplication and division of integers do not apply to addition and subtraction. Students most comfortable with concrete materials might choose to use integer tiles.
(Yellow represents positive and red represents negative.)

Adding Integers

Example:
–2 + (–3)



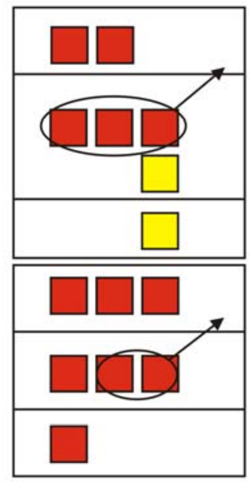
–2 + (–3) = –5

All of the tiles will always be red so the sum will always be negative.

Therefore, Janet's statement is inaccurate for adding integers.

Subtracting Integers

Example:
–2 – (–3)
–2 – (–3) = 1



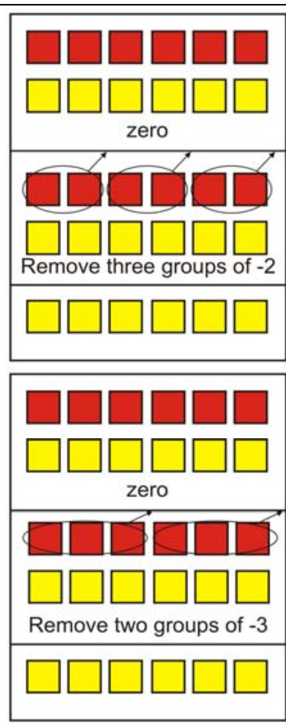
This example supports Janet's statement. Example:
–3 – (–2)
–3 – (–2) = –1

This example refutes Janet's statement.

Janet's statement is not always true for subtracting integers.

Multiplying Integers

Example:
–3 × (–2) = 6
Note: You must begin with tiles, representing zero, in order to be able to remove groups of –2.



This example supports Janet's statement.

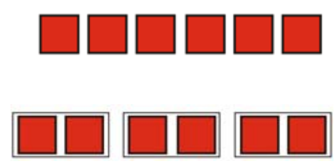
Example:
(–2) × (–3)

This example supports Janet's statement.

Janet's statement is true for multiplying integers.

Dividing Integers

Example:
(–6) ÷ (–2)



Separate into groups of –2

There are three groups.

(–6) ÷ (–2) = 3

This example supports Janet's statement.

Note: The negative divisor represents the number of negative tiles in each group. If the divisor were positive it would represent the number of groups.

The statement "A negative and a negative make a positive" is only consistently true for multiplication and division of integers. It is never true for addition and is only sometimes true for subtraction.

2. Students comfortable with drawing might choose to use integer line arrows.

Problem-Solving Strategies:
- Draw a diagram

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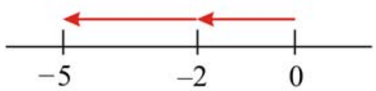
Adding Integers

Example: –2 + (–3)



–2 + (–3) = – 5

Since both arrows are left-pointing, the sum will always end left of zero, indicating a negative sum. Janet’s statement is not accurate for adding integers.

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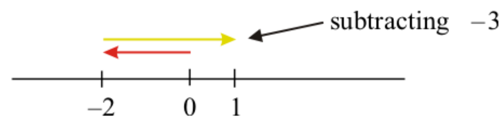
Subtracting Integers

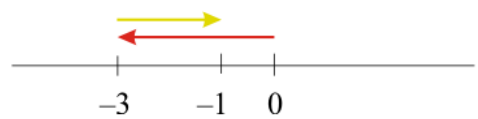
Subtraction is the opposite of addition, so we can model subtraction by pointing the subtracting integer’s arrow in the opposite direction.

Example: –2 – (–3)



–2 – (–3) = 1

This example supports Janet’s statement.
Example: –3 – (–2)



–3 – (–2) = –1

This example refutes Janet’s statement.

Janet’s statement is not always accurate for subtracting integers.

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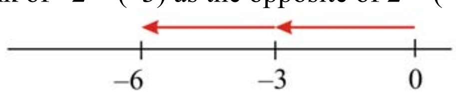
Multiplying Integers

Example: –2 × (–3)
–2 is the opposite of 2.

Think of –2 × (–3) as the opposite of 2 × (–3).



2 × (–3) = –6
The opposite of –6 is 6.
Therefore, –2 × (–3) = 6.

Since the arrows are both left-pointing, the product will always be negative. Therefore, the opposite will always be positive.

This example supports Janet’s statement.

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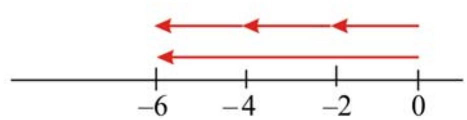
Dividing Integers

Example: –6 ÷ (–2)

Find the number of groups of –2 there are in –6.



Three –2 arrows are equivalent to one –6 arrow.
–6 ÷ (–2) = 3

This example supports Janet’s statement.

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Janet’s statement is only accurate for multiplication and division of integers. It is not accurate for addition and is only accurate for subtraction when the subtracting integer has a larger size.

</td>
</tr>
</table>

3. Some students may quote memorized rules and apply them without understanding. This is not recommended.

Problem-Solving Strategies:
- Apply a rule or algorithm

Addition:
negative + negative = negative and add the sizes (counter-example)

Subtraction: "add the opposite"
negative – negative
= negative + positive
= sign of the integer with larger size and subtract the sizes (inconclusive)

Multiplication:
The product of two integers with the same sign is a positive integer. (supports)

Division:
The quotient of two integers with the same sign is a positive integer. (supports)

4a. Students may use the Samples folder in The Geometer's Sketchpad®4. Select → Teaching Mathematics → Add Integers.

Problem-Solving Strategies:
- Using technology (GSP®4 and scientific calculators)

b. Students could experiment with a scientific calculator. Their investigation and summary would look similar to the other solutions.

Note: Different brands of calculators have different conventions/keys for entering negative numbers.

Problem-Solving Strategies:
- Using technology (GSP®4 and scientific calculators)

Is This Always True?
Grade 7

Name:
Date:

1. Patti says that if you add the same negative integer five times, the sign of the sum is negative and its size is five times the size of the original integer.

   For example:

   $-7 + (-7) + (-7) + (-7) + (-7)$
   $= -(5 \times 7)$ [7 is the size of –7]
   $= -35$

   Is Patti's statement true for all negative integers?

2. Dina says that if you add a negative integer to a positive integer, the sum is zero or less.

   Is Dina's statement true for all integers?

3. Suman says that if you subtract any two negative integers, the difference is always negative.

   Is Suman's statement true for all integers?

4. Lisa says that if you add two opposite integers, the sum is zero.

   Is Lisa's statement true for all integers?

5. Christopher says that if the mean (average) of a data set of integers is a negative integer, then there must be more negative integers than positive integers in the data set.

   Is Christopher's statement always true?

1. **Yes**. Adding several negative integers takes the sum farther from zero on the negative side of the number line, causing its sign to be negative. And as we know from multiplication of whole numbers, adding a number five times is equivalent to multiplying that number by 5. Using integer tiles, 5 × (–7) is represented by five groups of negative 7 and results in 35 negative tiles, which represent the product –35.

Problem-Solving Strategies:
- Use logic
- Use visualization

2. **No**. Counter-example: 8 + (–5) = 3.

Problem-Solving Strategies:
- Use a counter-example

3. **No**. Counter-example: (–2) – (–5) = 3.

Problem-Solving Strategies:
- Use a counter-example

4. **Yes**. Opposite integers are the same distance from zero on the number line but on opposite sides. When added, they cancel each other out, leaving the sum at zero. Using integer tiles, adding opposite integers is modelled by placing together the same number of positive tiles as negative tiles. This results in a model of 0.

Problem-Solving Strategies:
- Use logic
- Use visualization
- Use concrete materials

5. **No**. Counter-example: Choose 4 integers, e.g., –14, 1, 2, 3.

The mean is $\frac{-14+1+2+3}{4} = -2$ and there are more positive integers than negative integers.

Problem-Solving Strategies:
- Use a counter-example

Is This Always True? Grade 8

Name:
Date:

1. Bert says that to subtract one integer from another, simply add to the first number the opposite of the one you are subtracting.

   For example:
   9 – 5
   = 9 + (–5)
   = 4

   Is Bert's statement true for all integers?

2. Emmanuelle says that if you divide two negative integers, the quotient is a positive integer.

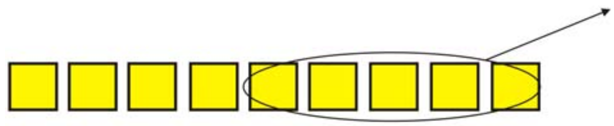
   Is Emmanuelle's statement true for all negative integers?

3. Sandra says that any negative rational number, say negative one-half, can be represented in 3 equal ways: $-\frac{1}{2}$ $\frac{-1}{2}$ $\frac{1}{-2}$

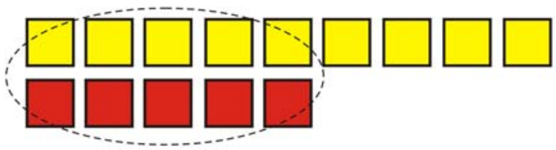
   Is Sandra's statement always true?

1. **Yes**. Addition is the opposite operation to subtraction (represented on the number line by motion in the opposite direction). To compensate for motion in the opposite direction, the opposite integer must be added to get to the same spot on the number line that would have been reached if subtraction had been used. Using integer tiles, interpreting 9 – 5 as positive 9 subtract positive 5 looks like:

Problem-Solving Strategies:
- Use concrete materials
- Use logical reasoning



and leaves an answer of positive 4. Interpreting 9 – 5 as positive 9 add negative 5 looks like:



and also results in an answer of positive 4, using the zero principle for the five pairs of positive/negative tiles. Illustrate as many examples using both positive and negative integers as are needed to convince students that to subtract an integer, you can, instead, add the opposite integer. Since addition is easier to do mentally, it is recommended that all strings of integers be interpreted as addition questions, e.g., 3 – 8 + 1 can be interpreted as positive 3 add negative 8, add positive 1.

2. **Yes**. There are several ways to reason this:
   a) A negative integer divided by a negative integer is the opposite of a positive integer divided by a negative integer. Since a positive integer divided by a negative integer is a negative integer, its opposite is a positive integer.

Problem-Solving Strategies:
- Use logical reasoning
- Use visualization of a number line
- Use principles of multiplication and division

   b) Using an example to look at it another way, you could think about –10 ÷ (–2). You would think "How many groups of –2 are there in –10?" Picturing the integer line, you would see five left-pointing 2-unit arrows lined up starting at zero and ending at –10. So the quotient has to be positive 5 (the five arrows). This holds true for all negative integers divided by a negative integer.

   c) A multiplication statement that corresponds to (–10) ÷ (–2) is (*n*) × (–2) = –10. Since we know that we need a positive to multiply by the negative to yield an answer of negative 10, *n* must be positive 5.

3. **Yes**. Express the 3 forms of the rational number as a decimal.
   For example:

Problem-Solving Strategies:
- Use the transitive property

$$-\frac{1}{2} = -\left(\frac{1}{2}\right) = -(0.5) = -0.5$$

$$\frac{-1}{2} = -1 \div 2 = -0.5$$

$$\frac{1}{-2} = 1 \div -2 = -0.5$$

Since all three forms equal –0.5, therefore $-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

Is This Always True?

Grade 9 Applied

Name:
Date:

1. Wen-Hao says that if the finite differences for a relation are all the same, that difference is the rate of change of the line.

   Is Wen-Hao's statement always true?

2. Nicholas says that the square of any negative integer is the same as taking the opposite of any square integer.

   Is Nicholas' statement always true?

3. Nimish says that when evaluating expressions with rational number bases and natural-number exponents that are even (e.g., $\left(\frac{2}{3}\right)^2$, $\left(-\frac{2}{3}\right)^4$), the result can never be negative.

   Is Nimish's statement always true?

4. Anoush says that the opposite of the terms in any polynomial is the same as multiplying each term of the polynomial by –1.

   Is Anoush's statement always true?

1. **No**, not necessarily.

Problem-Solving Strategies:
- Use a counter-example

x	*y* = 2*x* + 3	Finite differences
3	9	
6	15	6
9	21	6
12	27	6

Slope for $y = 2x + 3$ is 2.

If the increment in *x* values is 1, however, then a constant first difference would be the same as the rate of change.

2. **No**. Counter-example: $(–3)^2 = 9$ but $– (3^2) = –9$.

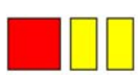
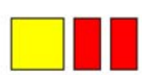
Problem-Solving Strategies:
- Use a counter-example

3. **Yes**. If the exponent is a natural number and is even, then the repeated multiplication performed to the rational number base (regardless if negative or positive) will occur an even number of times. The result will always be positive since two positive integers that are the same multiplied together is a positive integer, and two negative integers that are the same multiplied together is also a positive integer.

Problem-Solving Strategies:
- Use logical reasoning

4. **Yes**. Taking the opposite of each term in a polynomial is the same as multiplying each term by –1.

Problem-Solving Strategies:
- Use the distributive property

The opposite of  is 

In general, the opposite of a polynomial can be represented by algebra tiles of the same size but opposite colour.

In general, when you multiply each term by –1 you are not changing the size of the term but only the sign. This is the same as the opposite polynomial.

Is This Always True?

Grade 10 Applied

Name:
Date:

1. Portia says that if the second differences of a quadratic relation are negative, then the parabola has a minimum value.

   Is Portia's statement always true?

2. Vikas claims that when given an equation of a line, if the coefficient in front of x is negative, the slope is negative.

   Is Vikas' statement always true?

3. Lee states that when you square any binomial, i.e., of the form $(ax + b)^2$, the x^2 term and the constant term are always positive.

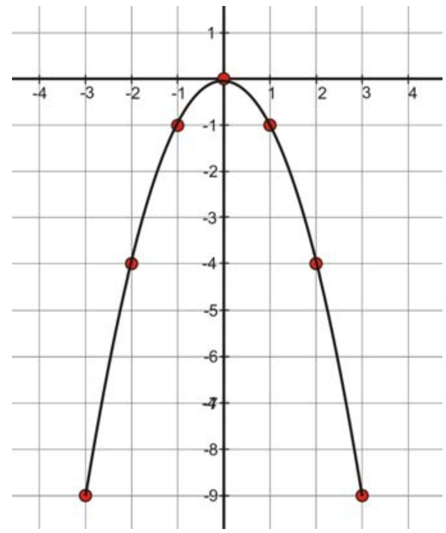
   Is Lee's statement always true?

4. To determine the slope of a line using the ratio $\frac{rise}{run}$, Dimetros says it does not matter which of two points is identified as "start point" and which point is identified as "end point."

   Is Dimetros' statement always true?

1. **No**. Counter-example: $y = -x^2$.

x	y	First Differences	Second Differences
–3	–9		
		5	
–2	–4		–2
		3	
–1	–1		–2
		1	
0	0		–2
		–1	
1	–1		–2
		–3	
2	–4		–2
		–5	
3	–9		



Problem-Solving Strategies:
- Use a counter-example

2. **No**. There are several ways of expressing an equation of a line. For example:

$2y - 3x = 12$	(counter-example)
$-2y - 3x - 2 = 0$	(supports)
$-y = -3x + 2$	(counter-example)
$y = -3x + 2$	(supports)

This statement is true if the equation is written in the form $y = mx + b$.

Problem Solving Strategies:
- Use a counter-example

3. **Yes**. For any square of binomial in the form $(ax + b)^2$.

$$(ax+b)^2 = (ax+b)(ax+b)$$
$$= a^2x^2 + abx + abx + b^2$$

That is, the coefficient of the x^2 term is always a^2, so if a is negative or positive, its square will make it positive. Also, if b is negative or positive, its square will make it positive. Thus, the x^2 term and the constant term are both positive.

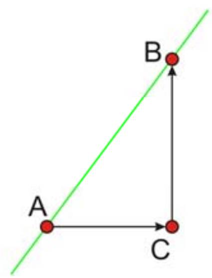
Problem Solving Strategies:
- Use algebra
- Use logical reasoning

4. **Yes**. The distances involved in $\frac{rise}{run}$ are the same, the only question is if the slope is positive or negative. Identify any two points on the line. Label points as A and B. If A is the start point and B is the end point, calculate the slope. Repeat with B as the start point and A as the end point. The slopes are equal.

Problem-Solving Strategies:
- Draw a diagram
- Use logic
- Apply the formula

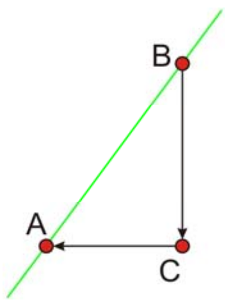
Case 1

When line rises as it moves to the right



Slope AB
A to C is +
C to B is +

Thus $\frac{rise}{run} = \frac{+}{+} = +$

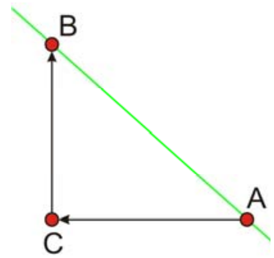


Slope BA
B to C is –
C to A is –

Thus $\frac{rise}{run} = \frac{-}{-} = +$

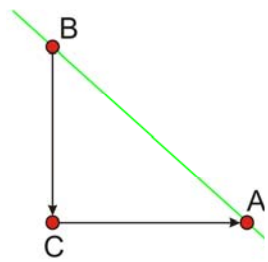
Case 2

When line descends as it moves to the right



Slope AB
A to C is –
C to B is +

Thus $\frac{rise}{run} = \frac{+}{-} = -$



Slope BA
B to C is –
C to A is +

Thus $\frac{rise}{run} = \frac{-}{+} = -$

Case 3

Horizontal line

The rise is 0, thus slope is 0, regardless.

Case 4

Vertical line

Run is 0, thus slope is undefined, regardless.